Nuclear matter and its quantum field theory

2018/22.

Nuclear droplet model – Weizsäcker ~1935

$$\frac{E(A,Z)}{A} = E_0 + a_{sym}\left(\frac{N-Z}{A}\right)^2 + \xi\, 4\pi R^2 \frac{1}{A} + \frac{3}{5}\frac{e^2 Z^2}{R}\cdot\frac{1}{A}$$

$\left(\frac{N-Z}{A}\right)^2$: deviation from zero isospin

R: nuclei radius

$\frac{3}{5}\frac{e^2 Z^2}{R}$: Coulomb repulsion

A nucleon number

Z proton number

N neutron number

$N + Z = A$

"Model" nuclei are composed of nucleons of radius r_0

$$\frac{4\pi}{3} r_0^3 \cdot A = \frac{4\pi}{3} R^3 \qquad E_0 = \frac{4\pi}{3} r_0^3 \cdot \epsilon_0$$

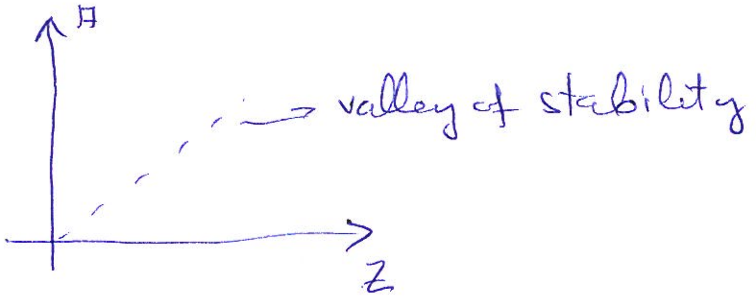
ϵ_0: energy density

$$R_0 = r_0 A^{1/3}$$

Binding energy for a nucleon $B/A = \left(E(A,Z) - Z m_p - N m_N\right)/A$

$\downarrow A \gg 1$

$-16{,}3$ MeV



Electron scattering on nuclei (Hofstädter, 1956)

R is measured as a function of A

$\Rightarrow r_0 \approx 1{,}16$ fm

Density of the "nuclear matter" $\boxed{\rho_0 = \frac{1}{\frac{4\pi}{3} r_0^3} = 0{,}153\,(\mathrm{fm})^{-3}}$

$$\frac{E_0}{A} \Rightarrow \frac{\epsilon_0}{\rho_0} = m_N + \frac{B}{A} \longrightarrow \boxed{\epsilon_0 = 141 \frac{\mathrm{MeV}}{(\mathrm{fm})^3}}$$

nat. units

"szaturációs energia ~~[illegible]~~ sűrűség"

Free Fermi-gas "model" 2018/23.

Assume isospin invariance — single Fermi-sphere filled up-to k_N^F

$$\rho_0 = \frac{g_*}{2\pi^2}\int_0^{k_N^F} k^2 dk = \frac{g_*}{6\pi^2}\left(k_N^F\right)^3 \qquad g_* = (2S+1)(2I_3+1) = 4$$

$$= \frac{2}{3\pi^2} k_N^{F\,3} \rightarrow \boxed{k_N^F = 1{,}31\ \text{fm}^{-1}}$$

The effective mass of nucleons in nuclear matter

$$\epsilon_0 = \frac{g_*}{2\pi^2}\int_0^{k_N^F} k^2 \sqrt{k^2+m_{eff}^2}\, dk \rightarrow \boxed{m_{eff} \sim (0{,}74 - 0{,}82)\, m_N}$$

~~Compressibility~~ — can be determined from fitting
Compression modulus density oscillations - deformable matter

$$K = 9\left[\rho^2\left(\frac{\epsilon(\rho)}{\rho}\right)''\right]_{\substack{\rho=\rho_0\\ \epsilon=\epsilon_0}} \approx 200\text{-}300\,\text{MeV}$$

Also informations are gathered from resonant excitation of oscillatory modes of nuclei - treatment within the droplet model

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Simplest field theoretic model: Johnson and Teller, 1955

Nucleons ψ_N bispinor field

Scalar meson field σ — responsible for the attraction

~~Scalar~~
Vector meson field ω_μ — responsible of very short range repulsion

$$L_N = \bar{\psi}_N\left[i\gamma_\mu\left(\partial^\mu + i g_\omega \omega^\mu\right) - \left(m - g_\sigma \sigma\right)\right]\psi_N$$

$$L_B = \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu$$ 2018/24.

