IR Fixed Point and Ghost Dominance in Landau Gauge Yang Mills Theory

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Non-Perturbative Functional Methods in Quantum Field Theory, Hévíz

SIC!QFT

Table of Contents

Infrared QCD

- Infrared cannot be described by perturbative methods $\longrightarrow$ non-perturbative methods like ERGE, nPI, lattice and DSEs
- Aspects of the IR: Chiral symmetry breaking, confinement
- In Landau gauge two promising confinement scenarios exist
  - Kugo-Ojima
  - Gribov-Zwanziger

$\longrightarrow$ ghost propagator divergent, gluon propagator finite/vanishing in IR

Behavior of the Gluon Propagator

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}$$

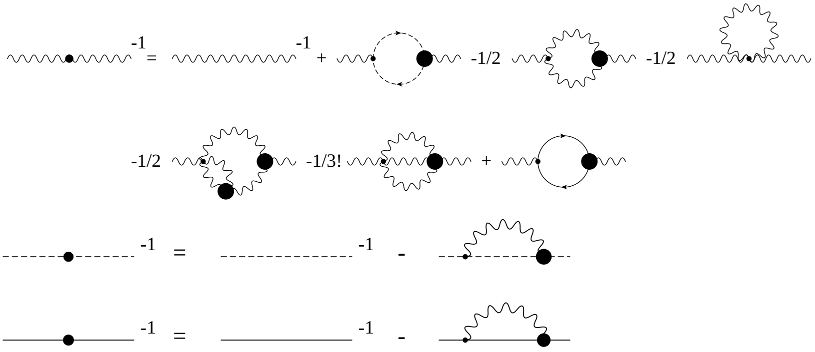
Divergent gluon propagator (IR slavery, confining gluons):

- $Z(p^2)$ singular
- at most $Z(p^2) \propto 1/p^2$
- linear rising quark potential
- result of DSEs without ghosts (Mandelstam approximation)

Vanishing gluon propagator (confined gluons):

- $Z(p^2)$ vanishes faster than p^2
- gluons do not propagate over long distances
- result of DSEs incl. ghosts [von Smekal, Alkofer, Hauck, Phys.Rev.Lett.79]

Propagator DSEs



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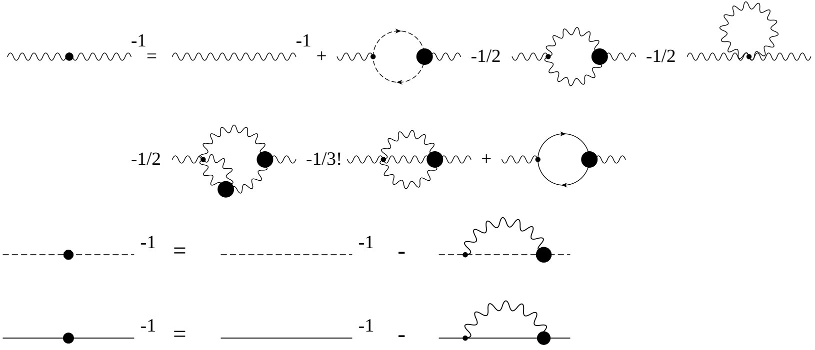

Simplifications:

- Restrict to pure Yang-Mills

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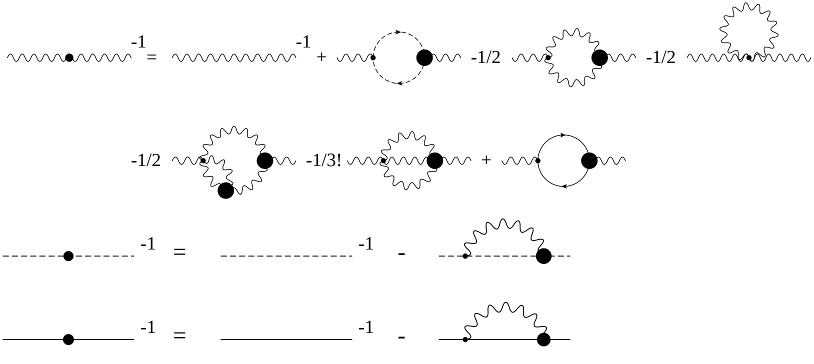

