Aspects of Dyson-Schwinger approach to light-quark pseudoscalar mesons at zero and finite temperature[a]

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[a] Talk at "Non-Perturbative Functional Methods in Quantum Field Theory"

A Triangle Workshop in Theoretical Particle Physics, Jan. 23 - 25, 2008, Hévíz, Hungary

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Introduction and motivation

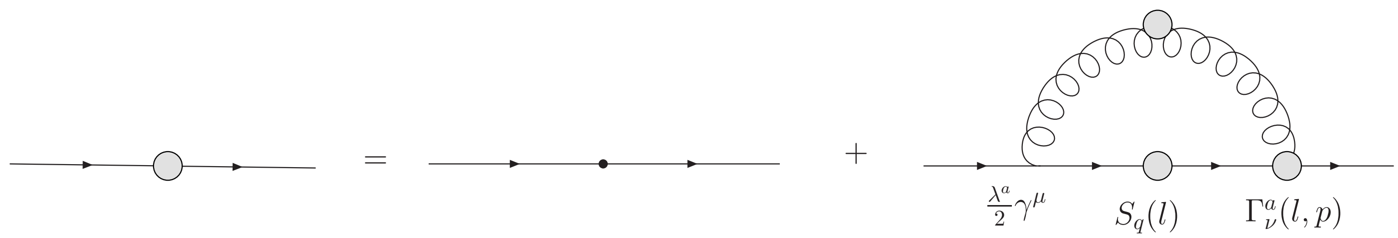
- Recent RHIC results: hot QCD matter has very intricate properties ... & still no direct signal of deconfinement
- If hot QCD matter called "QGP", this cannot be the once expected perturbatively interacting quark-guon gas until much higher T
- Also, lattice (& other): J/Ψ and η_c stay bound till $\sim 2T_{cri}$, maybe higher ... + similar indications about light-quark mesons = motivation to study *bound-state equations*
- RHIC's STAR collab.: 'A compelling, "smoking gun" signal for production of a new form of matter needed!'
- E.g., a change in symmetries obeyed by the strong interaction: the restoration of the chiral and $U_A(1)$ symmetry $\longrightarrow$ a good understanding of the *light pseudoscalar nonet* is needed

Dyson-Schwinger approach to quark-hadron physics

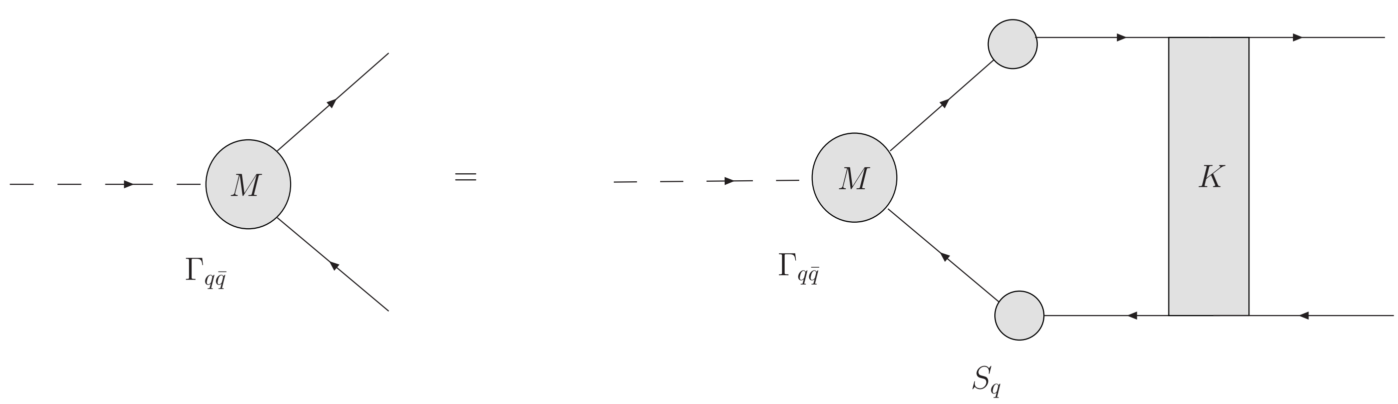
- = the bound state approach which is nopertubative, covariant and chirally well behaved (e.g., GMOR relation: $\lim_{\widetilde{m}_q \to 0} \; M^2_{q\bar{q}}/2\widetilde{m}_q = -\langle \bar{q}q \rangle / f^2_\pi$)
- a) direct contact with QCD through *ab initio* calculations
- b) phenomenological modeling of hadrons as quark bound states (e.g., here)
- coupled system of integral equations for Green functions of QCD
- ... but ... equation for n-point function calls (n+1)-point function ... $\to$ cannot solve in full the growing tower of DS equations
- $\to$ various degrees of truncations, approximations and modeling is unavoidable (more so in phenomenological modeling of hadrons, as here)

Dyson-Schwinger approach to quark-hadron physics

- Gap equation for propagator S_q of dressed quark q



- Homogeneous Bethe-Salpeter (BS) equation for a Meson $q\bar{q}$ bound state vertex $\Gamma_{q\bar{q}}$



Gap and BS equations in ladder truncation

$$S_q(p)^{-1} = i\gamma\cdot p + \widetilde{m}_q + \frac{4}{3}\int \frac{d^4\ell}{(2\pi)^4}\, g^2 G^{\rm eff}_{\mu\nu}(p-\ell)\gamma_\mu S_q(\ell)\gamma_\nu$$

$$\to S_q(p) = \frac{1}{i\not p A_q(p^2) + B_q(p^2)} = \frac{-i\not p A_q(p^2) + B_q(p^2)}{p^2 A_q(p^2)^2 + B_q(p^2)^2} = \frac{1}{A_q(p^2)}\frac{-i\not p + m_q(p^2)}{p^2 + m_q(p^2)^2}$$

$$\Gamma_{q\bar q'}(p,P) = -\frac{4}{3}\int\frac{d^4\ell}{(2\pi)^4} g^2 G^{\rm eff}_{\mu\nu}(p-\ell)\gamma_\mu S_q(\ell+\frac{P}{2})\Gamma_{q\bar q'}(\ell,P)S_q(\ell-\frac{P}{2})\gamma_\nu$$

- Euclidean space: $\{\gamma_\mu,\gamma_\nu\}=2\delta_{\mu\nu}$, $\gamma_\mu^\dagger=\gamma_\mu$, $a\cdot b = \sum_{i=1}^4 a_i b_i$
- P is the total momentum, $M^2 = -P^2$ meson mass[2]
- $G^{\rm eff}_{\mu\nu}(k)$ an “effective gluon propagator” - modeled !

From the gap and BS equations ...

- solutions of the gap equation $\rightarrow$ the <u>dressed</u> quark mass function

$$m_q(p^2) = \frac{B_q(p^2)}{A_q(p^2)}$$

- propagator solutions $A_q(p^2)$ and $B_q(p^2)$ pertain to <u>confined</u> quarks if

$$m_q^2(p^2) \neq -p^2 \quad \text{for real} \quad p^2$$

- The BS solutions $\Gamma_{q\bar{q}'}$ enable the calculation of the properties of $q\bar{q}$ bound states, such as the decay constants of pseudoscalar mesons:

$$\begin{aligned} f_{PS}\, P_\mu &= \langle 0|\bar{q}\, \frac{\lambda^{PS}}{2}\, \gamma_\mu\, \gamma_5\, q|\Phi_{PS}(P)\rangle \\ \longrightarrow \quad f_\pi P_\mu &= N_c\, \mathrm{tr_s} \int \frac{d^4\ell}{(2\pi)^4}\, \gamma_5 \gamma_\mu\, S(\ell + P/2)\, \Gamma_\pi(\ell; P)\, S(\ell - P/2) \end{aligned}$$

Renormalization-group improved interactions

$$\text{Landau gauge gluon propagator}: \quad g^2 G^{\rm eff}_{\mu\nu}(k) = G(-k^2)(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}),$$

$$G(Q^2) \equiv 4\pi \frac{\alpha_s^{\rm eff}(Q^2)}{Q^2} = G_{\sf UV}(Q^2) + G_{\sf IR}(Q^2), \qquad Q^2 \equiv -k^2 \; .$$

$$G_{\sf UV}(Q^2) = 4\pi \frac{\alpha_s^{\rm pert}(Q^2)}{Q^2} \approx \frac{4\pi^2 d}{Q^2 \ln(x_0 + \frac{Q^2}{\Lambda^2_{\sf QCD}})} \{1 + b \, \frac{\ln[\ln(x_0 + \frac{Q^2}{\Lambda^2_{\sf QCD}})]}{\ln(x_0 + \frac{Q^2}{\Lambda^2_{\sf QCD}})}\} \; ,$$

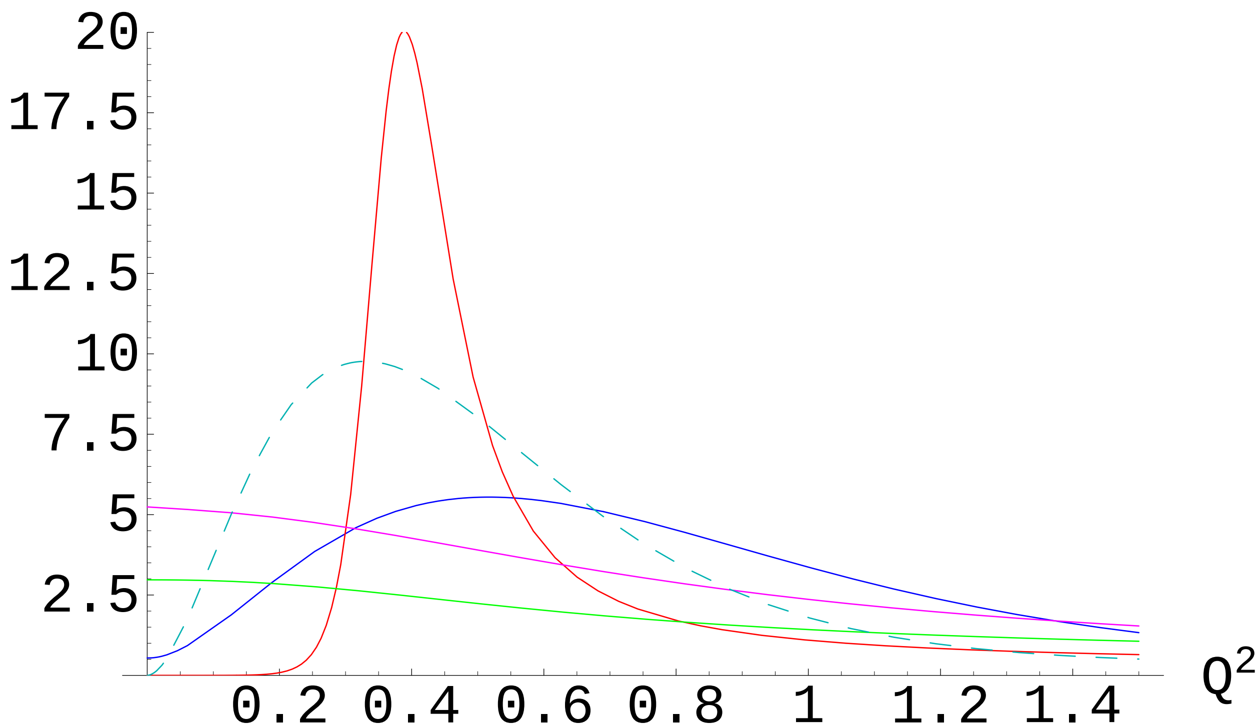
- but modeled non-perturbative part, e.g., Jain & Munczek:

$$G_{\sf IR}(Q^2) = G_{\sf non\text{-}pert}(Q^2) = 4\pi^2 \, a \, Q^2 \; \exp(-\mu Q^2) \quad (\text{similar}: \; \text{Maris, Roberts...})$$