$\omega_{\mu\nu} \to \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$

Field-equations

$$(\Box + m_\sigma^2)\sigma(x) = g_\sigma \bar{\psi}_N \psi_N \qquad (\Box + m_\omega^2)\omega_\mu(x) - \partial_\mu(\partial^\nu \omega_\nu(x)) = g_\omega \bar{\psi}_N \gamma_\mu \psi_N$$

Conservation of nucleon current $\partial^\mu \bar{\psi}\gamma_\mu \psi = 0$

Equation for $\partial^\mu \omega_\mu$

$$(\Box + m_\omega^2)\partial^\mu \omega_\mu - \Box(\partial^\nu \omega_\nu) = 0 \Longrightarrow m_\omega^2 \partial^\mu \omega_\mu = 0$$

$$\downarrow$$

$$\partial^\mu \omega_\mu = 0$$

Solution of Dirac-equation in a mean-field

$\sigma_{mean} = \Sigma \qquad \omega_{\mu\, mean} = \Omega_\mu$ space-independent homogeneous

Filling Fermi-sphere with the modes determined

$$\langle \bar{\psi}_N \Gamma_{Dirac} \psi_N \rangle = \int_{k<k_N^F} \frac{d^3k}{(2\pi)^3} \bar{\psi}_N(\underline{k}) \Gamma_{Dirac} \psi_N(\underline{k})$$

Static equations determining the mean fields

$$m_\sigma^2 \Sigma = g_\sigma \langle \bar{\psi}_N \psi_N \rangle \qquad m_\omega^2 \Omega_\mu = g_\omega \langle \bar{\psi}_N \gamma_\nu \psi_N \rangle$$

$\psi_N(k)$ modified plane waves $= N_k e^{-ik_0 x_0 + i\underline{k}\underline{x}}$

After substitution $[\gamma_\mu k^\mu - \gamma_\mu g_\omega \Omega^\mu - (m_N - g_\sigma \Sigma)] N_k = 0$

$m_{eff} = m_N - g_\sigma \Sigma$ meaning of Σ!

$K^\mu = k^\mu - g_\omega \Omega^\mu$ Effective dispersion relation

$$K_\mu K^\mu = m_{eff}^2$$

$E_N(\underline{k})$ $\underline{k}$ hullámvektorú nukleonállapot energiája 2018/25.

$$k_0 = g_\omega \Omega_0 + \underbrace{\left[(\underline{k} - g_\omega \underline{\Omega})^2 + (m_N - g_\sigma \Sigma)^2 \right]^{1/2}}_{E(\underline{k})}$$

Finite baryon density $k_0(k_N^F) = \mu_B$ baryochemical potential

Equation of state $p(\epsilon)$ pressure (energy density)

Evaluation with help of the energy-momentum tensor

$$T^{\mu\nu} = \frac{\partial}{\partial(\partial_\nu \phi)} \partial_\mu \phi - g^{\mu\nu} L$$ ϕ stands for all fields

Mean field approximation: only fermions contribute to the first term

$$\frac{\partial L_F}{\partial(\partial_\mu \psi)} = \bar{\Psi} i \gamma_\mu \qquad T_F^{\mu\nu} = \bar{\Psi}_N i \gamma^\mu \partial^\nu \psi - g^{\mu\nu} L_F$$

Mode contribution to the energy density T^{00}

$$\bar{N}_k \gamma^0 k^0 N_k - g^{00} \bar{N}_k (\gamma^0 k^0 - \underline{\gamma}\,\underline{k} - g_\omega \gamma^\mu \Omega_\mu - m_{eff}) N_k$$

$$= \bar{N}_k (\underline{\gamma}\,\underline{k} + g_\omega \gamma_\mu \Omega^\mu + m_{eff}) N_k$$

