Field theories – construction principles

$\downarrow$

transformation in internal (abstract) space

$$\phi(x) \longrightarrow \phi^{\overbrace{a_1 \ldots a_n}}_{\underbrace{\mu_1 \ldots \mu_m}}(x)$$

x – space time location, 4-vector – x^μ

transformation under space-time coordinate variation

Examples of linear transformations

$$\phi^{(\omega)a}(x) = \omega^{ab}\phi^b(x)$$

$$\phi^{(\Lambda)}_\mu(x) = \Lambda_{\mu\nu}\phi_\nu(\Lambda^{-1}x)$$

ω, Λ } independent of x "global"

Principle: Lagrangian density should be invariant under Lorentz-transformations and possibly some internal transformations

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[Consequences of continuous global symmetries conservation laws E. Noether, 1918]

see Patkós-Polónyi p.74

Simplest example scalar field – invariant under Lorentz- or any other internal transformation

$$S_{cl} = \int dt \int d^3x \left[\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi(x))\right] \qquad \partial_\mu = \frac{\partial}{\partial x^\mu}$$

$$= -\int d^4x \left[\frac{1}{2}\phi\,\Box\phi + V(\phi)\right] \qquad \partial^\mu = g^{\mu\nu}\partial_\nu$$

Dynamics – generalisation of Hamilton's principle

$$\delta_\phi S_{kl} = 0 = -\int d^4x \left[\frac{1}{2}\delta\phi\,\Box\phi + \frac{1}{2}\Box\phi\cdot\delta\phi + \frac{\partial V(\phi)}{\partial\phi}\delta\phi\right]$$

$$\longrightarrow \quad \Box\phi + \frac{\partial V}{\partial\phi} = 0 \quad \text{if } V = \frac{1}{2}m^2\phi^2 \qquad (\Box + m^2)\,\phi(x) = 0$$

free scalar field Klein-Gordon eq.

Dirac-field $\psi(x) \to \psi_\alpha(x)$ $\alpha = 1,..,4$ bispinor index

2018/2.

Dirac-current $\bar\psi(x)\gamma^\mu\psi(x) = j_D^\mu$ | $\bar\psi(x) = \psi^+(x)\gamma^0$

γ^μ – Dirac matrices

2x2 blocs! $\to$ $\gamma^0 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$ $\gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix}$

σ^j $j = 1,2,3$ Pauli-matrices

j_D^μ transforms like x^μ (4-vector)

Dirac-action $S_D = \int d^4x \left[\bar\psi (i\partial_\mu\gamma^\mu - m)\psi \right]$

$\psi, \bar\psi$ independent variables $\delta_{\bar\psi} S_D = 0 = (i\partial_\mu\gamma^\mu - m)\psi(x)$

$\delta_\psi S_D = 0 = (-i\bar\psi\overleftarrow{\partial}_\mu\gamma^\mu - \bar\psi m)$

Local (x-dependent) transformations

Example: color of quarks $q^c(x)$ $c = 1,2,3$

$$q^{(\omega)c}(x) = \omega(x)^{cd} q^d(x) \to q^{(\omega)c^+}(x) = q^{+d}(x)\, \omega^+(x)^{dc}$$

$$\bar q^{(\omega)c}(x) = \bar q^d(x)\, \omega^+(x)^{dc}$$

Invariance of $-m\bar q q \Rightarrow -m\,\bar q\,\omega^+(x)\cdot\omega(x)\,q \to \omega^+(x)\omega(x) = 1$

$\omega^+(x) = \omega^{-1}(x)$ unitary matrix <u>group</u> $\mathcal{G}$

$$\partial_\mu q^c(x) = \lim_{\Delta x^\mu \to 0} \frac{q^c(x) - q^c(x-\Delta x^\mu)}{\Delta x^\mu} \to \frac{\omega(x)q(x) - \omega(x-\Delta x)q(x-\Delta x)}{\Delta x^\mu}$$