- or, the dressed propagator with dim. 2 gluon condensate $\langle A^2 \rangle$-induced dynamical gluon mass (Kekez & Klabučar):

$$G(Q^2) = 4\pi \frac{\alpha_s^{\rm pert}(Q^2)}{Q^2} \left(\frac{Q^2}{Q^2 - M^2_{\sf gluon} + \frac{c_{\sf ghost}}{Q^2}} \right)^2 \frac{Q^2}{Q^2 + M^2_{\sf gluon} + \frac{c_{\sf gluon}}{Q^2}} \; .$$

Some effective strong couplings $\alpha_s^{\rm eff}(Q^2) \equiv Q^2\, G(Q^2)/4\pi$



- Blue = Munczek & Jain model. Red = K & K propagator with $\langle A^2 \rangle$-induced dynamical gluon mass. Green = Alkofer. Magenta = Bloch. Turquoise dashed: Maris, Roberts & Tandy model. **Important**: integrated IR strength must be sufficient for **DChSB**!

Separable model = good, + easier at $T > 0$

- Calculations simplify with the separable Ansatz for $G^{\rm eff}_{\mu\nu}$:

$$G^{\rm eff}_{\mu\nu}(p-q) \to \delta_{\mu\nu}\, G(p^2, q^2, p\cdot q)$$

$$G(p^2, q^2, p\cdot q) = D_0\ f_0(p^2) f_0(q^2) + D_1\ f_1(p^2)(p\cdot q) f_1(q^2)$$

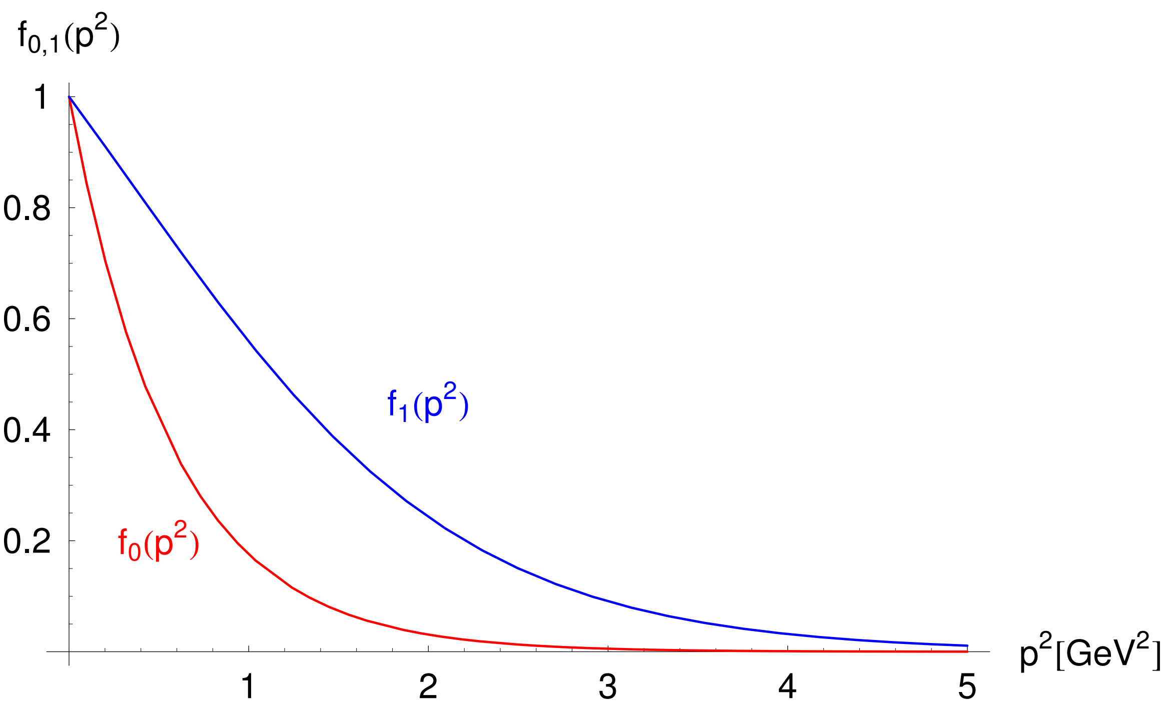
- two strength parameters D_0, D_1, and corresponding form factors $f_i(p^2)$. In the separable model, gap equation yields

$$\begin{aligned} B_f(p^2) &= \widetilde{m}_f + \frac{16}{3}\int \frac{d^4q}{(2\pi)^4} G(p^2, q^2, p\cdot q) \frac{B_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} \\ \left[A_f(p^2) - 1\right] p^2 &= \frac{8}{3}\int \frac{d^4q}{(2\pi)^4} G(p^2, q^2, p\cdot q) \frac{(p\cdot q) A_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} \,. \end{aligned}$$

- This gives $B_f(p^2) = \widetilde{m}_f + b_f\ f_0(p^2)$ and $A_f(p^2) = 1 + a_f\ f_1(p^2)$, reducing to nonlinear equations for constants b_f and a_f.

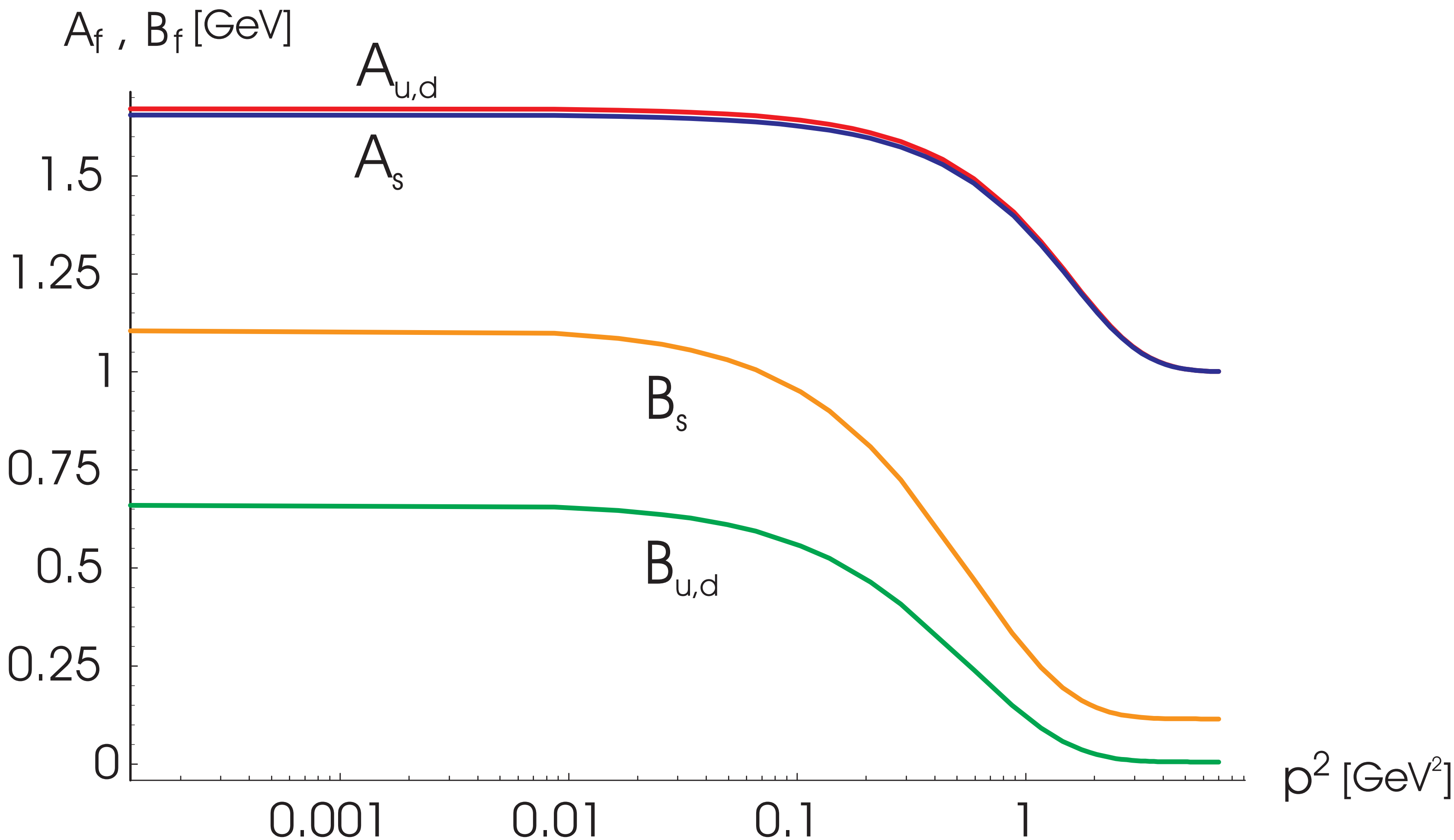
A simple choice for 'interaction form factors' of the separable model:

- $f_0(p^2) = \exp(-p^2/\Lambda_0^2)$
- $f_1(p^2) = [1 + \exp(-p_0^2/\Lambda_1^2)]/[1 + \exp((p^2 - p_0^2))/\Lambda_1^2]$
  gives good description of pseudoscalar properties if the interaction is strong enough for realistic DChSB, when $m_{u,d}(p^2 \sim small) \sim$ the typical constituent quark mass scale $\sim m_\rho/2 \sim m_N/3$.