Mode contribution to the pressure

$$-3p = T^i{}_i \qquad - \bar{N}_k \underline{\gamma}\,\underline{k} N_k - 3 \bar{N}_k (\gamma^0 k^0 - \underline{\gamma}\,\underline{k} - g_\omega \gamma_\mu \Omega^\mu - m_{eff}) N_k$$

An important auxiliary quantity

$$T_F^{i0} = g_* \int \frac{d^3k}{(2\pi)^3} \bar{N}_k \gamma^i k_i N_k \Big|_{\text{Fermi sphere}}$$

Evaluation of $\bar{N}_k \Gamma_{Dirac} N_k$ 2018/26.

with help of parametric derivatives of $\hat{H}_D$ Dirac-Hamiltonian

$$\hat{H}_D N_{\underline{k}} = k_0 N_{\underline{k}} \qquad \hat{H}_D = \gamma_0 (\underline{\gamma}\,\underline{k} + g_\omega \gamma_\mu \Omega^\mu + m_{eff})$$

$$= E_N(k) N_{\underline{k}}$$

$$E_N(k) = N_k^+ \hat{H}_D N_{\underline{k}}$$

$$\frac{\partial \hat{H}_D}{\partial \xi} \Rightarrow \quad \boxed{\frac{\partial N_{\underline{k}}^+ \hat{H}_D N_{\underline{k}}}{\partial \xi} = \frac{\partial \hat{H}_D}{\partial \xi}} \underbrace{N_k^+ N_k}_{1} + E_N(k) \underbrace{\frac{\partial N_k^+ N_k}{\partial \xi}}_{0}$$

$$= \frac{\partial E_N(\underline{k})}{\partial \xi} = \frac{\partial}{\partial \xi}\left(g \Omega_0 + E(\underline{k})\right)$$

For T_F^{ic} we need $N_k^+ \gamma^0 \gamma^i N_k \longrightarrow \xi \Leftrightarrow k^i$

$$\frac{\partial H_D}{\partial k^i} = \gamma^0 \gamma^i \qquad N_k^+ \gamma^0 \gamma^i N_k = \frac{k_i - g_\omega \Omega_i}{E(k)}$$

$$T_F^{i0} = 0 = g_\alpha \int \frac{d^3k}{(2\pi)^3} k_0 \frac{k_i - g_\omega \Omega_i}{E(k)} \rightarrow$$

Isotropy
Consistent solution
$\Omega_i = 0$

Full energy density (including also mean fields)

$$\epsilon_{MF} = \frac{1}{2} m_\sigma^2 \Sigma^2 - \frac{1}{2} m_\omega^2 \Omega_0^2 + 4 \int_{FS} \bar{N}_k \gamma^l k_l N_k + g_\omega \Omega_0 \int_{FS} N_k^+ N_k$$

$$+ m_{eff} \int_{FS} N_k^+ \gamma^0 N_k$$

$$= \frac{1}{2} m_\sigma^2 \Sigma^2 - \frac{1}{2} m_\omega^2 \Omega_0^2 + \frac{2}{\pi^2} \int_0^{k_N^F} k^2 dk \frac{k^2}{(k^2 + m_{eff}^2)^{1/2}} + g_\omega \Omega_0 \rho_0 +$$

$$+ \frac{2}{\pi^2} m_{eff} \int_0^{k_N^F} k^2 dk \frac{m_{eff}}{(k^2 + m_{eff}^2)^{1/2}}$$

2018/27.

$$\epsilon_{MF} = \frac{1}{2} m_\sigma^2 \Sigma - \frac{1}{2} m_\omega^2 \Omega_0^2 + g_\omega \Omega_0 \rho_0 + \frac{2}{\pi^2} \int_0^{k_N^F} k^2 dk \sqrt{k^2 + m_{eff}^2}$$