Not well-defined symmetry transformation

$\tilde q(x) \equiv e^{-i\Delta x_\mu A^\mu(x)} q(x-\Delta x_\mu)$ | $e^{-i\Delta x_\mu A^\mu} \in \mathcal{G}$ $A^\mu = A^{\mu a} t^a$

t^a generators of $\mathcal{G}$

$\boxed{\tilde q^{(\omega)}(x) = \omega(x)\tilde q(x)}$ Requirement

$$\tilde{q}^{(\omega)}(x) = e^{-i\Delta x_\mu A^{\mu(\omega)}(x)} q^{(\omega)}(x-\Delta x_\mu)$$

$A^\mu(x)$ compensating field

2018/3.

$$\downarrow$$

$$\tilde{q}(x) = \omega^+(x) e^{-i\Delta x_\mu A^{\mu(\omega)}(x)} \omega(x-\Delta x_\mu)\, q(x-\Delta x_\mu)$$

$$= e^{-i\Delta x_\mu A^\mu(x)} q(x-\Delta x_\mu)$$

Transformation rule for the group element

$$\boxed{e^{-i\Delta x_\mu A^{\mu(\omega)}(x)} = \omega(x) e^{-i\Delta x_\mu A^\mu(x)} \omega^+(x-\Delta x_\mu)}$$

Covariant derivative $D_\mu q(x) = \lim_{\Delta x_\mu \to 0} \frac{q(x) - \tilde{q}(x)}{\Delta x^\mu}$

$$= \frac{q(x) - (1 - i\Delta x_\mu A^\mu(x))(q(x) - \Delta x_\mu \partial^\mu q(x))}{\Delta x_\mu} = \underbrace{(\partial^\mu + i A^\mu(x))}_{D^\mu} q(x) + O(\Delta x_\mu)$$

Expanding the transformation rule of the exponential to linear order in Δx_μ

$$1 - i\Delta x_\mu A^{\mu(\omega)}(x) = \omega(x)(1 - i\Delta x_\mu A^\mu(x))(\omega^+(x) - \Delta x_\mu \partial^\mu \omega^+(x))$$

$$\approx 1 - i\Delta x_\mu \omega(x) A^\mu(x) \omega^+(x) - \omega(x)\partial^\mu \omega^+(x) \Delta x_\mu$$

$$\Longrightarrow \quad \boxed{A^{\mu(\omega)}(x) = \omega(x) A^\mu(x) \omega^+(x) - i\omega(x)\partial^\mu \omega^+(x)}$$

Invariant Dirac-action

$$S_D = \int d^4x\, \bar{q}(x)[i\not{D} - m] q(x) \qquad \not{D} = \gamma^\mu D_\mu$$

$$D_\mu = \partial_\mu + i A_\mu$$

Multicomponent $\Phi(x)$ – scalar in Lorentz-sense $\quad \Phi^{ab}_{(x)} \to \omega^{ac}(x)\omega^{bd}(x)\Phi^{cd}(x)$

$$S_{\text{Scalar}} = \int d^4x \left[\mathrm{Tr}(D_\mu\Phi)^+(D^\mu\Phi) - V(\mathrm{tr}(\Phi^+\Phi))\right]$$

Summary – local internal symmetry 2018/4

$$\partial_\mu \phi \longrightarrow D_\mu \phi = (\partial_\mu + i A_\mu)\,\phi(x) \qquad A_\mu = A^a_\mu(x)\, t^a$$

transformations $\phi(x) \to \omega(x)\phi(x)$ $\qquad t^a$ generator of $\mathcal{G}$

