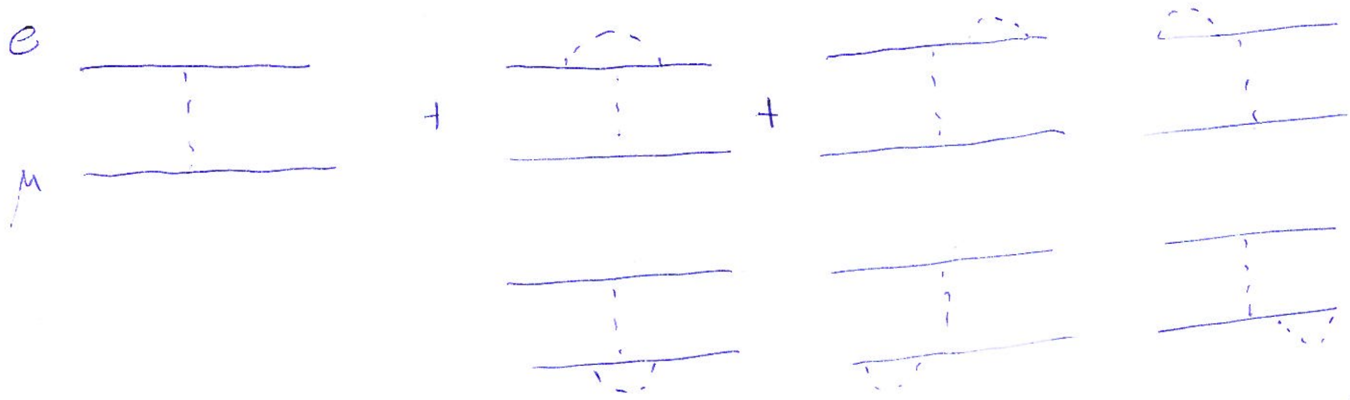
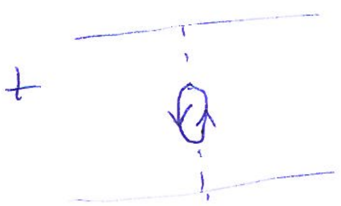
Effect of vacuum polarisation on the Coulomb-potential
(Landau, Abrikosov, Khalatnikov 1954)   2018/12.

Feynman-diagrams modifying one-photon exchange in μ-e scattering



modification of interaction vertex

self-energy corrections to the free electron propagation



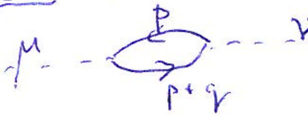
vacuum polarisation

Virtual electron-positron pair $\Delta E \cdot \Delta \tau \sim \hbar$ allows its existence for $\Delta \tau$ (charge separation under the influence of the photon field)

The last diagram contributes to the modification of the Coulomb law.

$$V_{eff} = \frac{e^2(r_0)}{r} d(r) \left(\frac{1}{4\pi\varepsilon_0}\right) \qquad d(r) = 1 + e^2(r_0) d_1(r) + e^4(r_0) d_2(r) + \dots$$

$d(r)$ → dielectric function of the vacuum

Vacuum polarisation tensor  $S_F(p)$ electron propagator

$$\Pi_{\mu\nu}(q) = -Tr_S \int \gamma_\mu S_F(p+q) \gamma_\nu S_F(p) \frac{d^4p}{(2\pi)^4} e^2(r_0)$$ 4x4 matrix

$$S_F(p+q) = (\not p + \not q - m)^{-1} \approx \frac{1}{\not p - m} + \frac{1}{\not p - m}(-\not q)\frac{1}{\not p - m} +$$ 2018/13.

$$+ (\not p - m)^{-1}(-\not q)(\not p - m)^{-1}(-\not q)(\not p - m)^{-1} + \dots$$