Mean field equations

$$m_\sigma^2 \Sigma = g_\sigma \int N_k^+ \gamma_0 N_k = g_\sigma m_{eff} \int k^2 dk \frac{1}{(k^2 + m_{eff}^2)^{1/2}}$$

$$= g_\sigma \frac{2}{\pi^2} \int^{k_N^F} k^2 dk \frac{m_N - g_\sigma \Sigma}{(k^2 + (m_N - g_\sigma \Sigma)^2)^{1/2}}$$

$$m_\omega^2 \Omega_0 = g_\omega \int N_k^+ N_k = g_\omega \rho_0 \leftarrow \text{substituting into } \epsilon_{MF}$$

$$\epsilon_{MF} = \frac{1}{2} m_\sigma^2 \Sigma^2 + \frac{1}{2} m_\omega^2 \Omega_0^2 + \frac{2}{\pi^2} \int_0^{k_N^F} k^2 dk \sqrt{k^2 + m_{eff}^2}$$

Pressure

$$P_{MF} = -\frac{1}{2} m_\sigma^2 \Sigma^2 + \frac{1}{2} m_\omega^2 \Omega_0^2 + \frac{2}{3\pi^2} \int_0^{k_N^F} \frac{k^4}{\sqrt{k^2 + m_{eff}^2}} dk$$

Mean field equations can be solved for $g_\sigma \Sigma$, $g_\omega \Omega_0$ as functions of $\frac{g_\sigma^2}{m_\sigma^2}, \frac{g_\omega^2}{m_\omega^2}, k_N^F$

$$k_N^F \updownarrow \rho_B$$

$$\epsilon\left(\frac{g_\sigma^2}{m_\sigma^2}, \frac{g_\omega^2}{m_\omega^2}, \rho_B\right)$$

$\downarrow$

$\epsilon(\rho_0)$ display minimum when varying $\frac{g_\sigma^2}{m_\sigma^2}, \frac{g_\omega^2}{m_\omega^2}$

$$\rho_{min,0} \equiv \rho_{sat} \leftrightarrow \rho_0$$

$\epsilon(\rho_{sat}) \equiv \epsilon_0 \longleftrightarrow B/A$ (Binding energy / nucleon)

These 2 conditions fix $\frac{g_\sigma^2}{m_\sigma^2}, \frac{g_\omega^2}{m_\omega^2}$

Predictions $K \simeq 500\,MeV$, $\frac{m_{eff}}{m_N} \sim 0{,}5$ Improved model is necessary

① higher power in the σ potential energy

2018/29.

$$\frac{1}{2}m^2\sigma^2 + \frac{1}{3}b\,m_N(g_\sigma\sigma)^3 + \frac{1}{4}c\,(g_\sigma\sigma)^4$$

② symmetry energy — isospin symmetry breakdown

ρ^μ $I=1$ vector meson

③ baryon resonances

$$L = \sum_B \bar{\Psi}_B\left[i\gamma^\mu\partial_\mu - m_B - g_{\omega B}\gamma_\mu\omega^\mu - \frac{1}{2}g_{\rho B}\gamma_\mu \underline{I}\rho^\mu\right]\psi_B$$

$$+\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{3}b\,m_N(g_\sigma\sigma)^3 - \frac{1}{4}c\,(g_\sigma\sigma)^4$$

$$-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\rho^\mu$$

Solution searched for in MF approximation

$$\sigma \neq 0,\ \omega^0 \neq 0,\ \rho_z^0 \neq 0$$

$$\downarrow$$

$$\mathbb{R}_{03}$$

$\frac{g_\sigma}{m_\sigma}$, $\frac{g_\omega}{m_\omega}$, b, c can be expressed analytically in terms of ρ, $\frac{B}{A}$, K, m^* at $\rho = \rho_0$

+ isospin splitting $\Leftrightarrow \frac{g_\rho}{m_\rho}$

Extended model for the theory of neutron stars

Baryons $B = N, \Lambda, \Sigma$ (triplet), Ξ (dublet) 2018/30.