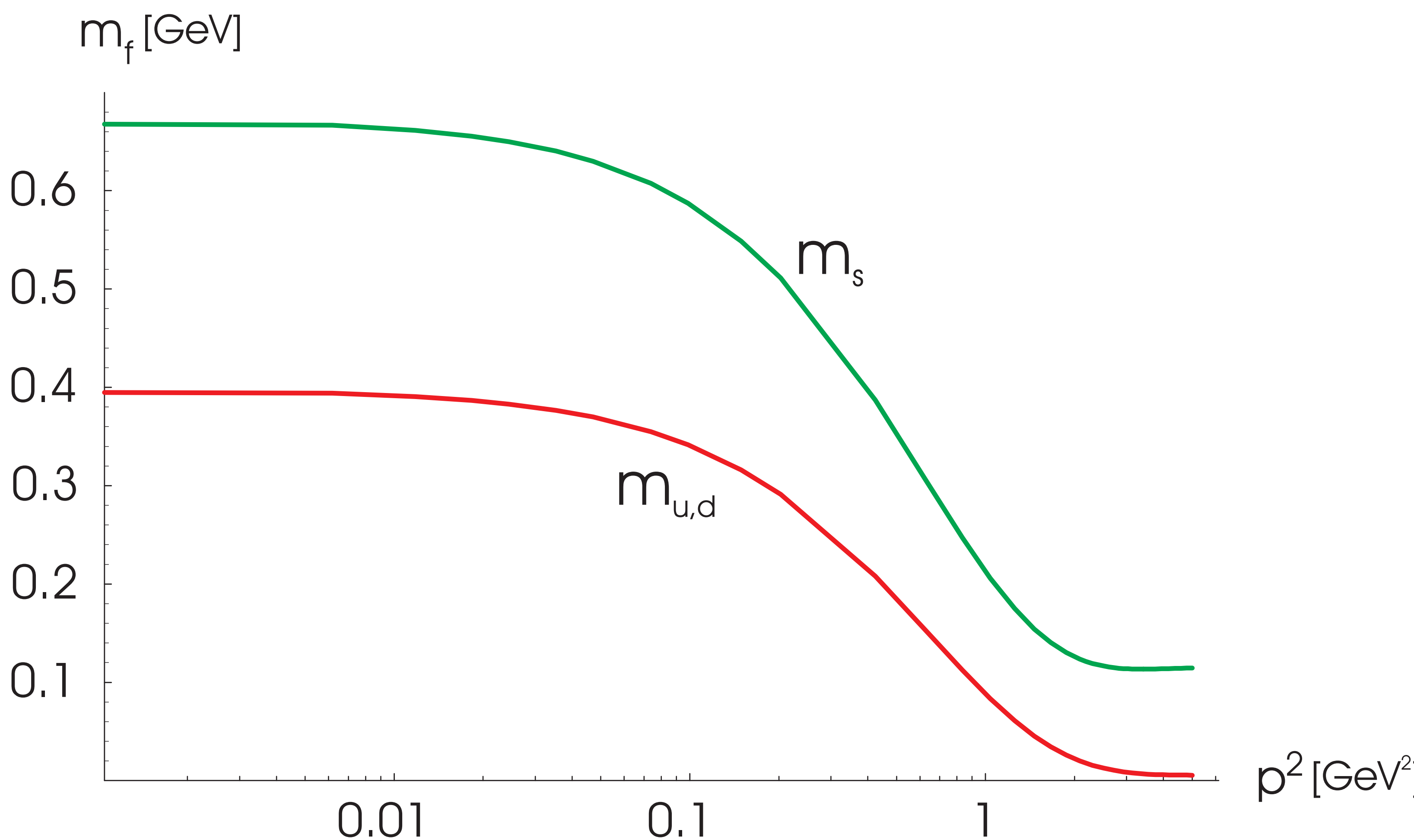
Nonperturbative dynamical propagator dressing

- $\longrightarrow$ Dynamical Chiral Symmetry Breaking (DChSB)



DChSB = nonperturb. generation of large quark masses ...

- ... even in the chiral limit ($\widetilde{m}_f \to 0$), where the octet pseudoscalar mesons are Goldstone bosons of DChSB!



At $T = 0$, good DS results; e.g., "non-anomalous":

- Separable model parameter values reproducing experimental data:
- $\widetilde{m}_{u,d} = 5.5$ MeV, $\Lambda_0 = 758$ MeV, $\Lambda_1 = 961$ MeV, $p_0 = 600$ MeV, $D_0\Lambda_0^2 = 219$, $D_1\Lambda_1^4 = 40$ (fixed by fitting M_π, f_π, M_ρ, $g_{\rho\pi^+\pi^-}$, $g_{\rho e^+e^-}$ $\rightarrow$ pertinent predictions $a_{u,d} = 0.672$, $b_{u,d} = 660$ MeV, i.e., $m_{u,d}(p^2)$, $\langle\bar{u}u\rangle$)
- $\widetilde{m}_s = 115$ MeV (fixed by fitting $M_K \rightarrow$ predictions $a_s = 0.657$, $b_s = 998$ MeV, i.e., $m_s(p^2)$, $\langle\bar{s}s\rangle$, $M_{s\bar{s}}$, f_K, $f_{s\bar{s}}$)
- Summary of results (all in GeV) for $q = u, d, s$ and pseudoscalar mesons without the influence of gluon anomaly:

PS	M_{PS}	M_{PS}^{exp}	f_{PS}	f_{PS}^{exp}	$m_q(0)$	$-\langle q\bar{q}\rangle_0^{1/3}$
π	0.140	0.1396	0.092	0.0924 ± 0.0003	0.398	0.217
K	0.495	0.4937	0.110	0.1130 ± 0.0010		
$s\bar{s}$	0.685		0.119		0.672	

Extension to $T \neq 0$

- At $T \neq 0$, the quark 4-momentum $p \longrightarrow p_n = (\omega_n, \vec{p})$, where $\omega_n = (2n+1)\pi T$ are the discrete ($n = 0, \pm 1, \pm 2, \pm 3, \dots$) Matsubara frequencies, so that $p_n^2 = \omega_n^2 + \vec{p}^2$.
- Gap equation solution for the dressed quark propagator

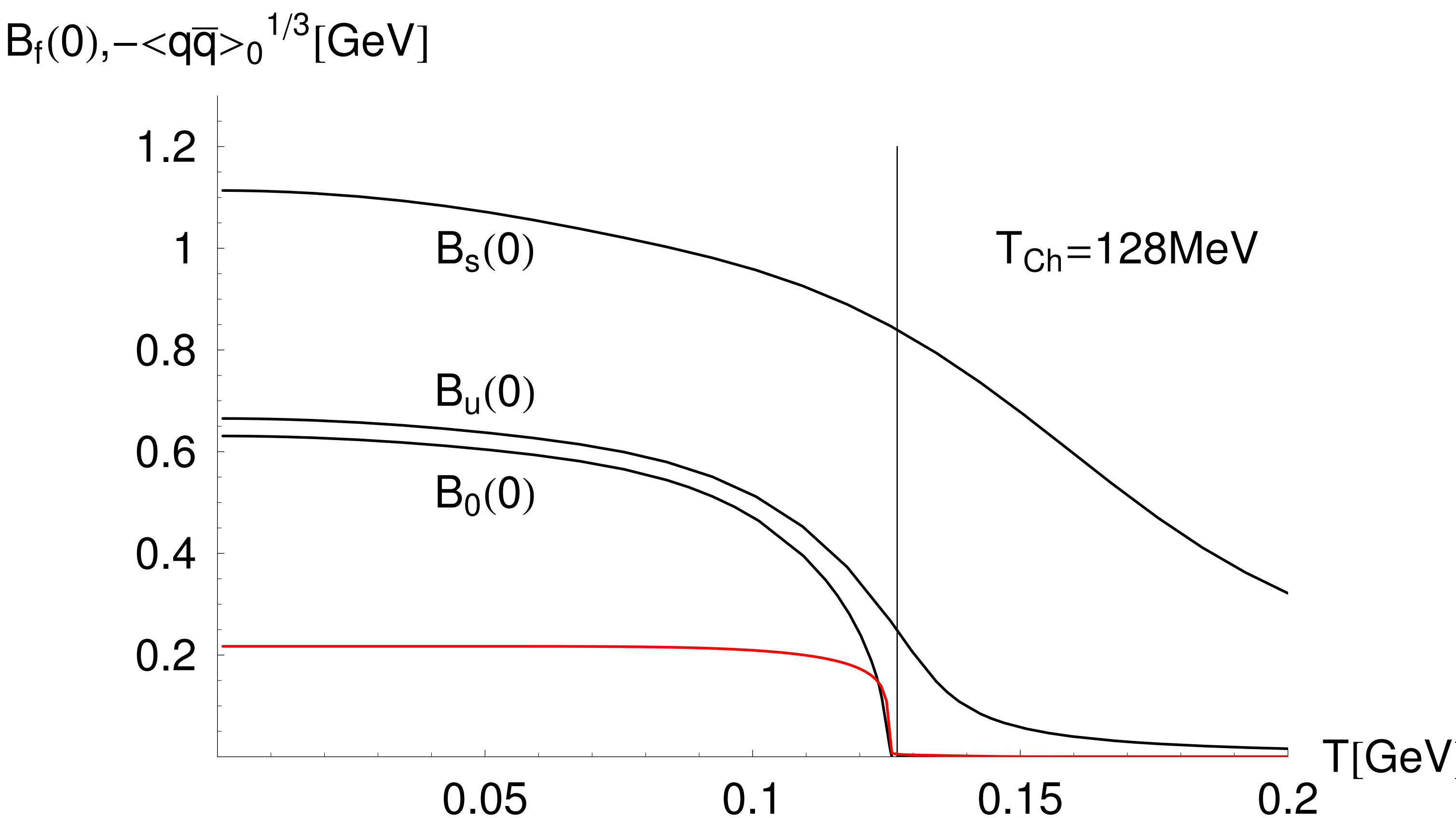
$$S_f(p_n, T) = [i\vec{\gamma}\cdot\vec{p}\, A_f(p_n^2, T) + i\gamma_4\omega_n\, C_f(p_n^2, T) + B_f(p_n^2, T)]^{-1}$$

$$= \frac{-i\vec{\gamma}\cdot\vec{p}\, A_f(p_n^2, T) - i\gamma_4\omega_n\, C_f(p_n^2, T) + B_f(p_n^2, T)}{\vec{p}^2\, A_f^2(p_n^2, T) + \omega_n^2\, C_f^2(p_n^2, T) + B_f^2(p_n^2, T)}\,.$$

- There are now three amplitudes due to the loss of $O(4)$ symmetry, and at sufficently high $T \geq T_d$ denominator CAN vanish. $\longrightarrow$ For $T \geq T_d$ quarks can be deconfined!