Mass is negligible when $|p| \gg |q|, m$ $\quad S_F(p) \approx \frac{\not p}{p^2}$

$$\Pi_{\mu\nu}(q) = \Pi_{\mu\nu}(q=0) - e^2(r_c)\int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}_S\left(\gamma_\mu \frac{\not p}{p^2}(-\not q)\frac{\not p}{p^2}\gamma_\nu \frac{\not p}{p^2}\right) \rightarrow$$ odd function of p: $\emptyset$

$$-e^2(r_c)\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^8}\mathrm{Tr}_S(\gamma_\mu \not p \not q \not p \not q \not p \gamma_\nu \not p)$$

Further terms: convergent, not depend on the upper limit of integration $\Lambda = p_{max}$

Evaluation of the Dirac-trace $\{\gamma^\mu, \gamma^\nu\}_+ = 2g^{\mu\nu}$

$$\not p \not p = p^2 \qquad \not p \gamma_\lambda \not p = -p^2\gamma_\lambda + 2p_\lambda \not p$$

$$\mathrm{Tr}_S\{\gamma_\mu \not p \gamma_\nu \not p\} = 4(-p^2 g_{\mu\nu} + 2p_\mu p_\nu)$$

$$\mathrm{Tr}_S\{\gamma_\mu \not p \not q \not p \not q \not p \gamma_\nu \not p\} = 4g_{\mu\nu}(p^2q^2 - 2(pq)^2 p^2) - 8p_\mu p_\nu p^2 q^2$$
$$- 8p^2(pq)(p_\nu q_\mu + p_\mu q_\nu)$$

The polarisation tensor (after subtracting $\Pi_{\mu\nu}(q=0)$) is transversal

$$\Pi^{(div)}_{\mu\nu} = \Pi_{\mu\nu} - \Pi_{\mu\nu}(0) = (q^2 g_{\mu\nu} - q_\mu q_\nu)\,\pi(q^2)$$ $\hookrightarrow$ polarisation function

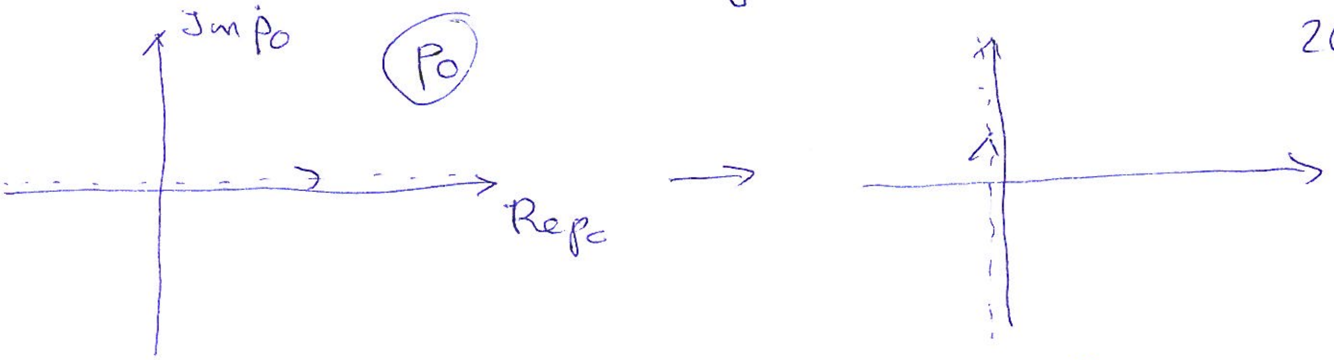
$$\Pi^{(div)\mu}_{\ \mu} = 3q^2\pi(q^2) = -8e^2(r_0)\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^6}(p^2q^2 - 2(pq)^2)$$

$\hookrightarrow$ expectation for the component parallel to q

$$= -4e^2(r_c)q^2\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^4} \longleftarrow \frac{1}{4}p^2q^2$$

Computation of the integral with Wick rotation

2018/14.