Mesons σ, ω, ρ

Leptons e, μ

$$L = \sum_B \bar{\psi}_B \left[i\gamma_\mu \partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu \omega^\mu - \frac{1}{2} g_{\rho B}\gamma_\mu \tau^a \rho_a^\mu \right] \psi_B$$

$$+ \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2\right) - \frac{1}{3} b m_N (g_\sigma \sigma)^3 - \frac{1}{4} c (g_\sigma \sigma)^4$$

$$- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu^a \rho_a^\mu$$

$$+ \sum_{\lambda = e,\mu} \bar{\psi}_\lambda (i\gamma_\mu \partial^\mu - m_\lambda)\psi_\lambda$$

Non-zero mean fields Σ, Ω_0, R_{03}

Energy of the baryon modes $E_B(k) = g_{\omega B}\Omega_0 + g_{\rho B} R_{03} I_{3B} + (k^2 + m_B^{*2})^{1/2}$

$$m_B^* = m_B - g_B \Sigma$$

Each baryon fills its Fermi sphere $\mu_B = E_B(k_B^F)$

Leptons free Fermi gas $e_\lambda(k) = (k^2 + m_\lambda^2)^{1/2}$ $\mu_\lambda = e_\lambda(k_\lambda^F)$

Mean-field approximation to energy density

$$\epsilon = \frac{1}{2} m_\sigma^2 \Sigma^2 + \frac{1}{3} b m_N (g_\sigma \Sigma)^3 + \frac{1}{4} c (g_\sigma \Sigma)^4 + \frac{1}{2} m_\omega^2 \Omega_0^2 + \frac{1}{2} m_\rho^2 R_{03}^2$$

$$+ \sum_B \frac{2J_B + 1}{2\pi^2} \int_0^{k_B^F} (k^2 + m_B^{*2})^{1/2} k^2 dk$$

(k_B^F are not the same because electric interaction violates isospin symmetry)

$$+ \sum_\lambda \frac{1}{\pi^2} \int_0^{k_\lambda^F} (k^2 + m_\lambda^2)^{1/2} k^2 dk$$

Pressure (same approximation) 2018/31.

$$p = -\frac{1}{3} b m_N (g_\sigma \Sigma)^3 - \frac{1}{4} c_N (g_\sigma \Sigma)^4 - \frac{1}{2} m_\sigma^2 \Sigma^2$$ ← attractive interaction

$$+ \frac{1}{2} m_\omega^2 \Omega_0^2 + \frac{1}{2} m_\rho^2 R_{03}^2$$ ← repulsion

$$+ \frac{1}{3} \sum_B \frac{2J_B+1}{2\pi^2} \int_0^{k_B^F} \frac{k^4}{(k^2+m_B^{*2})^{1/2}} dk + \frac{1}{3} \sum_\lambda \frac{1}{\pi^2} \int_0^{k_\lambda^F} \frac{k_\lambda^4}{(k^2+m_\lambda^2)^{1/2}} dk$$

"Chemical" equilibrium

β-decay (weak interaction)

$n + \nu_e \leftrightarrow p + e^-$ density of neutrinos vanishes because they fly out of the neutron matter $\underline{\mu_\nu = 0}$

Equilibrium condition

$\mu_n = \mu_p + \mu_{e^-}$

Strange - antistrange production

$n + p \leftrightarrow p + \Lambda + K^0$, $K^0 \to 2\gamma$

Strangeness is not conserved, no separate chemical potential

$$\mu_{K^0} = \mu_\gamma = 0$$

$$\mu_n = \mu_\Lambda$$

$\mu_n + \mu_p = \mu_p + \mu_\Lambda$

$p + p \leftrightarrow p + \Lambda + K^+$, $K^+ \to \mu^+ + \nu_\mu$ $\quad \underline{\mu_{K^+} = \mu_{\mu^+}}$ $\quad \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ $\quad \underline{\mu_{\mu^+} = \mu_{e^+}}$

$n + n \leftrightarrow p + \Lambda + K^-$, $K^- \to \mu^- + \bar{\nu}_\mu$ $\quad \underline{\mu_{K^-} = \mu_{\mu^-}}$ $\quad \mu^- \to e^- + \bar{\nu}_e + \nu_\mu$