$\hookrightarrow \in \mathcal{G}$ gauge group

$$\tilde{\phi}(x) = e^{-i\Delta x^\mu A_\mu}\,\phi(x-\Delta x^\mu)$$

$$\tilde{\phi}^{(\omega)}(x) = \omega(x)\,\tilde{\phi}(x) = \omega(x)\, e^{-i\Delta x^\mu A_\mu}\,\phi(x-\Delta x^\mu)$$

$$= e^{-i\Delta x^\mu A^{(\omega)}_\mu}\,\omega(x-\Delta x^\mu)\,\phi(x-\Delta x^\mu)$$

$$\rightarrow \quad e^{-i\Delta x^\mu A^{(\omega)}_\mu} = \omega(x)\, e^{-i\Delta x^\mu A_\mu}\,\omega^+(x-\Delta x^\mu)$$

(it gives back the transformation law of A_μ derived differently)

Matter - gauge field interaction

Dirac-theory $\quad S_D = \int d^4x\; \bar{q}(x)\left[i\not{D} - m\right] q(x) \qquad \not{D} = D_\mu \gamma^\mu$

Scalar matrix theory

$$\phi(x) \longrightarrow \omega_L(x)\,\phi(x)\,\omega_R^+(x) \qquad \omega_L \in \mathcal{G}_L \quad \omega_R \in \mathcal{G}_R$$

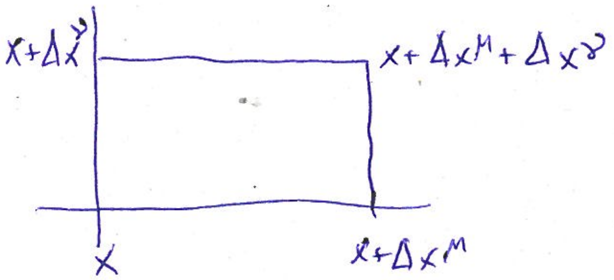
Symmetry group $\mathcal{G}_L \times \mathcal{G}_R$

$$D_\mu \phi = \partial_\mu \phi + i A_\mu \phi - i \phi B_\mu$$

Példa: $\quad S = \int d^4x \left\{ \mathrm{Tr}_\phi\, (D_\mu\phi)^+ (D^\mu\phi) - V\left(\mathrm{tr}_{\mathcal{G}}\, \phi^+\phi\right)\right\}$

Dynamics of the compensating field 2018/5.

Wilson loop - oriented closed curve



$$U_\mu(x-\Delta x^\mu) = e^{i\Delta x_\mu A^\mu(x)}$$

$$U_\mu^{(\omega)}(x-\Delta x^\mu) = \omega(x-\Delta x^\mu)U_\mu(x)\omega^+(x)$$

Ordered product $W_{\mu\nu}(x) = U_\mu(x)U_\nu(x+\Delta x^\mu)\,U_{-\mu}(x+\Delta x^\mu+\Delta x^\nu)\,U_{-\nu}(x+\Delta x^\nu)$

$U_{-\nu}(x) = U_\nu^+(x-\Delta x^\nu)$ opposite transport is the inverse operation

$U_{-\mu}(x) = U_\mu^+(x-\Delta x^\mu)$

$$W_{\mu\nu}(x) = U_\mu(x)U_\nu(x+\Delta x^\mu)U_\mu^+(x+\Delta x^\nu)U_\nu^+(x)$$

$$W_{\mu\nu}^{(\omega)}(x) = \omega(x)U_\mu(x)\omega^+(x+\Delta x^\mu)\,\omega(x+\Delta x^\mu)\cdot U_\nu(x+\Delta x^\mu)\omega^+(x+\Delta x^\mu+\Delta x^\nu)$$

$$\cdot\,\omega(x+\Delta x^\mu+\Delta x^\nu)U_\mu^+(x+\Delta x^\nu)\omega^+(x+\Delta x^\nu)$$

$$\cdot\,\omega(x+\Delta x^\nu)\,U_\nu^+(x)\,\omega^+(x)$$

$$= \omega(x)U_\mu(x)U_\nu(x+\Delta x^\mu)U_\mu^+(x+\Delta x^\nu)U_\nu^+(x)\omega^+(x)$$