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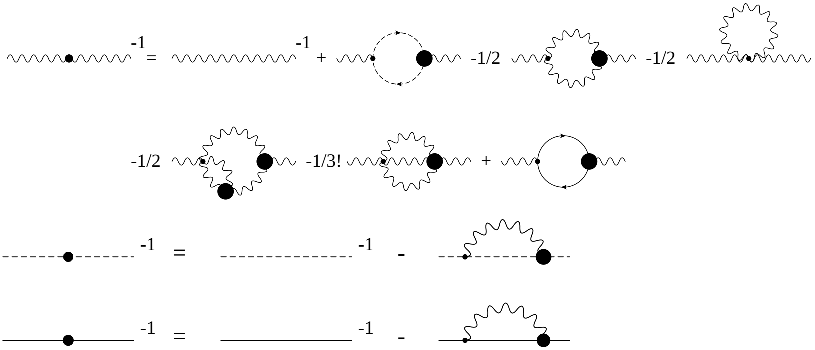

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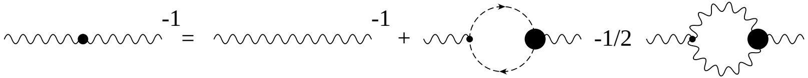

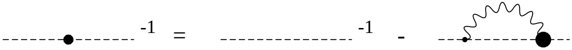

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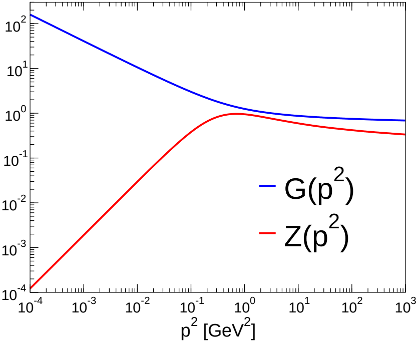
- Neglect two-loop graphs
- Ansätze for vertices

Solution at the Level of the Propagators

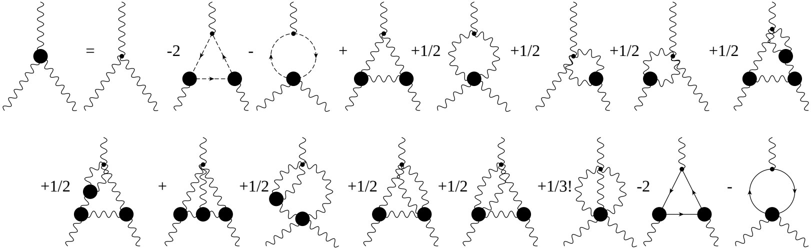


Fischer, Alkofer, Phys. Lett. B 536

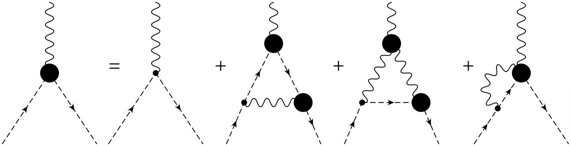
- Vanishing gluon propagator, IR exponent 2κ
- Diverging ghost propagator, IR exponent $-\kappa$

Vertex DSEs

Three-gluon vertex:



Ghost-gluon vertex:



Ghost-Gluon Vertex

Starting point is transversal gluon propagator in Landau gauge:

$$k_\mu D_{\mu\nu}(k) = k_\mu \frac{Z(k)}{k^2}\left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right] = 0$$

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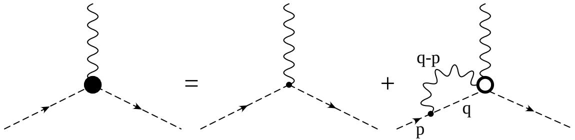
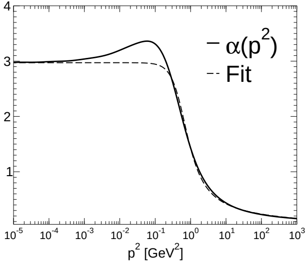