$\underline{\mu_{\mu^-} = \mu_{e^-}}$

$\mu_n + \mu_n = \mu_p + \mu_\Lambda + \mu_{\mu^-}$

$= \mu_p + \mu_n + \mu_{e^-}$ fullfilled in view of the above relations

General statement $\boxed{\mu_B = \mu_n - q\mu_{e^-}}$ two conserved charges

Presence of a certain particle when

$$\mu_{\mu^\pm} > (k^2 + m_{\mu^\pm}^2)^{1/2}$$

$$\mu_B > \cancel{m_B^*} + g_{\omega B}\Omega_0 + g_{\rho B} R_{03} I_{3B} + (m_B^{*2} + k^2)^{1/2}$$

(non-zero radius of its Fermi sphere)

Conserved quantities

Vanishing of the electric charge (also locally)

$$0 = \sum_\lambda \langle q_\lambda \rangle + \sum_B \langle q_B \rangle = \sum_i (2J_i + 1) q_i \frac{k_i^{F\,3}}{6\pi^2}$$

Baryon-density

$$\rho^B = \sum_B (2J_B + 1) b_B \frac{k_B^{F\,3}}{6\pi^2}$$

These two equations are used for the determination of μ_n, μ_{e^-} (ρ_B is a sort of control parameter)

Equations for the determination of the mean meson fields

$$\Omega_0 = \sum_B \frac{g_{\omega B}}{m_\omega^2} \rho_B \qquad R_{03} = \sum_B \frac{g_{\rho B}}{m_\rho^2} I_{3B} \rho_B$$

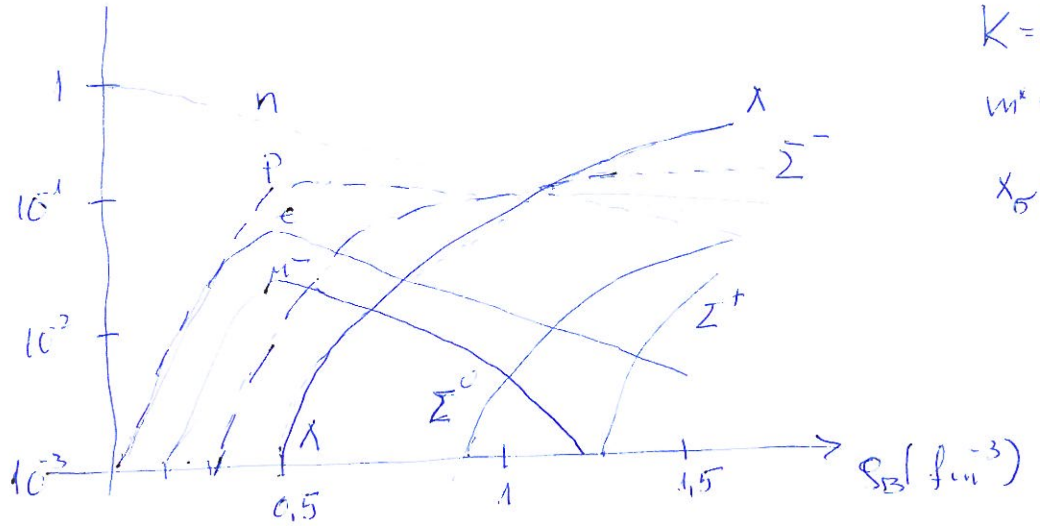
$$m_\sigma^2 \Sigma + g b m_N (g_\sigma \Sigma)^2 + c (g_\sigma \Sigma)^3 g_\sigma = \sum_B \frac{2J_B + 1}{2\pi^2} \int_0^{k_B^F} \frac{m_B^*}{\sqrt{k^2 + m_B^{*2}}} k^2 dk$$

N Dirac-equations to determine k_B^F $\qquad \mu_B = E_B(k_B^F)$

2 Dirac-equations to determine k_λ^F $\qquad \mu_\lambda = (k_\lambda^{F\,2} + m_\lambda^2)^{1/2}$

Result of the relative populations as a function of ρ_B

2018/33.



$K = 240\ MeV$

$m^* = 0.78\, m_N$

$x_\sigma = \frac{g_{\sigma H}}{g_\sigma} \cdot 0.6$