$$= \omega(x)W_{\mu\nu}(x)\omega^+(x)$$

$\Delta x_\mu, \Delta x_\nu \to 0$ expansion to quadraticorder

$$W_{\mu\nu}(x) \cong 1 + i\Delta x^\mu\Delta x^\nu\left(\partial_\nu A_\mu(x) - \partial_\mu A_\nu(x) + i[A_\nu(x), A_\mu(x)]\right)$$

$$\equiv 1 + i\Delta x^\mu\Delta x^\nu F_{\nu\mu}$$

$F_{\nu\mu}^{(\omega)} = \omega(x)F_{\nu\mu}(x)\omega^+(x)$ $\quad$ $\mathrm{tr}\,F_{\nu\mu}(x)F^{\nu\mu}(x)$ invariant!

$$L_{gauge} = -\frac{1}{2g^2}\mathrm{tr}_g F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4g^2}F^a_{\mu\nu}(x)F^{\mu\nu a}(x)$$

using $\mathrm{tr}\,t^a t^b = \frac{1}{2}\delta^{ab}$

Conventional form 2018/6.

$$F^{\mu\nu a} = -g f^{\mu\nu a}$$

$$A^{\mu a} = -g a^{\mu a}$$

Dirac action for quarks $q_f(x)$ f flavor index

$$\int d^4x\, \bar{q}_f(x) \left[(i\partial_\mu + g a_\mu)\gamma^\mu - m_f \right] q(x)$$

the strong interactions are flavor (f) blind.

Interaction vertex $g t^a \gamma^\mu$

Gauge-field action after rescaling

$$\int d^4x \left(-\frac{1}{2} \operatorname{tr} f^{\mu\nu}(x) f^{\mu\nu}(x) \right) = \int d^4x \left(-\frac{1}{4} f^{\mu\nu a}(x) f^{\mu\nu a}(x) \right)$$

$$f^{\mu\nu a}(x) = \partial^\mu a^{\nu a} - \partial^\nu a^{\mu a} + g f^{abc} a^{\mu b} a^{\nu c}$$

Electrodynamics $f^{\mu\nu} = \partial^\mu a^\nu - \partial^\nu a^\mu$ free photons!
$G = U(1)$

Standard Model W^+_μ, W^-_μ $\underbrace{W^{(0)}_\mu \text{ and } B_\mu}$
$G = SU(2)_L \otimes U(1)_R$ mixes into Z_μ, A_μ

QCD
$G = SU(3)$ $t^a = \frac{\lambda_a}{2}$

Next steps: scattering described in Q.M.
and in Q.E.D.

lowest degree approximation generalisation to non-Abelian gauge groups

Electromagnetic scattering amplitude and Coulomb potential 2018/7.

Scattering in Q.M. – Born approximation

$$\psi(x) = \underset{\substack{\uparrow \\ \text{incoming wave}}}{e^{i\underline{k}\underline{x}}} + \psi_{scat} \qquad \psi_{scat} = \underset{\substack{\uparrow \\ \text{spherical wave}}}{\frac{e^{ik|\underline{x}|}}{|\underline{x}|}} \cdot \overset{\substack{\text{scatt. amplitude} \\ \uparrow}}{f(\underline{n})} \qquad \underline{n} = \frac{\underline{x}}{|\underline{x}|}$$

Schrödinger-equation

$$(\Delta + k^2)\psi(x) = \frac{2m}{\hbar^2} V(x)\psi(x) \qquad E_{in} = \frac{\hbar^2 k^2}{2m}$$

Rewriting as integral equation with help $G_+(\underline{x}-\underline{x}') = -\frac{1}{4\pi}\frac{e^{ik|\underline{x}-\underline{x}'|}}{|\underline{x}-\underline{x}'|}$

$$(\Delta + k^2) G_+(\underline{x}-\underline{x}') = \delta(\underline{x}-\underline{x}')$$

$$\psi(x) = \psi_{in}(x) + \int d^3x' G_+(\underline{x}-\underline{x}')\frac{2m}{\hbar^2} V(x')\psi(x')$$