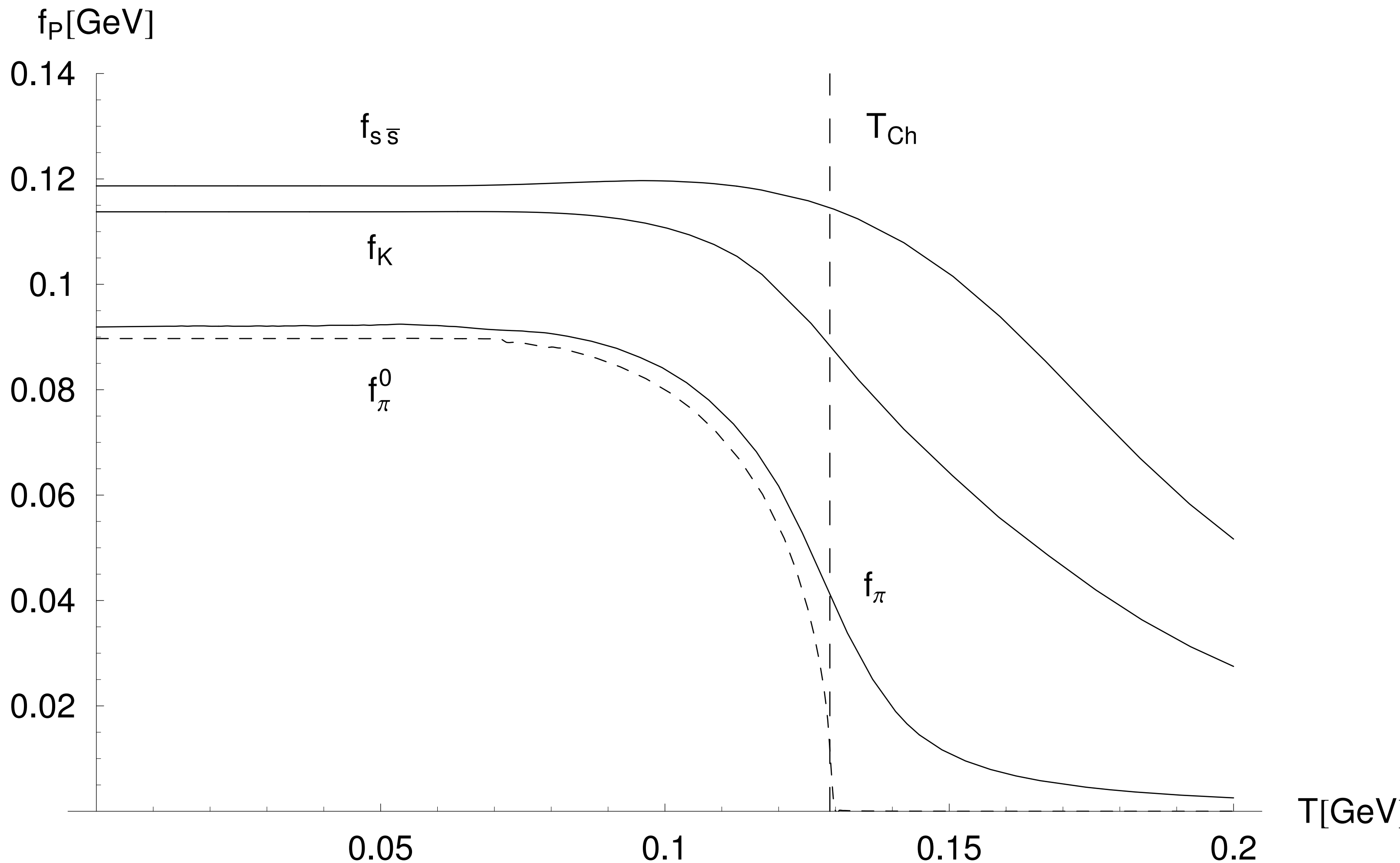
At $T > 0$, good and less good DS results

E.g., chiral symmetry restoration qualitatively good, but $T_{\rm Ch}$ lower than lattice (maybe up to 35%, and even more for 'more realistic' DS models unless they contain δ-function):



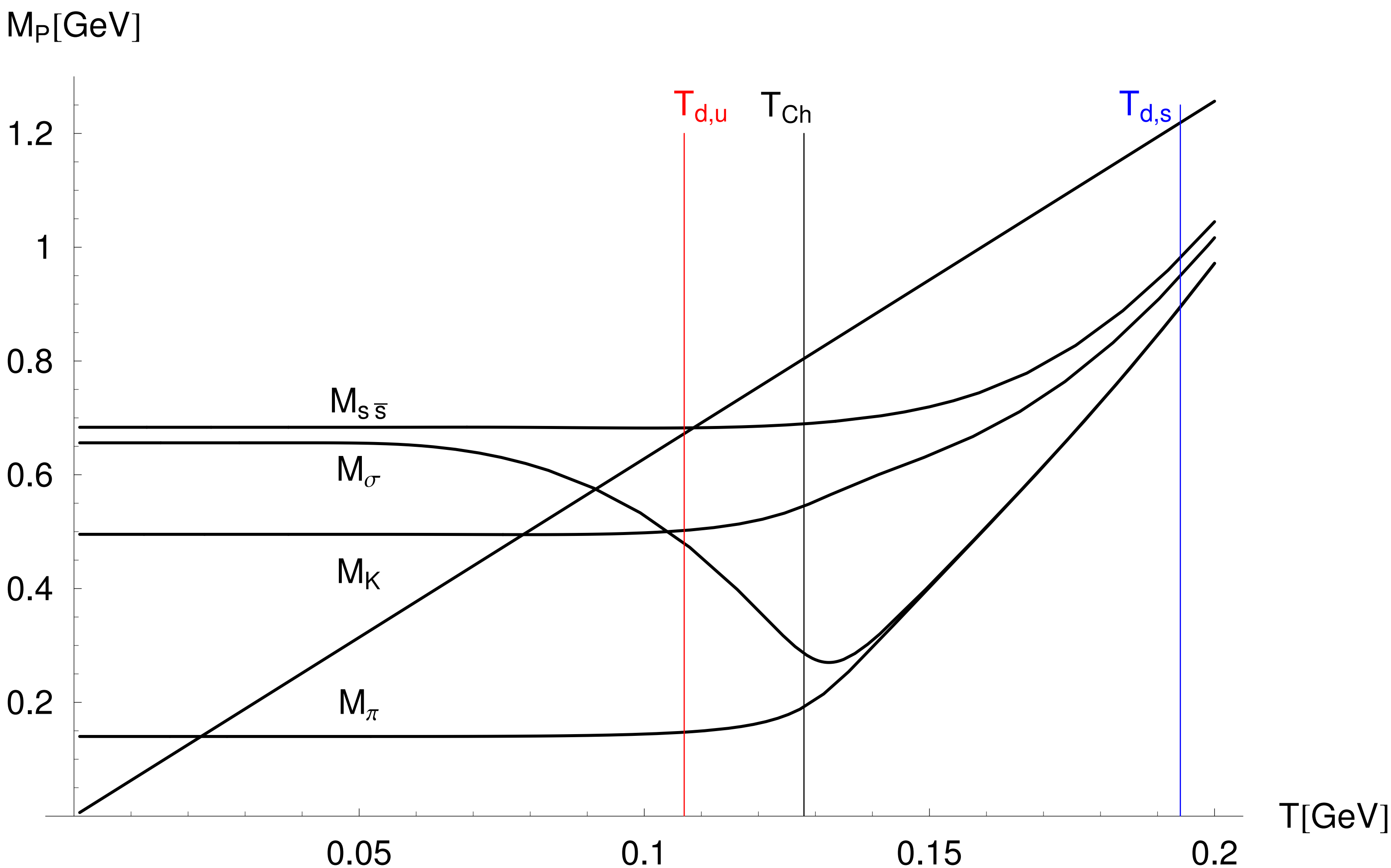
Same with pseudoscalar decay constants $f_P(T)$:

Both crossover and Ch-limit behavior OK, but
$T_{\rm Ch} = 128$ MeV ... cured by Polyakov loop (PL $\rightarrow T_{\rm Ch} = 195$ MeV)



Similarly with the T-dependence of $\pi, K, s\bar{s}, \sigma$ masses:

'Deconfinement' $T_{d,q}$ from S_q pole - very different $T_{d,u}$, $T_{d,s}$... should also be cured/synchronized with $T_{\rm Ch} (= T_{cri})$ by PL



Anomaly and mixing in $\eta - \eta'$ complex

- present approach yields mass^2 eigenvalues
$M^2_{u\bar{d}} = M^2_{\pi^+}, M^2_{u\bar{s}} = M^2_{K}, ..., \hat{M}^2_{NA} = \mathrm{diag}(M^2_{u\bar{u}}, M^2_{d\bar{d}}, M^2_{s\bar{s}})$
- $|u\bar{d}\rangle = |\pi^+\rangle, |u\bar{s}\rangle = |K^+\rangle$, ... but $|u\bar{u}\rangle$, $|d\bar{d}\rangle$ and $|s\bar{s}\rangle$ do not correspond to any physical particles (at $T = 0$ at least!), although in the isospin limit (adopted from now on) $M_{u\bar{u}} = M_{d\bar{d}} = M_{u\bar{d}} = M_{\pi}$. I is a good quantum number!
- $\longrightarrow$ recouple into the familiar $I_3 = 0$ octet-singlet basis

$$
\begin{aligned}
|\pi^0\rangle &= \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle) \ , \\
|\eta_8\rangle &= \frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle) \ , \\
|\eta_0\rangle &= \frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle) \ .
\end{aligned}
$$

Anomaly and mixing in $\eta - \eta'$ complex

- the "non-anomalous" (chiral-limit-vanishing!) part of the mass-squared matrix of π^0 and η's is (in π^0-η_8-η_0 basis)

$$\hat{M}^2_{NA} = \begin{pmatrix} M^2_\pi & 0 & 0 \\ 0 & M^2_{88} & M^2_{80} \\ 0 & M^2_{08} & M^2_{00} \end{pmatrix}$$

$$M^2_{88} \equiv \langle \eta_8 | \hat{M}^2_{NA} | \eta_8 \rangle \equiv M^2_{\eta_8} = \frac{2}{3}(M^2_{s\bar{s}} + \frac{1}{2} M^2_\pi),$$

$$M^2_{80} \equiv \langle \eta_8 | \hat{M}^2_{NA} | \eta_0 \rangle = M^2_{08} = \frac{\sqrt{2}}{3}(M^2_\pi - M^2_{s\bar{s}})$$

$$M^2_{00} \equiv \langle \eta_0 | \hat{M}^2_{NA} | \eta_0 \rangle = \frac{2}{3}(\frac{1}{2} M^2_{s\bar{s}} + M^2_\pi),$$

- in order to avoid the U$_A$(1) problem, U$_A$(1) symmetry must ultimately be broken by gluon anomaly at least at the level of the masses

Anomaly and mixing in $\eta - \eta'$ complex

- All masses in $\hat{M}^2_{NA}$ are calculated in the ladder approx., which cannot include the gluon anomaly!
- Large N_c: the gluon anomaly suppressed as $1/N_c$! $\to$ Include its effect just at the level of masses: break the $U_A(1)$ symmetry and avoid the $U_A(1)$ problem by shifting the η_0 (squared) mass by anomalous contribution 3β.
- complete mass matrix is then $\hat{M}^2 = \hat{M}^2_{NA} + \hat{M}^2_{A}$ where

$$\hat{M}^2_A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3\beta \end{pmatrix} \quad \text{does not vanish in the chiral limit.}$$

- 3β, the anomalous mass of η_0, is related to the topological susceptibility of the vacuum. It is fixed by phenomenology or taken from the lattice calculations.