$$p^2 = p_0^2 - \vec{p}^2 \longrightarrow p_E^2 = -p_L^2$$

$$-4 e_0^2 \frac{1}{2} \int_{q^2}^{\Lambda^2} \frac{p_E^2 \, dp_E^2}{p_E^4} \underbrace{\int d\Omega_4 \frac{1}{16\pi^4}}_{\frac{1}{8\pi^2}} = -\frac{e^2(r_c)}{4\pi^2} \ln \frac{\Lambda^2}{q^2}$$

$$D^{-1}_{\mu\nu} = q^2 g_{\mu\nu} \left(1 - \Pi(q^2)\right) \leftarrow \boxed{\Pi(q^2) = -\frac{\alpha(r_c)}{3\pi} \ln \frac{\Lambda^2}{q^2}}$$

$$D_{\mu\nu} = \frac{g_{\mu\nu}}{q^2} \frac{1}{1 + \frac{\alpha(r_c)}{3\pi}\left(\ln \frac{\mu^2}{q^2} + \ln \frac{\Lambda^2}{\mu^2}\right)}$$

$$= \frac{g_{\mu\nu}}{q^2} \frac{1}{1 + \frac{\alpha(r_c)}{3\pi} \ln \frac{\Lambda^2}{\mu^2}} \frac{1}{1 + \frac{\frac{\alpha(r_0)}{3\pi} \ln \frac{\mu^2}{q^2}}{1 + \frac{\alpha(r_0)}{3\pi} \ln \frac{\Lambda^2}{\mu^2}}}$$

$$D_{\mu\nu} = \langle 0 | T(A_\mu(x) A_\nu(y)) | 0 \rangle \qquad A^{ren}_\mu = A_\mu Z_3^{-1/2} \qquad Z_3^{-1} = 1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{\mu^2}$$

$$D^{ren}_{\mu\nu} = \langle 0 | T(A^{ren}_\mu(x) A^{ren}_\nu(y)) | 0 \rangle$$

$$= \frac{g_{\mu\nu}}{q^2} \frac{1}{1 + \frac{\frac{\alpha(r_0)}{3\pi} \ln \frac{\mu^2}{q^2}}{1 + \frac{\alpha(r_0)}{6\pi} \ln \frac{\Lambda^2}{\mu^2}}} = \frac{g_{\mu\nu}}{q^2} \frac{1}{1 + \frac{\alpha(r \sim \mu^{-1})}{3\pi} \ln \frac{\mu^2}{q^2}}$$

$$\alpha(r) = \frac{\alpha(r_0)}{1 + \frac{\alpha(r_0)}{3\pi} \ln \frac{\Lambda^2}{\mu^2}}$$

$r_0 \sim \Lambda^{-1}$

$r \sim \mu^{-1}$

$$= \frac{\alpha(r_0)}{1 + \frac{\alpha(r_0)}{3\pi} \ln \frac{r^2}{r_0^2}}$$

$r > r_0 \quad \alpha(r) < \alpha(r_0)$

árnyékolás

<u>screening</u>

by vacuum polarisation

Inverse relation

$$\alpha(r_0) = \frac{\alpha(r)}{1 - \frac{\alpha(r)}{3\pi} \ln \frac{r^2}{r_0^2}}$$

Try to define $r_0 \to 0$ $(\Lambda \to \infty)$ limit

keeping $\alpha(r)$ fixed at $r = \lambda_{Compton}$

Pole appears and prevents r_0 to reach 0

$$r_{Landau} = \underbrace{r(Compton)}_{\lambda_{Compton}} e^{-\frac{3\pi}{\alpha(r_{Compton})}} \qquad \alpha(r_{Compton}) \sim \frac{1}{137}$$

Electroweak scale $r_{EW} \sim M_Z^{-1}$ is reached much before the Landau-pole is reached