$\Rightarrow$ Ghost-gluon vertex bare in the IR

Running Coupling

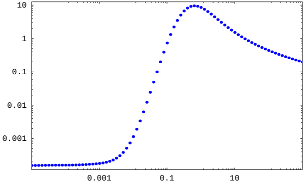


Fischer, Alkofer, Phys. Lett. B 536

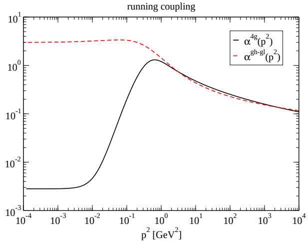
$$\alpha^{gh}(p^2) = \frac{g^2}{4\pi}\,[Z_{2,1}(p^2)]^2\; G^2(p^2)\; Z(p^2) \xrightarrow[p^2\to 0]{} \frac{8.92}{N_c}$$

$$\alpha^{3g}(p^2) = \frac{g^2}{4\pi}\,[Z_{0,3}(p^2)]^2\; Z^3(p^2) \xrightarrow[p^2\to 0]{} c_2$$

$$\alpha^{4g}(p^2) = \frac{g^2}{4\pi}\,[Z_{0,4}(p^2)]\; Z^2(p^2) \xrightarrow[p^2\to 0]{} c_3$$



Schwenzer, to be published

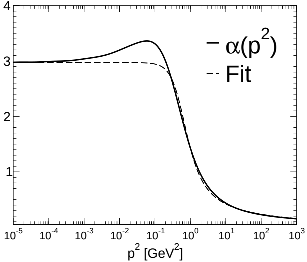


Kellermann, Fischer, arxiv:0801.2697

Running Coupling

Gluon vertices have a running coupling orders smaller than the ghost gluon vertex

→ Indication that ghosts dominate in the IR.



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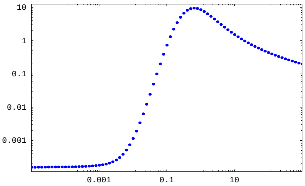

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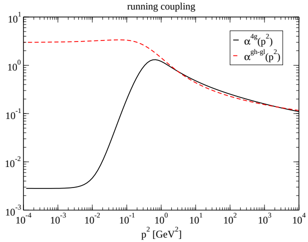

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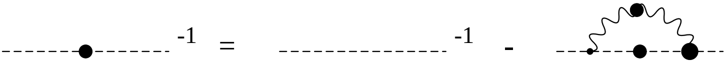
Propagators

- IR behavior can be determined from the ghost DSE:



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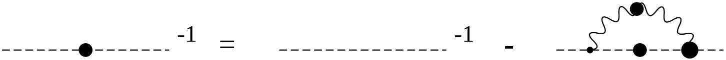

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- Power law ansätze for dressing functions in the IR

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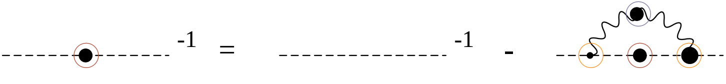

- Use bare ghost-gluon vertex
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$$\left(\frac{B\cdot(p^2)^\beta}{p^2}\right)^{-1} \sim \int \frac{d^d q}{(2\pi)^d} P_{\mu\nu} \frac{A\cdot(q^2)^\alpha}{q^2} \frac{B\cdot((p-q)^2)^\beta}{(p-q)^2} (p-q)_\mu q_\nu$$

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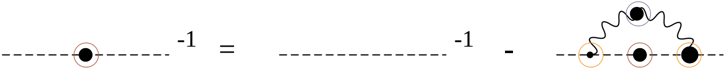