Anomaly and mixing in $\eta - \eta'$ complex

- we can also rewrite $\hat{M}_A^2$ in the $q\bar{q}$ basis $|u\bar{u}\rangle$, $|d\bar{d}\rangle$, $|s\bar{s}\rangle$

$$\hat{M}_A^2 = \beta \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow[\text{breaking}]{\text{flavor}} \hat{M}_A^2 = \beta \begin{pmatrix} 1 & 1 & X \\ 1 & 1 & X \\ X & X & X^2 \end{pmatrix}$$

- We introduced the effects of the flavor breaking on the anomaly-induced transitions $|q\bar{q}\rangle \rightarrow |q'\bar{q}'\rangle$ $(q, q' = u, d, s)$. $s\bar{s}$ transition suppression estimated by $X \approx f_\pi / f_{s\bar{s}}$.
- Then, $\hat{M}_A^2$ in the octet-singlet basis is modified to

$$\hat{M}_A^2 = \beta \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{2}{3}(1-X)^2 & \frac{\sqrt{2}}{3}(2-X-X^2) \\ 0 & \frac{\sqrt{2}}{3}(2-X-X^2) & \frac{1}{3}(2+X)^2 \end{pmatrix}$$

- $\rightarrow$ In the isospin limit, one can always restrict to 2×2 submatrix of etas

Anomaly and mixing in $\eta - \eta'$ complex

- nonstrange (*NS*) – strange (*S*) basis

$$
\begin{aligned}
|\eta_{NS}\rangle &= \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle) = \frac{1}{\sqrt{3}}|\eta_8\rangle + \sqrt{\frac{2}{3}}|\eta_0\rangle \, , \\
|\eta_S\rangle &= |s\bar{s}\rangle = -\sqrt{\frac{2}{3}}|\eta_8\rangle + \frac{1}{\sqrt{3}}|\eta_0\rangle \, .
\end{aligned}
$$

- the η–η' matrix in this basis is

$$
\hat{M}^2 = \begin{pmatrix} M^2_{\eta_{NS}} & M^2_{\eta_S \eta_{NS}} \\ M^2_{\eta_{NS}\eta_S} & M^2_{\eta_S} \end{pmatrix} = \begin{pmatrix} M^2_{u\bar{u}} + 2\beta & \sqrt{2}\beta X \\ \sqrt{2}\beta X & M^2_{s\bar{s}} + \beta X^2 \end{pmatrix} \xrightarrow{\phi} \begin{pmatrix} m^2_\eta & 0 \\ 0 & m^2_{\eta'} \end{pmatrix}
$$

- *NS*–*S* mixing relations

$$
|\eta\rangle = \cos\phi |\eta_{NS}\rangle - \sin\phi|\eta_S\rangle \, , \quad |\eta'\rangle = \sin\phi|\eta_{NS}\rangle + \cos\phi|\eta_S\rangle \, .
$$

$$
\theta = \phi - \arctan\sqrt{2}
$$

Anomaly and mixing in $\eta - \eta'$ complex

- Let lowercase m_M's denote the empirical mass of meson M. From our calculated, model mass matrix in *NS*–*S* basis, we form its empirical counterpart $\hat{m}^2_{\rm exp}$ by
- *i)* obvious substitutions $M_{u\bar{u}} \equiv M_\pi \to m_\pi$, $M_{s\bar{s}} \to m_{s\bar{s}}$
- *ii)* by noting that $m_{s\bar{s}}$, the “empirical" mass of the unphysical $s\bar{s}$ pseudoscalar bound state, is given in terms of masses of physical particles as $m^2_{s\bar{s}} \approx 2m^2_K - m^2_\pi$ due to GMOR. Then,

$$\hat{m}^2_{\rm exp} = \left[\begin{array}{cc} m^2_\pi + 2\beta & \sqrt{2}\beta X \\ \sqrt{2}\beta X & 2m^2_K - m^2_\pi + \beta X^2 \end{array} \right] \xrightarrow[\phi_{\rm exp}]{} \left[\begin{array}{cc} m^2_\eta & 0 \\ 0 & m^2_{\eta'} \end{array} \right].$$

Finally, fix anomalous contribution to η-η':

- requiring that the experimental trace $(m_\eta^2 + m_{\eta'}^2)_{exp} \approx 1.22$ GeV^2 be reproduced by the theoretical $\hat{M}^2$, yields $\beta_{\rm fit} = \frac{1}{2+X^2}[(m_\eta^2 + m_{\eta'}^2)_{exp} - (M_{u\bar{u}}^2 + M_{s\bar{s}}^2)]$
- But better get β from lattice χ! Then no free parameters!
- the trace of the empirical $\hat{m}^2_{\rm exp}$ demands the 1^{st} equality in

$$\beta(2+X^2) = m_\eta^2 + m_{\eta'}^2 - 2m_K^2 = \frac{2N_f}{f_\pi^2}\chi_{\rm YM} \quad (2^{\rm nd}\text{equality} = \text{WV relation})$$

- then, the $NS - S$ mixing angle ϕ

$$\tan 2\phi = \frac{2\, M^2_{\eta_S\eta_{NS}}}{M^2_{\eta_S} - M^2_{\eta_{NS}}} = \frac{2\sqrt{2}\beta X}{M^2_{\eta_S} - M^2_{\eta_{NS}}},$$

$$M^2_{\eta_{NS}} = M^2_{u\bar{u}} + 2\beta = M^2_\pi + 2\beta, \quad M^2_{\eta_S} = M^2_{s\bar{s}} + \beta X^2 = M^2_{s\bar{s}} + \beta\frac{f_\pi^2}{f_{s\bar{s}}^2}$$

Anomaly and mixing in $\eta - \eta'$ complex

- The diagonalization of the $NS - S$ mass matrix then finally gives us the *calculated* η and η' masses:

$$
\begin{aligned}
M_\eta^2 &= \cos^2\phi\, M_{\eta_{NS}}^2 - \sqrt{2}\beta X \sin 2\phi + \sin^2\phi\, M_{\eta_S}^2 \\
M_{\eta'}^2 &= \sin^2\phi\, M_{\eta_{NS}}^2 + \sqrt{2}\beta X \sin 2\phi + \cos^2\phi\, M_{\eta_S}^2
\end{aligned}
$$

- Equivalently, from the secular determinant,

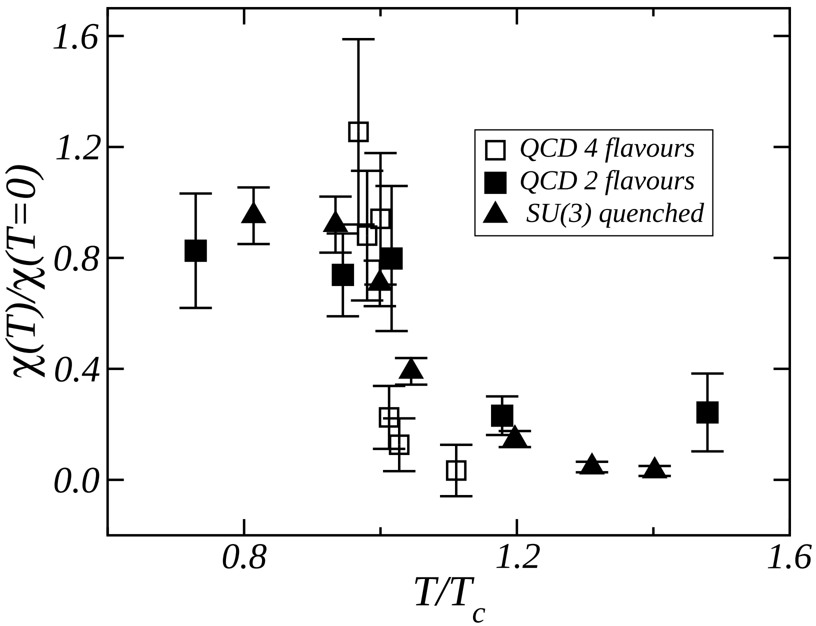
$$
\begin{aligned}
M_\eta^2 &= \frac{1}{2}\left[M_{\eta_{NS}}^2 + M_{\eta_S}^2 - \sqrt{(M_{\eta_{NS}}^2 - M_{\eta_S}^2)^2 + 8\beta^2 X^2} \right] \\
&= \frac{1}{2}\left[M_\pi^2 + M_{s\bar{s}}^2 + \beta(2+X^2) - \sqrt{(M_\pi^2 + 2\beta - M_{s\bar{s}}^2 - \beta X^2)^2 + 8\beta^2 X^2} \right] \\
M_{\eta'}^2 &= \frac{1}{2}\left[M_{\eta_{NS}}^2 + M_{\eta_S}^2 + \sqrt{(M_{\eta_{NS}}^2 - M_{\eta_S}^2)^2 + 8\beta^2 X^2} \right] \\
&= \frac{1}{2}\left[M_\pi^2 + M_{s\bar{s}}^2 + \beta(2+X^2) + \sqrt{(M_\pi^2 + 2\beta - M_{s\bar{s}}^2 - \beta X^2)^2 + 8\beta^2 X^2} \right]
\end{aligned}
$$

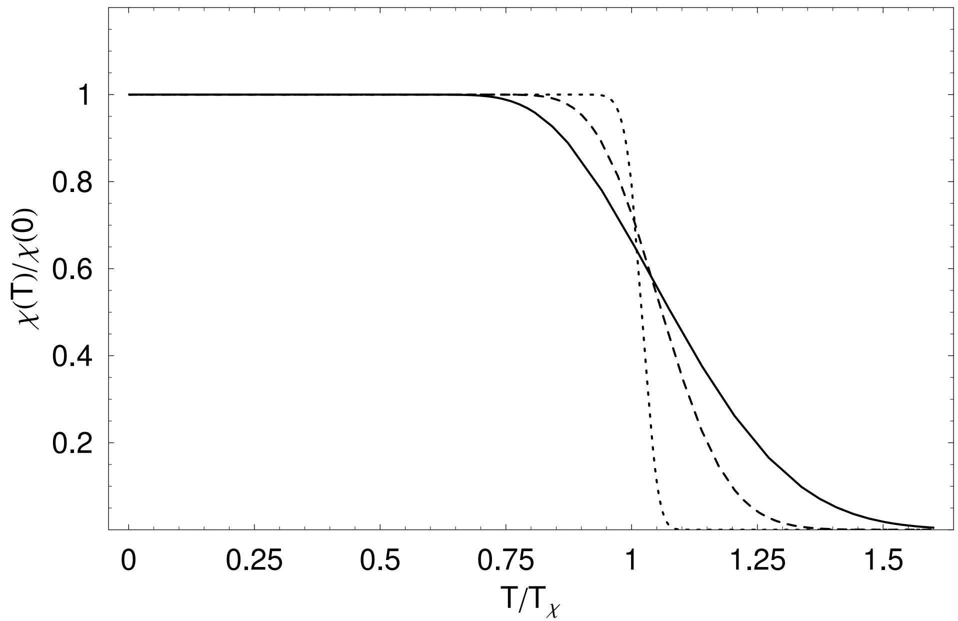
Separable model results on η and η' mesons (at $T = 0$)

	$\beta_{\rm fit}$	$\beta_{\rm latt.}$	Exp.
θ	-12.22°	-13.92°	
M_η	548.9	543.1	547.75
$M_{\eta'}$	958.5	932.5	957.78
X	0.772	0.772	
3β	0.845	0.781	

- masses are in units of MeV, 3β in units of GeV^2 and the mixing angles are dimensionless.
- $\beta_{\rm latt.}$ was obtained from $\chi_{\rm YM}(T = 0) = (175.7\ {\rm MeV})^4$
- $X = f_\pi / f_{s\bar{s}}$ as well as the whole $\hat{M}^2_{NA}$ (consisting of M_π and $M_{s\bar{s}}$) are calculated model quantities.