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The rate of variation — "beta-function"

$$\beta(QED) = \frac{d\,\alpha(r)}{d \ln r} = -\frac{2\alpha^2(r)}{3\pi}$$

its integration from $r = \lambda_{Compton}$ reproduces the above relation

Renormalisation group equation

Asymptotic freedom of non-Abelian "charges" 2018/16.

$$g^2(r_0) = \frac{g^2(r)}{1 + \gamma g^2(r) \ln \frac{r}{r_0}}$$

$$\gamma_{gluon} = \frac{11}{6\pi} n - \nu \frac{1}{3\pi}$$

$SU(n)$

$n = 3$ QCD

ν d.o.f. of matter

Until $\frac{11}{6\pi} n > \nu \frac{1}{3\pi}$

when $r_0 \to 0$ $g^2(r_0) \to 0$ (starting with $g^2(r)$ = finite)

<u>anti-screening</u>

Asymptotic behavior

$$g^2(r_0) \simeq \frac{1}{\gamma \ln \frac{r}{r_0}}$$ independent of the starting value

Opposite direction – amplifying effect of vacuum polarisation

$$g^2(r) = \frac{g^2(r_0)}{1 - \gamma g^2(r_0) \ln \frac{r}{r_0}}$$

increasing strength of the interaction

$\downarrow$

<u>quark confinement</u>

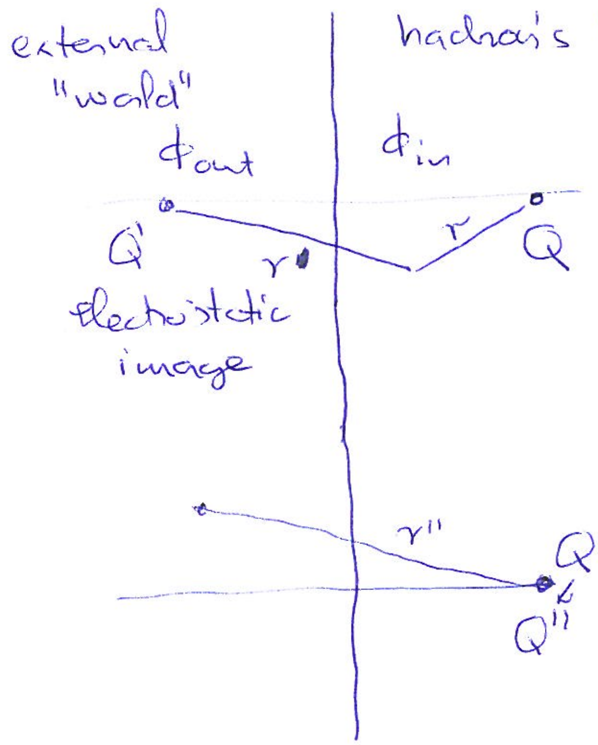
Next steps: Review of the quark confinement mechanism

2018/17.

Simple ideas and models of quark confinement

quark - confinement $\Longleftrightarrow$ confinement of color flux

Electrostatic analogy



$$\phi_{in} = \frac{1}{4\pi\varepsilon_i}\left(\frac{Q}{r} + \frac{Q'}{r'}\right)$$

$$Q' = \frac{\varepsilon_i - \varepsilon_o}{\varepsilon_i + \varepsilon_o} Q$$

$$\phi_{out} = \frac{1}{4\pi\varepsilon_o}\,\frac{Q''}{r''}$$

$$Q'' = \frac{2\varepsilon_o}{\varepsilon_o + \varepsilon_i} Q$$

If $\varepsilon_{out} = 0$ perfect screening – flux is closed into the interior of the hadron

$\varepsilon = 1 + \chi_e$ $\quad$ χ_e electric susceptibility $\longrightarrow$ $\chi_e = -1$

perfect <u>dia</u>-electric medium

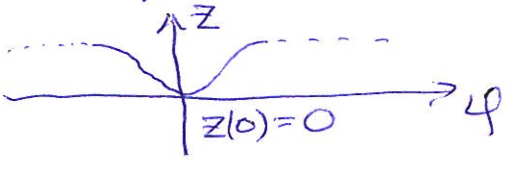
(analogy with Meissner effect!)