First iteration $\boxed{\psi^{(Born)}_{scatt} = \frac{2m}{\hbar^2}\int d^3x' G_+(\underline{x}-\underline{x}') V(x')\psi_{in}(x')}$

$|\underline{x}-\underline{x}'| \gg |\underline{x}'| \longrightarrow |\underline{x}-\underline{x}'| \approx |\underline{x}| - \underline{n}\,\underline{x}'$ "hullámzóna"

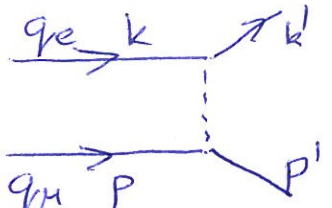
$$\psi^{(Born)}_{scat} \cong \frac{e^{ik|\underline{x}|}}{|\underline{x}|}\left(-\frac{1}{4\pi}\int d^3x' \frac{2m}{\hbar^2} V(x') e^{i(\underline{k}-k\underline{n})\underline{x}'}\right)$$

$$= \frac{e^{ik|\underline{x}|}}{|\underline{x}|}\underbrace{\left(-\frac{1}{4\pi}\frac{2m}{\hbar^2}\tilde{V}(\underline{k}-\underline{n}k)\right)}_{f(\underline{n})}$$

If one succeeds computing $f(\underline{n})$ by this correspondence one finds the Fourier-transform of the potential

$\underline{q} = \underline{k} - k\underline{n}$ momentum transfer

Electron-muon scattering in quantum electrodynamics



q_μ ↑ for particle (not a Lorentz-index)

Momentum transfer ~~q~~ $q^2=(p'-p)^2=(k'-k)^2$

non-relativistic scattering   2018/8.

$p_\mu \simeq (m_\mu, \underline{p})$ $k_\mu \simeq (m_e, \underline{k})$ etc

(↑ Lorentz, ↑ particle)

$$q^2 \simeq -(\underline{p}-\underline{p}')^2 = -(\underline{k}-\underline{k}')^2$$

Scattering amplitude – S-matrix
in one-photon exchange approximation
(one has to choose a gauge – Lorentz-Feynman gauge)

$$S_{fi} = i(2\pi)^4 \delta(p+k-p'-k')\, q_e q_\mu \underbrace{\bar{u}^{(e)}_{r'}(k')\gamma^\lambda u^{(e)}_r(k)}_{\text{transition current}} \frac{1}{q^2} \bar{u}^{(\mu)}_{s'}(p')\gamma_\lambda u^{(\mu)}_s(p)$$

$$j^{(e)\lambda} \cdot D_\gamma(q^2) \cdot j^{(\mu)}_\lambda$$

$\boxed{\text{Patkós-Polónyi p.105}}$

$$\bar{u}^{(e)}_r(p) = \sqrt{k_0+m_e}\begin{pmatrix} \chi_r \\ \frac{\underline{\sigma}\underline{k}}{k_0+m_e}\chi_r \end{pmatrix}$$

χ_r Pauli spinor $\chi_{1/2}=\begin{pmatrix}1\\0\end{pmatrix}$ $\chi_{-1/2}=\begin{pmatrix}0\\1\end{pmatrix}$

$$\approx \sqrt{2m_e}\begin{pmatrix} \chi_r \\ \frac{\underline{\sigma}\underline{k}}{2m}\chi_r \end{pmatrix}$$