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$$\left(\frac{B\cdot(p^2)^\beta}{p^2}\right)^{-1} \sim \int \frac{d^dq}{(2\pi)^d} P_{\mu\nu} \frac{A\cdot(q^2)^\alpha}{q^2} \frac{B\cdot((p-q)^2)^\beta}{(p-q)^2}(p-q)_\mu q_\nu$$

- Only one momentum scale → simple power counting is possible:

$$1-\beta = \frac{d}{2}+\alpha-1+\beta-1+\frac{1}{2}+\frac{1}{2} \Longrightarrow -2\beta = \alpha+\frac{d}{2}-2$$

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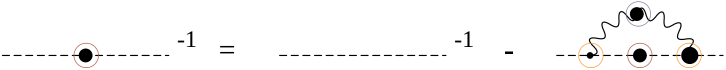

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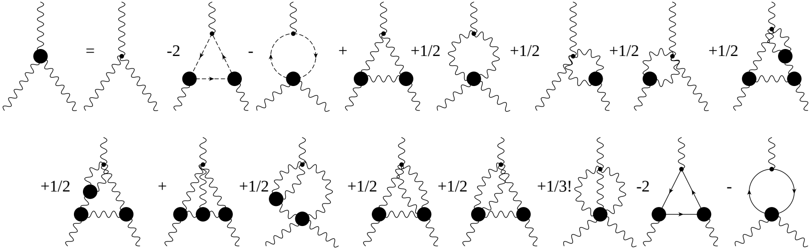
- Only one momentum scale $\rightarrow$ simple power counting is possible:
- Analytic result: $-\beta \equiv \kappa = 0.59\ldots$

Skeleton Expansion

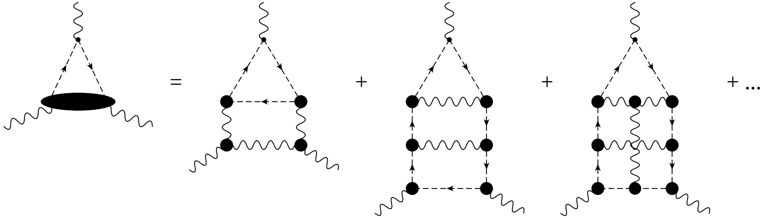
How to treat more complicated graphs?



Skeleton Expansion

Employ skeleton expansion

$\sim$ loop expansion with dressed quantities



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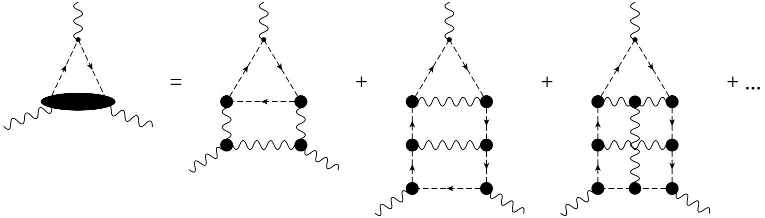
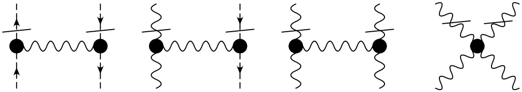


All orders have the same IR exponent, (insertions generating higher orders give no additional contributions).

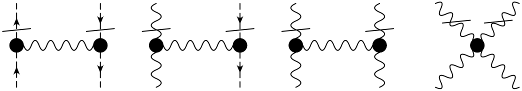


Skeleton Expansion

Skeleton expansion $\Longrightarrow$ calculation of the IR exponent of an arbitrary vertex function with $2n$ external ghosts and m external gluons:

$$\boxed{\delta_{2n,m} = (n-m)\kappa + (1-n)\left(\frac{d}{2}-2\right)}$$

All orders have the same IR exponent, (insertions generating higher orders give no additional contributions).