Field theoretic model of G 't Hooft

$$L = -\frac{1}{4} Z(\varphi) F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi) - J_\mu A^\mu$$

φ-field represents the medium, where the electric field "tries" to penetrate into

$Z(\varphi)$ given function



$\varphi \sim 0$
$Z(\varphi) \sim \varphi^2$

2018/18.

Maxwell's equations

$$-\frac{Z}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(ZE^2 - ZB^2\right)$$

$c \equiv 1$ <u>unit choice</u>

$$\equiv \frac{1}{2}\left(\varepsilon E^2 - \frac{1}{\mu}B^2\right) = \frac{1}{2}\left(\underline{E}\,\underline{D} - \underline{B}\,\underline{H}\right)$$

$\underline{D} = Z\underline{E}$ $\quad \underline{H} = Z\underline{B}$

$\boxed{\varepsilon = Z}$ $\quad \mu = \frac{1}{Z}$

$Z \sim$ dielectric function

$Z^{-1} \sim$ magnetic permeability

$\varepsilon\mu = 1 \; (\equiv c^2)$

$\mathrm{div}\,\underline{D} = j_0 \qquad -\dot{\underline{D}} + \mathrm{rot}\,H = \underline{j}$

Static case $j_0 \neq 0 \quad \underline{j} = 0 \Leftrightarrow \dot{D} = 0$

$\Leftrightarrow \underline{H} = 0$

$$\downarrow$$

$\underline{D} \neq 0$

The complete energy density

$$\mathcal{E} = \frac{1}{2\varepsilon}\underline{D}^2 + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}m^2\varphi^2 + \lambda\varphi^4 + j_0 A^0$$

Finite energy density for $\varepsilon = 0$ implies $\underline{D}^2 = 0$

The converse statement: Since $j_0 \neq 0 \Rightarrow \underline{D} \neq 0 \Rightarrow |\varphi| > 0$

The field equation of φ from $L = \frac{1}{2}Z(\varphi)\underline{E}^2 - \frac{1}{2}m^2\varphi^2 - \lambda\varphi^4$

$+ \frac{1}{2}(\nabla\varphi)^2 - j_0 A_0$

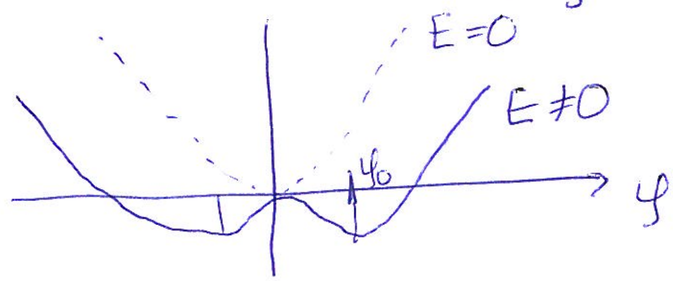
$Z(\varphi) = K\varphi^2$

"Homogenous" solution near the charge where $|E|$ is large

$(KE^2 - m^2)\varphi - 4\lambda\varphi^3 = 0 \qquad KE^2 - m^2 > 0 \rightarrow \varphi_0^2 \neq 0$ solution

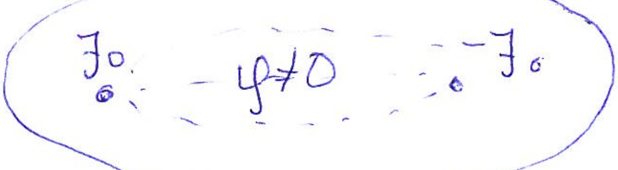
$KE^2 - m^2 < 0 \rightarrow \varphi_0 = 0$

The potential "felt" by φ changes with $|E|$



Soft hadron bag $\longleftrightarrow$ hadron with "color" source (quark) constituents

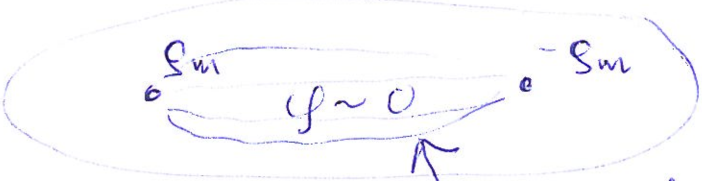
2018/19.