Transition current in the non-relativistic limit

$$j^0 = \bar{u}_{r'}(k')\gamma^0 u_r(k) = u^+_{r'}(k')u_r(k) = 2m\left(\chi^+_{r'}, \chi^+_{r'}\frac{\underline{\sigma}\underline{k}'}{2m}\right)\begin{pmatrix} \chi_r \\ \frac{\underline{\sigma}\underline{k}}{2m}\chi_r \end{pmatrix}$$

$$= 2m\Big(\delta_{rr'} + \chi_{r'} \underbrace{\underline{\sigma}\underline{k}' \cdot \underline{\sigma}\underline{k}}_{k'_i k_j(\delta_{ij}+i\varepsilon_{ijl}\sigma_l)} \chi_r\Big)$$

$$= 2m\left[\delta_{rr'}\left(1+\frac{\underline{k}\underline{k}'}{4m^2}\right) + \frac{i}{4m^2}(\underline{k}'\times\underline{k})\,\chi^+_{r'}\underline{\sigma}\chi_r\right] \simeq 2m\delta_{rr'}$$

spin preserving >> spin-flip part of the amplitude

2018/9.

$$\bar{u}_{r'}(k')\gamma^0\gamma^\ell u_r(k) = 2m\left(\chi^+_{r'},\ \chi^+_{r'}\frac{\underline{\sigma}\underline{k}'}{2m}\right)\begin{pmatrix}0 & \sigma_\ell\\ \sigma_\ell & 0\end{pmatrix}\begin{pmatrix}\chi_r\\ \frac{\underline{\sigma}\underline{k}}{2m}\chi_r\end{pmatrix}$$

$$= (k+k')_\ell\,\delta_{rr'} + i\left[\underline{k}\times\chi^+_{r'}\underline{\sigma}\chi_r - \underline{k}'\times\chi^+_{r'}\underline{\sigma}\chi_r\right]_\ell$$

$O\left(\frac{k, k'}{m}\right)$ smaller than the zeroth compent of the transition current

In the S-matrix element $O\left(\frac{k^2, k'^2}{m^2}\right)$ suppression

$$S_{fi} \approx i\,(2\pi)^4\,\delta^{(4)}(p+k-p'-k')\,q_e q_\mu\,4m_e m_\mu\,\delta_{rr'}\delta_{ss'}\left(-\frac{1}{q^2}\right)$$

Comparison of the differential cross sections

Q.M $\quad \dfrac{d\sigma}{d\Omega_{\underline{n}}} = \underline{\dfrac{m^2}{4\pi^2}\,|\tilde{V}(\underline{k}-\underline{n}k)|^2}\ \left(\equiv|f(\underline{n})|^2\right) \quad \underline{k}'=\underline{n}k$

$(\hbar=1)$

Q.E.D. $\quad T_{fi} = 4\,m_e m_\mu\,q_e q_\mu\,\delta_{rr'}\delta_{ss'}\left(-\frac{1}{q^2}\right)$

$$\frac{d\sigma}{d\Omega_{\underline{k}'}} = \frac{1}{4\pi^2}\cdot\frac{1}{16m_\mu^2}|T_{fi}|^2 = \underline{\frac{m_e^2}{4\pi^2}\left(-\frac{1}{q^2}\right)^2 q_e^2 q_\mu^2}$$

$$\tilde{V}_{e\mu}(q) = -\frac{q_e q_\mu}{q^2} \longleftrightarrow V(r) = \frac{q_e q_\mu}{4\pi r} = \frac{\alpha^2}{r}\quad (q_e = q_\mu)$$

α_e^2 finomszerkezeti állandó $= \frac{q_e^2}{4\pi}$

<u>Next step</u> repeating the calculation for colored quarks.

Potential from gluon exchange – color interaction

photon | gluons | 2018/III.10.

$$\frac{g^{\mu\nu}}{q^2} \qquad\qquad \frac{g^{\mu\nu}\delta^{ab}}{q^2}$$

electric current density | color current density

$$q\bar{\psi}_e\gamma^\lambda\psi_e \qquad\qquad g\,\bar{q}_\alpha\gamma^\lambda t^{a}_{\alpha\beta}q_\beta$$