χ, topological susceptibility of QCD vacuum, at $T > 0$



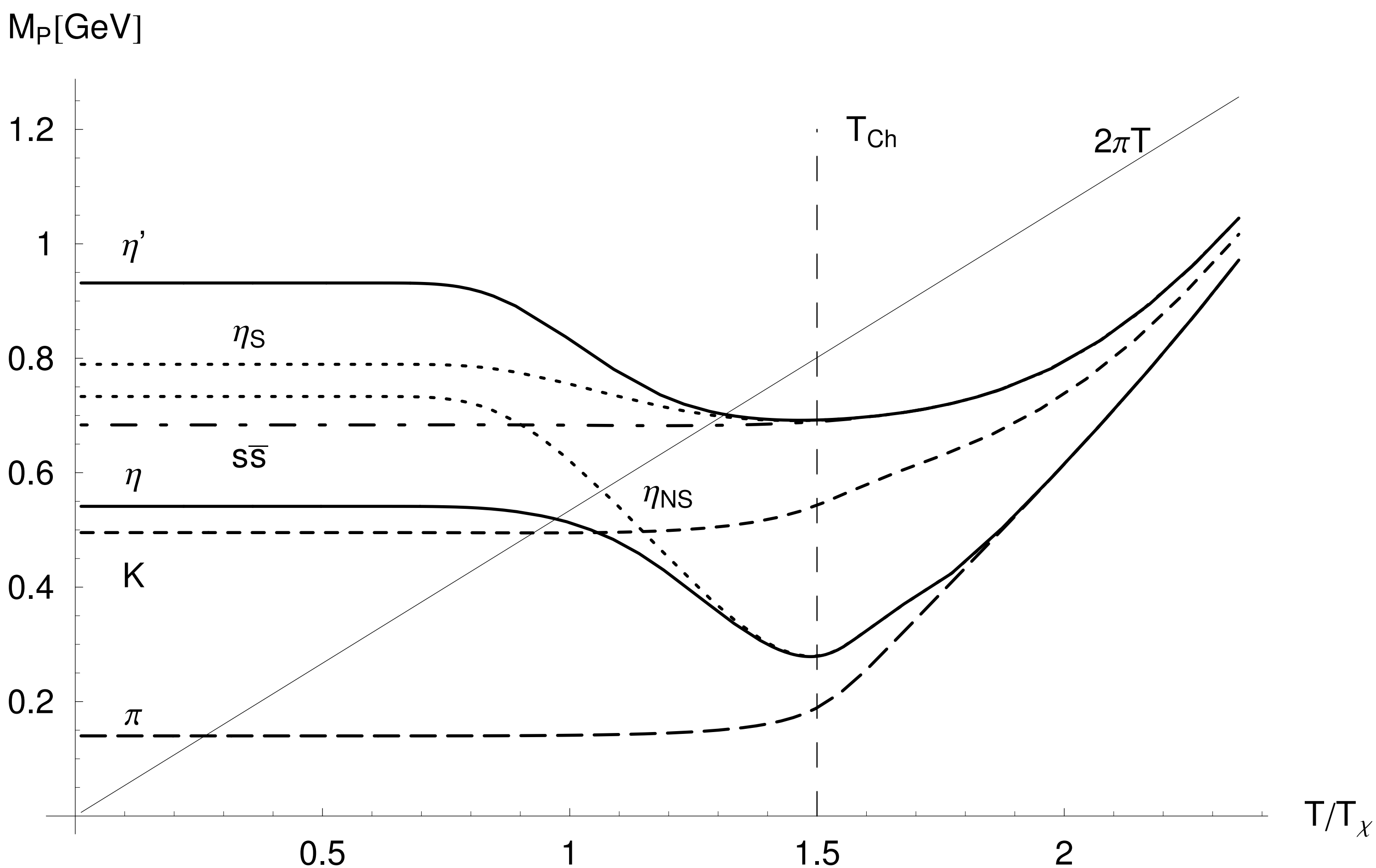


$$\chi = \int d^4x \, \langle q(x) q(0) \rangle \, , \qquad q(x) = \frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x)$$

- $q(x)$ = topological charge density operator

Relative temperature (T/T_χ) dependence of meson masses

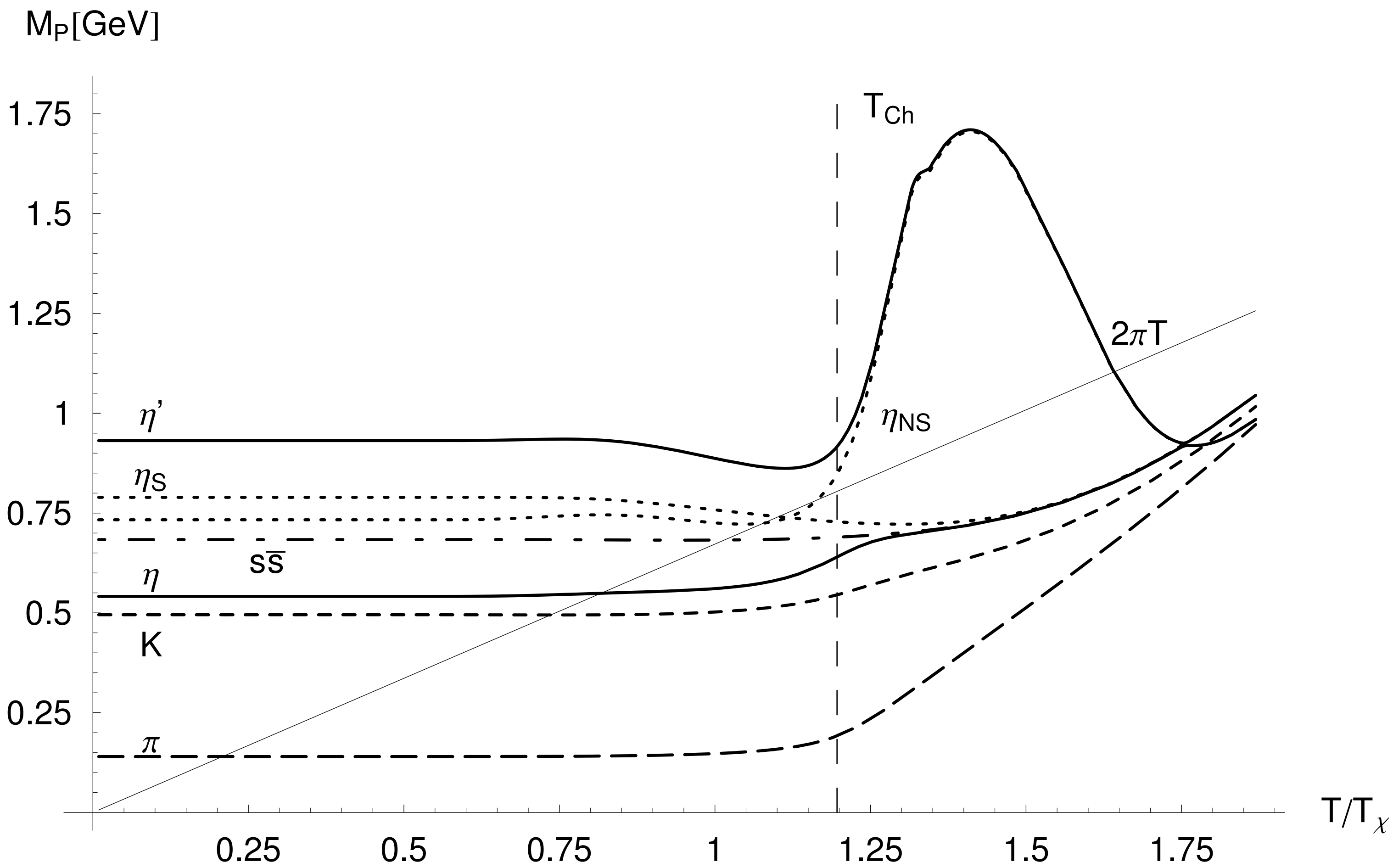
$\chi = \chi_{\rm YM} \rightarrow$ Pisarski-Wilczek scenario only for very unrealistically low $T_\chi/T_{\rm Ch}$ ratio