$\varphi = 0$

Dual superconductor model — "Electric" Meissner effect

$\varphi = \varphi_0$ (superconducting state)



magnetic field lines

Abrikosov vortex solution

$\Phi_m = n \frac{4\pi}{e}$ flux quantisation

At its end put corresponding ϱ_m charges

Duality $\quad \underline{D} \Leftrightarrow -\underline{B}$

$\quad \underline{E} \Leftrightarrow \underline{H} \qquad (\varrho_e, j_e) \longleftrightarrow (\varrho_m, j_m)$

$\operatorname{div} \underline{B} = -\varrho_m \qquad \operatorname{div} \underline{D} = \varrho_e$

$\operatorname{rot} \underline{H} = j_e + \frac{\partial D}{\partial t} \qquad \operatorname{rot} \underline{E} = -\frac{\partial B}{\partial t} + j_m$

Patkós–Polónyi p. 224

MIT Bag model $\qquad S = -\frac{1}{4} \underbrace{\int F_{\mu\nu} F^{\mu\nu}}_{\text{interior of the bag}} - B \underbrace{\int d^4x}_{\text{interior of the bag}}$

Variation of the bag surface $\frac{1}{2}(\underline{E}^2 - \underline{B}^2)\big|_F = B \leftarrow$ "bag constant"

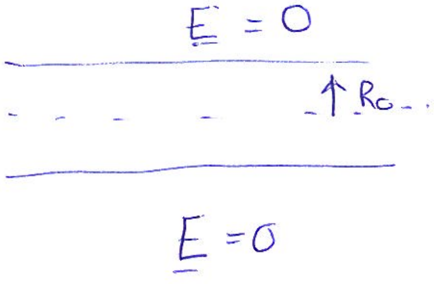
Meaning of B — energy density of the interior is higher than outside (the true vacuum)

$\mathcal{H}_{\text{static}} = \int d^3x \left(\frac{1}{2}(\nabla A_0)^2 + B + \varrho A_0 \right) \qquad \underline{E} = -\underline{\nabla} A_0$

Boundary conditions on the bag surface: flux confinement

$\underline{n} \cdot \underline{E}\big|_F = 0 \qquad n_0 \underline{E}\big|_F + \underline{n} \times \underline{B}\big|_F = 0$

Static flux-tube solution ($\underline{E} \neq 0$ $\underline{B} = 0$) 2018/20.



homogenous $\underline{E}$-distribution along the tube

flux $= g_{eff} = E R_0^2 \pi$

on the surface $\frac{1}{2}E^2 = B$

$$g_{eff} = \sqrt{2B}\, R_0^2 \pi$$

$$R_0 = \left(\frac{g_{eff}}{\pi\sqrt{2B}}\right)^{1/2}$$

String-tension $\leftrightarrow$ energy / unit length

$$L \cdot R_0^2 \pi \cdot \left(\frac{1}{2}\underline{E}^2 + B\right) = 2LBR_0^2\pi = 2LB\,\frac{g_{eff}}{\pi\sqrt{2B}}\,\pi$$

$$= g_{eff} L \sqrt{2B}$$

$$\rightarrow \sigma = g_{eff}\sqrt{2B}$$

Put charges of opposite sign to the ends of the tube. For asymptotic distances the energy increases <u>linearly</u> with the distance