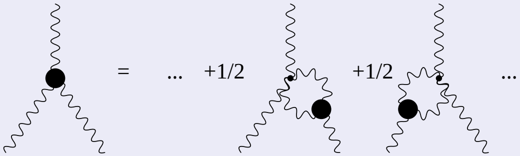
Refined Power Counting

- Assumption up to now: All external momenta vanish $\rightarrow$ unique solution
- Allow for subsets of external momenta to vanish, but still all momenta in the IR $\rightarrow$ different solution possible?
- Limits for the three-point vertices that do not violate the old solution:
  - vanishing gluon momentum: $1 - 2\kappa$
  - vanishing ghost momentum: 0
- Power counting for single diagrams $\rightarrow$ system of inequalities that can be solved.
- Necessary assumption: existence of an skeleton expansion.

Example of Power Counting

Single graphs either dominate the IR behavior or are subleading → inequality relations

Constraint on the Gluon Propagator



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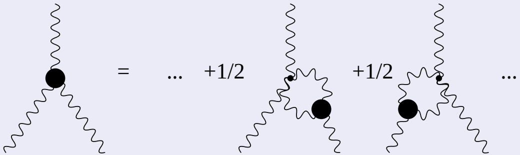

$$\Rightarrow \delta_{3g} \leq \delta_{3g} + 2\delta_{gl} \Rightarrow \delta_{gl} \geq 0$$

Results from Power Counting

- Singularities possible if only one momentum goes to 0.
- The singularities saturate the maximal allowed limit from the uniform solution (0 and $1-2\kappa$) .
- The possible value of κ can be restricted to $1/2 \leq \kappa \leq 3/4$.
- Gluon propagator exponent cannot be negative $\rightarrow$ no "confining" gluons.
- Unique solution for the IR exponents.

Qualitative Behavior

- Ghost propagator is divergent.

δ_{gh}	δ_{gl}	δ_{gg}	δ_{3g}	δ_{gg}^{gh}	δ_{gg}^{gl}	δ_{3g}^{gl}	$\forall$
$-\kappa$	2κ	0	-3κ	0	$1-2\kappa$	$1-2\kappa$	$1/2 \leq \kappa \leq 3/4$

Qualitative Behavior

- Ghost is divergent.
- Gluon propagator is vanishing.

δ_{gh}	δ_{gl}	δ_{gg}	δ_{3g}	δ_{gg}^{gh}	δ_{gg}^{gl}	δ_{3g}^{gl}	$\forall$
$-\kappa$	2κ	0	-3κ	0	$1-2\kappa$	$1-2\kappa$	$1/2 \leq \kappa \leq 3/4$

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- A vanishing ghost momentum gives a constant ghost-gluon vertex.

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- A vanishing gluon momentum gives a light divergence for the three-point vertices.

δ_{gh}	δ_{gl}	δ_{gg}	δ_{3g}	δ_{gg}^{gh}	δ_{gg}^{gl}	δ_{3g}^{gl}	$\forall$
$-\kappa$	2κ	0	-3κ	0	$1-2\kappa$	$1-2\kappa$	$1/2 \leq \kappa \leq 3/4$

Propagators

Loop integrals can be calculated for arbitrary dimensions d and exponents ν_i using

$$\int \frac{d^d q}{(2\pi)^d}(q^2)^{\nu_1}((q-p)^2)^{\nu_2} =$$

$$= (4\pi)^{-\frac{d}{2}} \frac{\Gamma(\frac{d}{2}+\nu_1)\Gamma(\frac{d}{2}+\nu_2)\Gamma(-\nu_1-\nu_2-\frac{d}{2})}{\Gamma(-\nu_1)\Gamma(-\nu_2)\Gamma(d+\nu_1+\nu_2)}(p^2)^{\frac{d}{2}+\nu_1+\nu_2}.$$

→ Power law as expected

Three-Point Functions

Solution for 3-point functions in terms of Appell's functions $F_4(x, y)$

$$x = p_2^2/p_1^2, \quad y = p_3^2/p_1^2$$

Analytic continuation necessary

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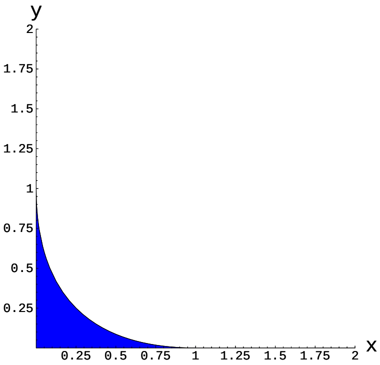