Extremely low $T_\chi = 2/3\, T_{\rm Ch}$ $(T_{\rm Ch} = 1.50\, T_\chi)$

Relative temperature (T/T_χ) dependence of meson masses

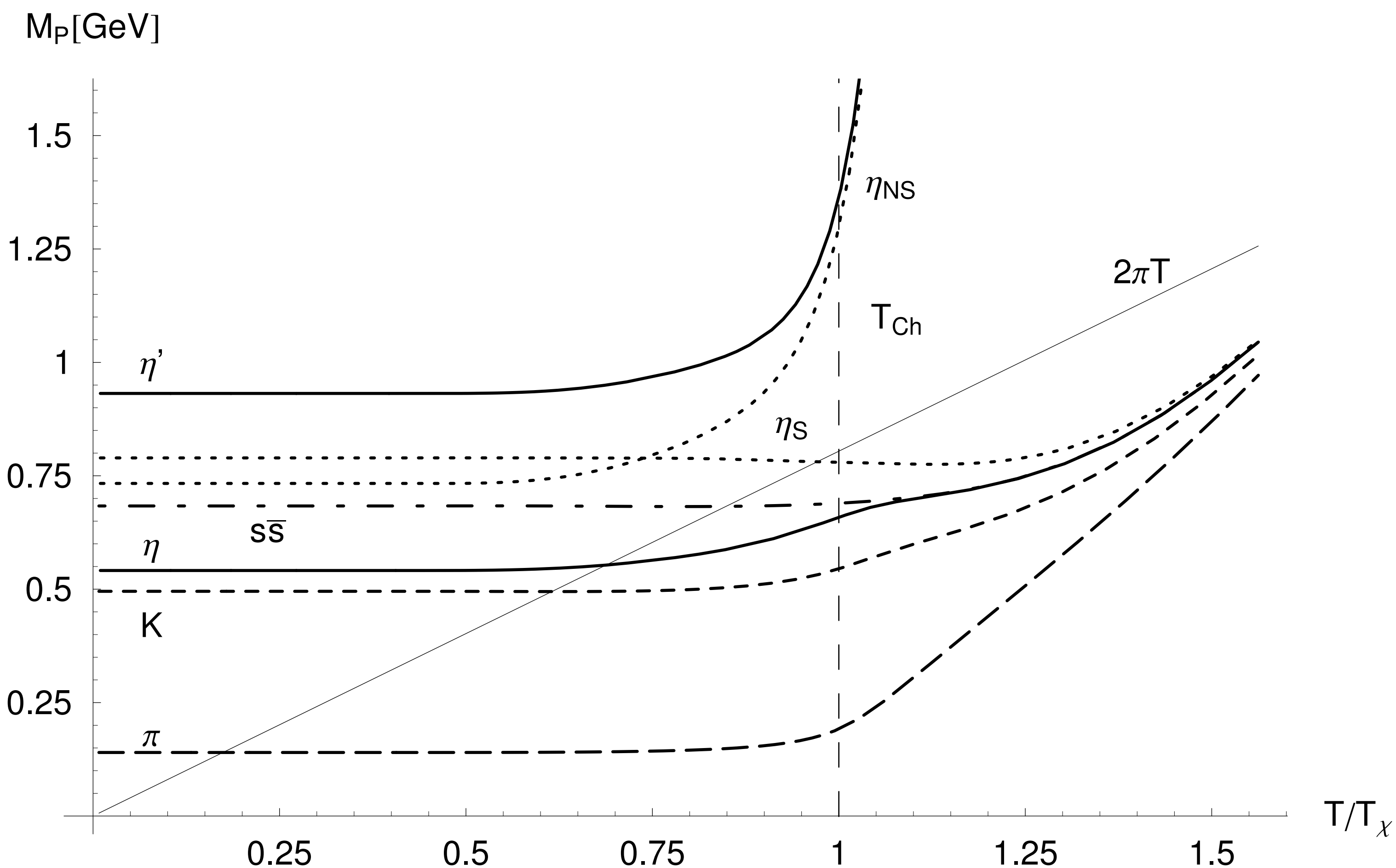
$\chi = \chi_{\rm YM}$ implies η' mass increase $\rightarrow$ suppression of the η' multiplicity already for still unrealistically low topological susceptibility-melting temperature $T_\chi \gtrsim 0.8\, T_{\rm Ch}$



Still unrealistically low $T_\chi = 0.836\, T_{\rm Ch}$ $(T_{\rm Ch} = 1.20\, T_\chi)$

Relative temperature (T/T_χ) dependence of meson masses

$\chi = \chi_{\rm YM}$ implies a huge η' mass increase, to 5 GeV for $T_\chi = T_{\rm Ch}$, and even more for $T_\chi > T_{\rm Ch} \rightarrow$ total suppression of the η' multiplicity = signature of Ch symm. restoration **OR** of the failure of WV relation



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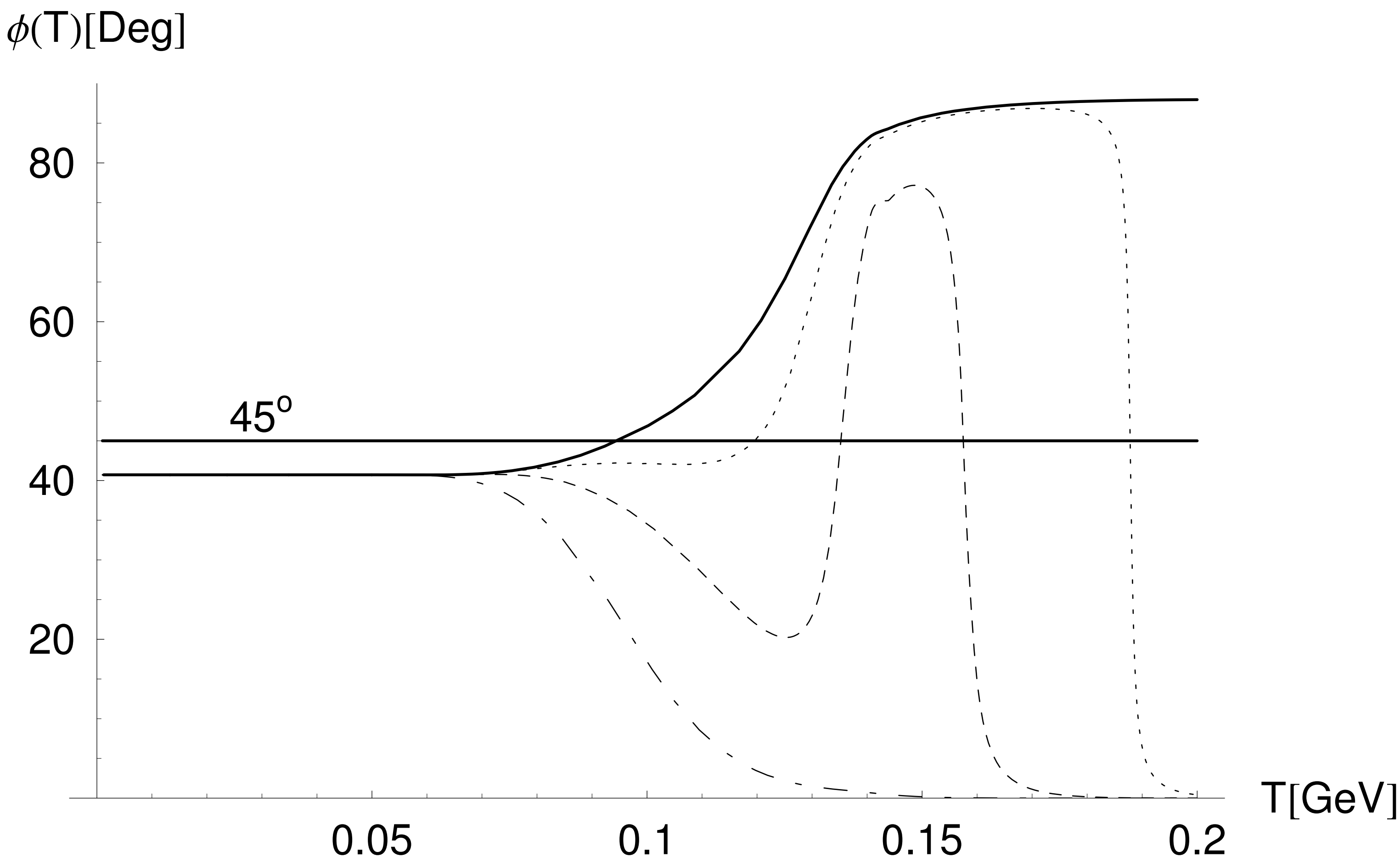
The T-dependence of the NS-S mixing angle ϕ for $\chi_{\rm YM}$



$\phi(T)$ for $T_\chi = 2/3\, T_{\rm Ch}$ (dash-dotted curve), $T_\chi = 0.758\, T_{\rm Ch}$ (dashed curve), $T_\chi = 0.836\, T_{\rm Ch}$ (dotted curve), and $T_\chi = T_{\rm Ch}$ (solid curve).

Shore's generalization of WV – valid to all orders in $1/N_c$

- Inclusion of gluon anomaly in DGMOR relations $\rightarrow$

$$(f^{0\eta'})^2 m_{\eta'}^2 + (f^{0\eta})^2 m_\eta^2 = \frac{1}{3}\left(f_\pi^2 m_\pi^2 + 2 f_K^2 m_K^2\right) + 6A \quad (1)$$

$$f^{0\eta'} f^{8\eta'} m_{\eta'}^2 + f^{0\eta} f^{8\eta} m_\eta^2 = \frac{2\sqrt{2}}{3}\left(f_\pi^2 m_\pi^2 - f_K^2 m_K^2\right) \quad (2)$$

$$(f^{8\eta'})^2 m_{\eta'}^2 + (f^{8\eta})^2 m_\eta^2 = -\frac{1}{3}\left(f_\pi^2 m_\pi^2 - 4 f_K^2 m_K^2\right) \quad (3)$$

$A = \chi_{\mathrm{YM}} + \mathcal{O}(\frac{1}{N_c})$ = full QCD topological charge.

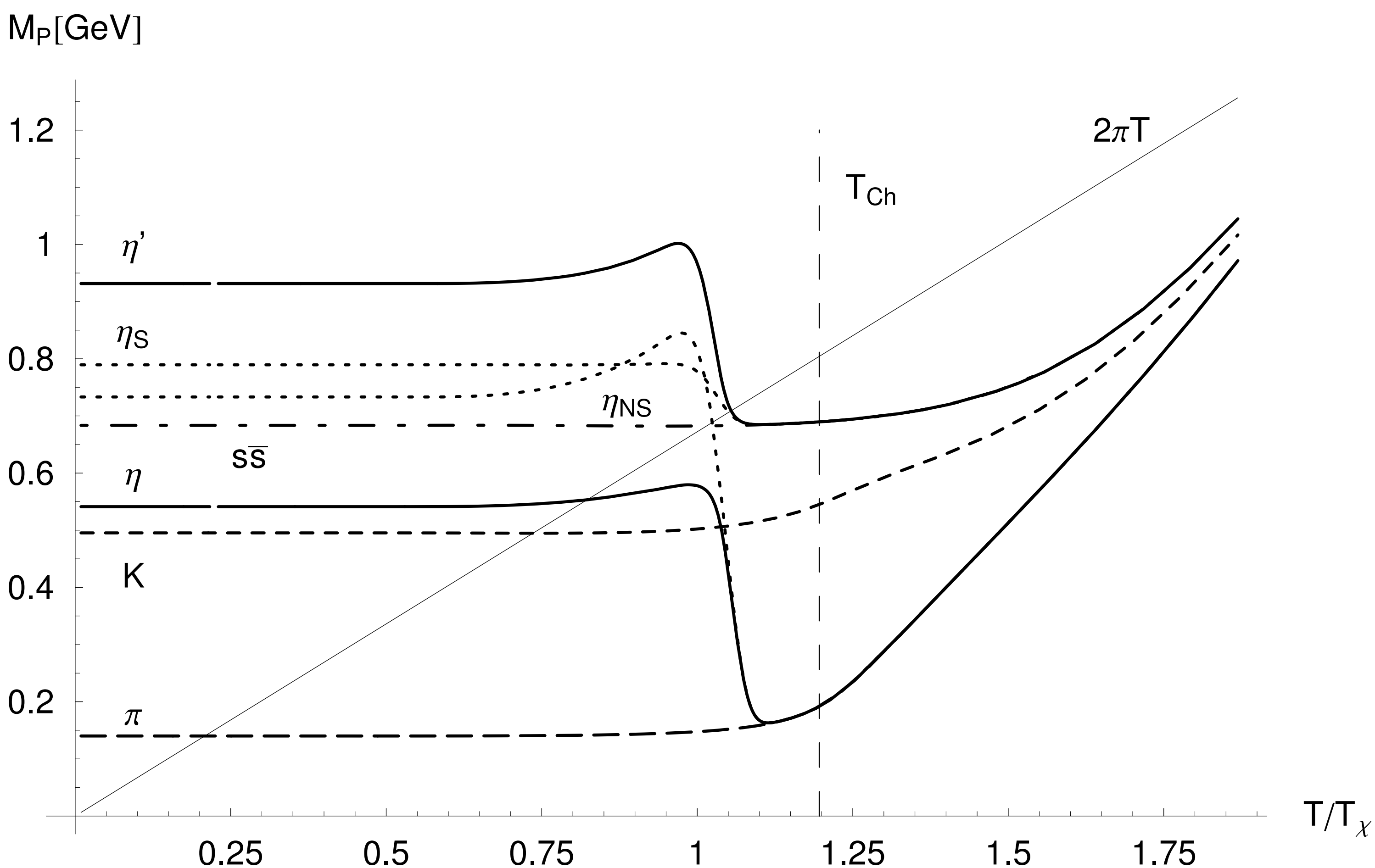
(1)+(3)$\rightarrow$

$$(f^{0\eta'})^2 m_{\eta'}^2 + (f^{0\eta})^2 m_\eta^2 + (f^{8\eta})^2 m_\eta^2 + (f^{8\eta'})^2 m_{\eta'}^2 - 2 f_K^2 m_K^2 = 6A$$

- Then, large N_c limit and $f^{0\eta}, f^{8\eta'} \rightarrow 0$ as well as $f^{0\eta'}, f^{8\eta}, f_K \rightarrow f_\pi$ recovers the standard WV.