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Quark-antiquark spectroscopy for heavy quarks

$$V_{q\bar{q}}(r) = +\frac{4}{3}\frac{\alpha_s(r)}{r} + \sigma r + V_0 \quad \text{ground state of } c\bar{c}, b\bar{b}$$

$$\hat{H}_{J/\psi} = \frac{\hat{p}^2_{red}}{2m_{red}} + \sigma r + V_0 + \frac{4}{3}\frac{\alpha_s(r)}{r} + V_{spin\text{-}dependent}$$

In QED Breit-Fermi potential from the correspondence

$$\frac{4}{3}g^2 \leftarrow \frac{e^2}{q^2}\,\bar{u}_1(p_1')\gamma_\mu u_1(p_1) \cdot \bar{u}_2(p_2')\gamma^\mu u_2(p_2) \equiv (2m_{red})^2 \chi_1'^{+}\chi_2'^{+}\, V(q)\, \chi_1 \chi_2$$

Keep leading corrections to the non-relativistic bispinor

$$u = \sqrt{2m}\left(1 + \frac{p^2}{8m^2}\right)\begin{pmatrix}\chi_s \\ \frac{\sigma p}{2m}\chi_s\end{pmatrix} \quad ; \quad \frac{1}{q^2} = \frac{1}{(p'-p)^2} = -\frac{1}{\mathbf{q}^2} , \frac{1}{4m^2} + \frac{(qp')(qp)}{m^2(q^2)^2}$$

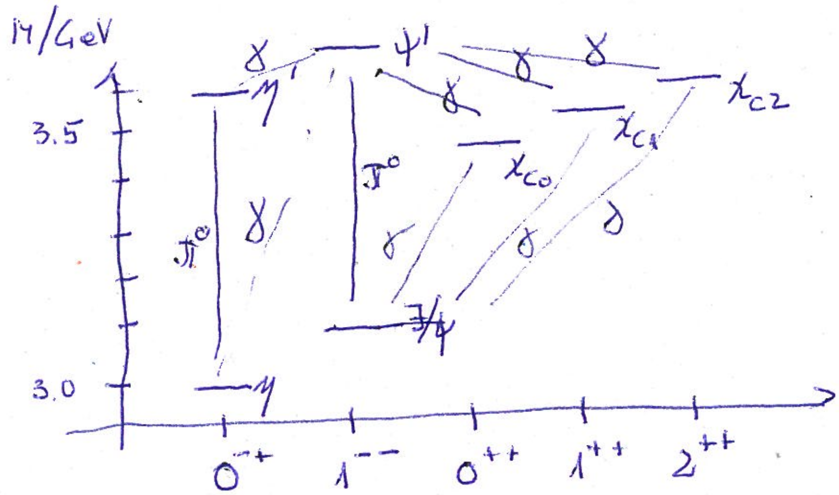
2018/21

$$V = \frac{4}{3}\alpha_s\Big[\frac{1}{q^2} - \frac{1}{4m^2} + \frac{i}{4q^2m^2}\Big((q\times p_1)\underline{\sigma}_1 - (q\times p_2)\underline{\sigma}_2 + 2(q\times p_1)\underline{\sigma}_2 - 2(q\times p_1)\underline{\sigma}_1\Big)$$

$$- \frac{(qp_1)(qp_2)}{m^2(q^2)^2} - \frac{p_1p_2}{m^2q^2} + \frac{(q\underline{\sigma}_1)(q\underline{\sigma}_2)}{4m^2q^2} - \frac{\underline{\sigma}_1\underline{\sigma}_2}{m^2}\Big]$$

Charmonium $c\bar{c}$ spectrum $\rightarrow$ fitting $\alpha_s(r_{charmonium})$

V_0, σ, m_c



Υ spectroscopy (9 states are known)

$\alpha_s(b\bar{b})$ obtained from $\alpha_s(c\bar{c})$ with renormalisation group equation

Originally Breit-Fermi Hamiltonian tested with positronium spectroscopy