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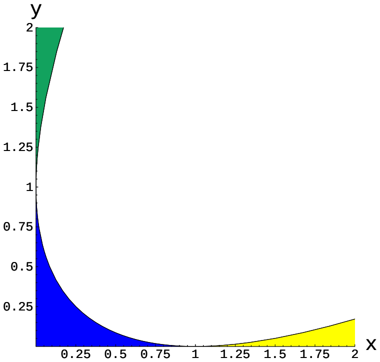

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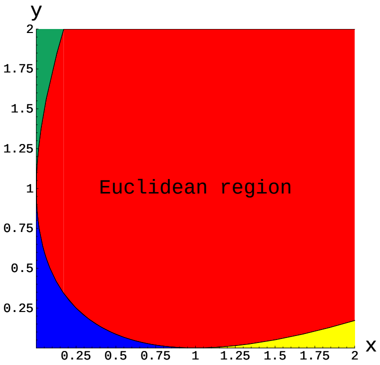

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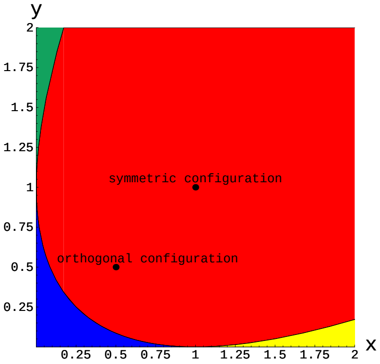
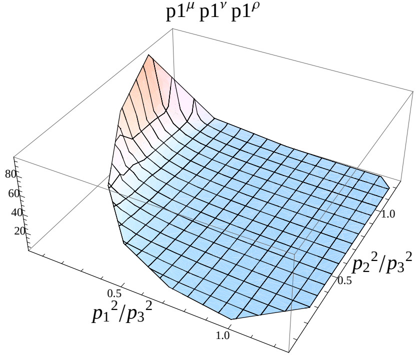
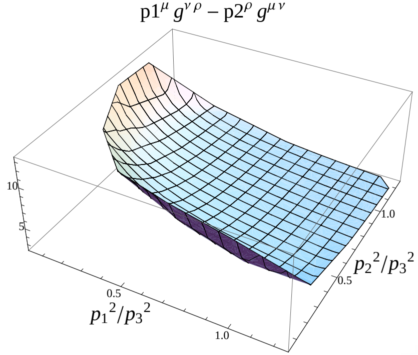


Tensors of the Ghost Triangle (Three-Gluon Vertex)

Tensor decomposition á la Davydychev: 10 tensors (instead of 14) are relevant in the IR.





Singularities are in agreement with power counting constraints.

Dependence on Infrared Exponent

How much influence has the numerical value of κ on the ghost triangle?

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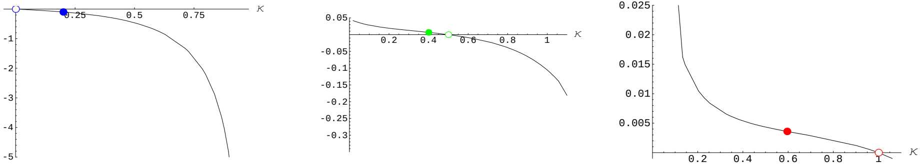
Overlap of the tree-level tensor with the ghost-triangle for $d = 2, 3$ and 4:



Dependence on κ is only weak in relevant region

$\rightarrow$ Ghost dominance seems to be a robust mechanism

Summary

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- Numerical results in agreement with power counting analysis.
- Results very stable when changing $0.5 \leq \kappa \leq 0.75$.

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Thank you for your attention!