M_P for 4-flavor QCD topological susceptibility

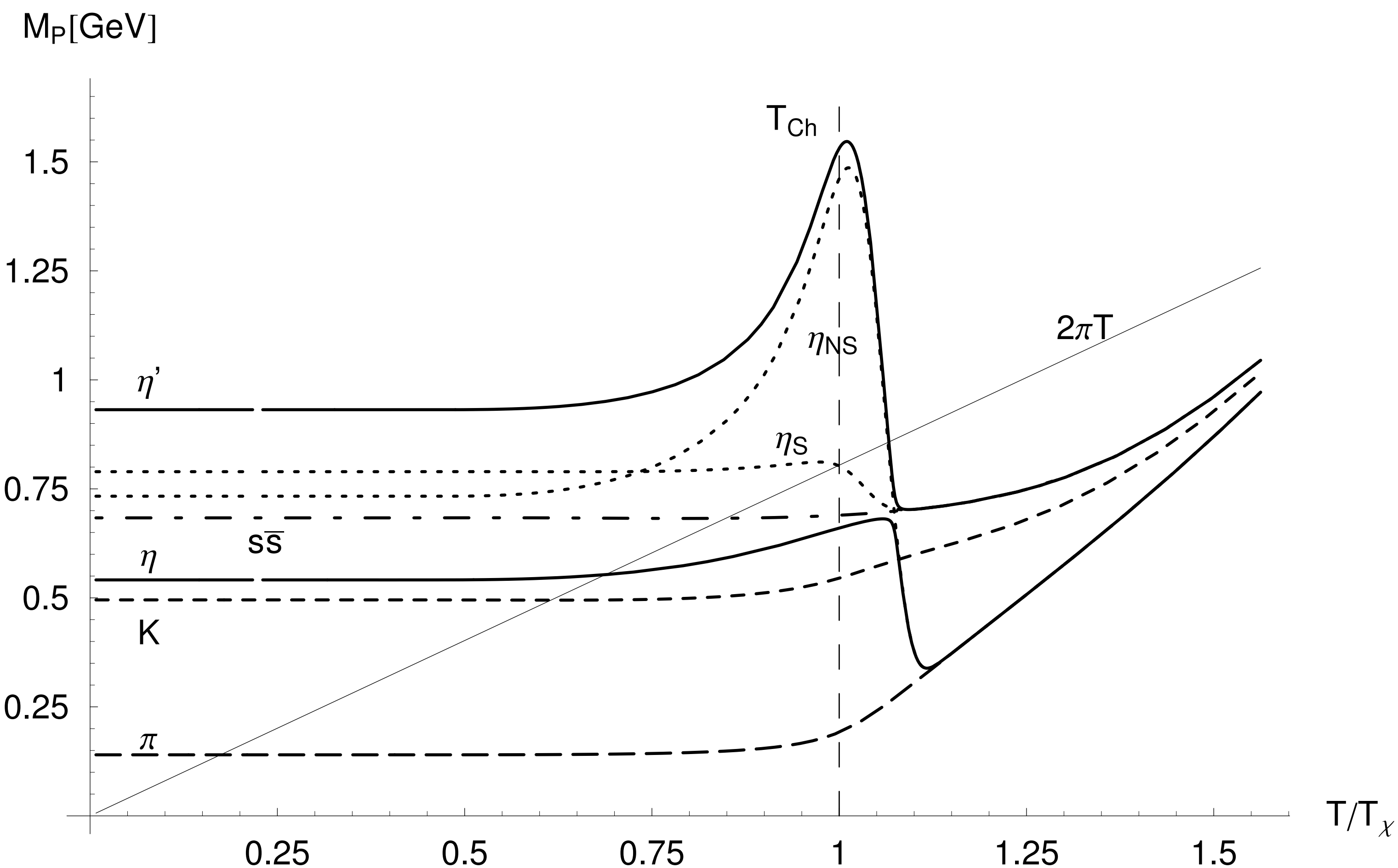
χ_{4fQCD} melts faster than χ_{YM}. Thus, $\chi = \chi_{4fQCD}$ enables η and η' multiplicity increase ('Pisarski-Wilczek scenario') at higher $T_\chi/T_{\rm Ch}$



Still unrealistically low $T_\chi = 0.836\, T_{\rm Ch}$ $\quad (T_{\rm Ch} = 1.20\, T_\chi)$

M_P for 4-flavor QCD topological susceptibility

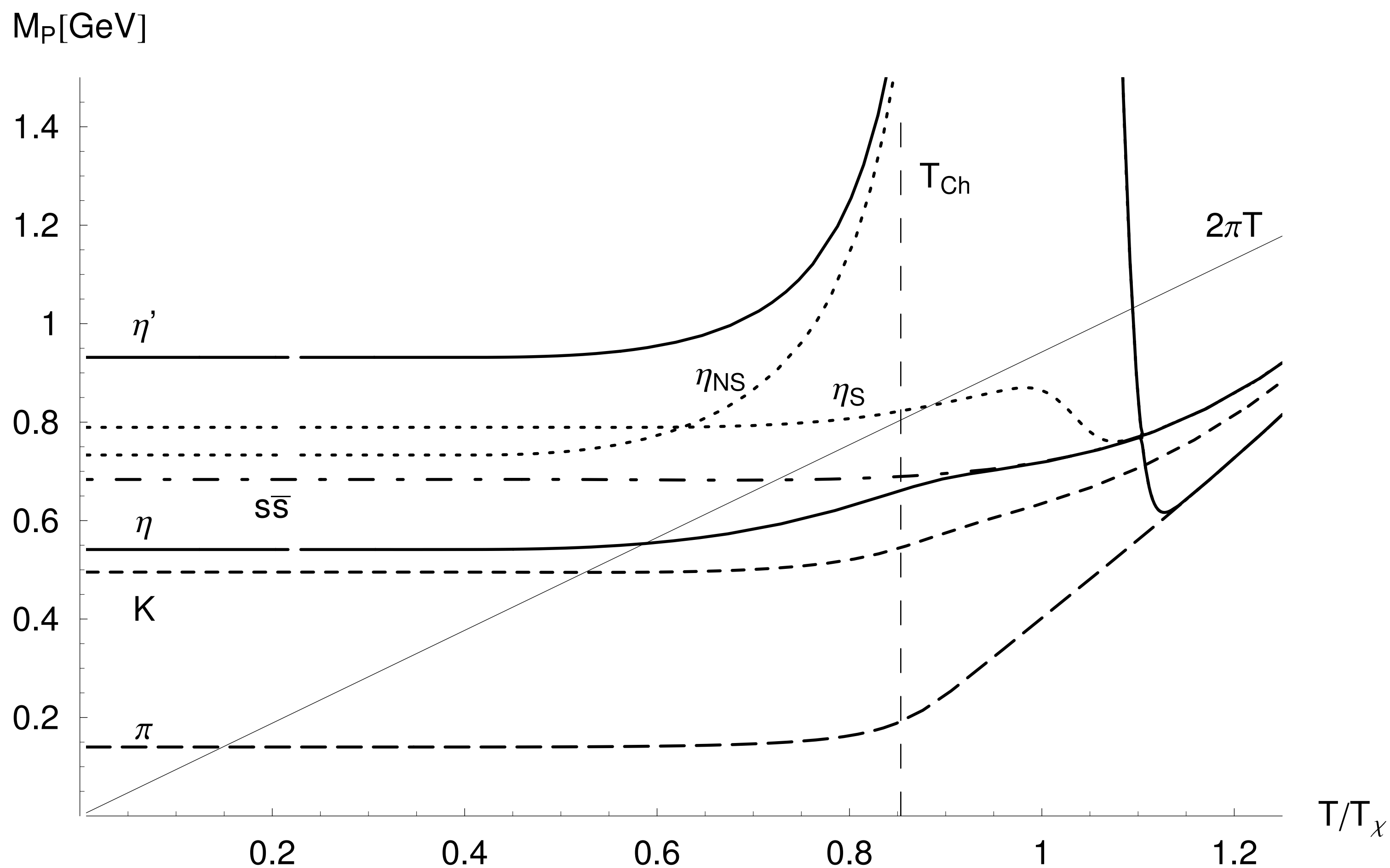
$\chi = \chi_{4fQCD}$ at the preferred choice $T_\chi = T_{\rm Ch}$ leads to the reasonable increases and then fall-offs of η and η' masses



$T_\chi = T_{\rm Ch}$

M_P for 4-flavor QCD topological susceptibility

$\chi = \chi_{4fQCD}$ for $T_\chi > T_{\rm Ch}$ leads to η' mass increases, but noticeably more moderate than in the YM case.



$T_\chi = 1.17\, T_{\rm Ch} = 150$ MeV

Summary

- At $T = 0$, well- and long-known successful DS description of $I \neq 0$ pseudoscalars in many models ... but ALSO of the $\eta - \eta'$ complex thanks to WV relation
- This includes the separable model, after successful extension to strange sector
- Separable model - easier for $T > 0$ calculations, but illustrates some general features
- Generally, DS approach has good features also at $T > 0$, but synchronization of deconfinement and chiral restoration temperatures needed (D. Horvatić talk on PL)
- Main point - the $T > 0$ extension of the DS treatment of $\eta - \eta'$ [PRD 76, 096009 (2007)].
- Results on $\eta - \eta'$ complex at $T \neq 0$ DIFFER VERY MUCH for various possible topological susceptibilities χ
- They also differ much for various possible relationships between the chiral restoration and χ-melting temperatures $\rightarrow$ stresses the importance of synchronization of various characteristic temperatures
- Possible signal, especially from η', in hot QCD matter: η' suppression $\rightarrow$ chiral restoration, otherwise indication of breakdown of strict WV relation (with χ_{YM})