$a = 1,2,\dots 8$; $\beta = 1,2,3$

$$S_{fi} = i(2\pi)^4\delta(p+k-p'-k')\,g^2\,\bar{u}^{(q)}_{r'\alpha'}(p')\gamma^\lambda t^{a}_{\alpha'\alpha}u^{(q)}_{r\alpha}(p)\frac{\delta^{ab}}{(p-p')^2}\cdot$$
$$\cdot\,\bar{u}^{(\bar{q})}_{s'\beta'}(k')\gamma_\lambda t^{b}_{\beta'\beta}u^{(\bar{q})}_{s\beta}(k)$$

Non-relativistic scattering

$$S_{fi} = i(2\pi)^4\delta(p+k-p'-k')\,g^2\left(\frac{1}{-q^2}\right)\delta_{rr'}\delta_{ss'}\,4m_q m_{\bar{q}}\,t^{a}_{\alpha'\alpha}t^{a}_{\beta'\beta}$$

$$\sum_{\alpha'\beta'}\left(\frac{1}{3}\sum_{\alpha}\right)\left(\frac{1}{3}\sum_{\beta}\right)|T_{fi}|^2 \equiv \overline{|T_{fi}|^2}$$

summation over final color state

averaging over initial color states

uncorrelated, random occurence

$$\overline{|T_{fi}|^2} = (4m_q m_{\bar{q}})^2\frac{1}{9}\sum_{\alpha\alpha'}t^{a}_{\alpha'\alpha}\left(t^{b}_{\alpha'\alpha}\right)^*\sum_{\beta\beta'}t^{a}_{\beta'\beta}\left(t^{b}_{\beta'\beta}\right)^*$$

Hermitian generators

$$t^{a}_{\alpha'\alpha} = (t^{a+})_{\alpha'\alpha} = \left(t^{a}_{\alpha\alpha'}\right)^* \longrightarrow t^{a}_{\alpha'\alpha}t^{b*}_{\alpha'\alpha} = t^{a}_{\alpha'\alpha}t^{b}_{\alpha\alpha'}$$
$$= \mathrm{tr}\,t^a t^b = \text{normalisation condition}$$
$$= \frac{1}{2}\delta^{ab}$$

2018/11.9.

$$\overline{|T_{fi}|^2} = (4m_q m_{\tilde{q}})^2 \, \frac{1}{9} \left(-\frac{g^2}{q^2}\right)^2 \frac{1}{4} \underbrace{\delta^{ab}\delta^{ab}}_{8} = \frac{2}{9} (4m_q m_{\tilde{q}})^2 \left(-\frac{g^2}{q^2}\right)^2$$

$$\Rightarrow \quad \tilde{V}_{q\tilde{q}} = \sqrt{\frac{2}{9}} \left(-\frac{g^2}{q^2}\right) \frac{1}{4\pi}$$

$\boxed{\text{coefficients depends on the color state of the colliding quarks}}$

In a quark-antiquark "molecule" — meson the color states are correlated — <u>colorless</u> state

$$\psi_{color} = \frac{1}{\sqrt{3}} \left(\chi_1^{anti} \psi_1 + \chi_2^{anti} \psi_2 + \chi_3^{anti} \psi_3 \right)$$

The generator of the antiquark color-current is t^{a*}

$$S_{fi} \sim \frac{1}{\sqrt{3}} t^a_{\alpha'\alpha} \frac{1}{\sqrt{3}} t^{a*}_{\alpha'\alpha} = \frac{1}{3} t^a_{\alpha'\alpha} t^a_{\alpha\alpha'} = \frac{4}{3}$$

$$\Rightarrow \quad \tilde{V}^{(meson)}_{q\bar{q}} = \frac{4}{3} \alpha_s \left(-\frac{1}{q^2}\right) \qquad \alpha_s = \frac{g^2}{4\pi}$$

↳ This factor is used in the $c\bar{c}$ - charmonium spectroscopy

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Next steps: effect of vacuum polarisation on the interaction potential

- Q.E.D.
- generalisation to non-Abelian gauge theories