The QCD transition temperature in a Polyakov-loop DSE model*

D. Horvatić[a], D. Blaschke[b,c,d], D. Klabučar[a], A. E. Radzhabov[c]

*Non-Perturbative Functional Methods in Quantum Field Theory, Héviz, Hungary, 2008.

[a]Theoretical Physics Department, University of Zagreb, Croatia
[b]University of Rostock, Germany
[c]JINR Dubna, Russia
[d]University of Wroclaw, Poland

January 24, 2008

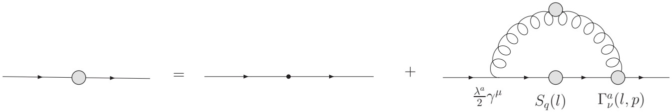
Outline

- Dyson-Schwinger approach to quark-hadron physics
- Separable model at $T = 0$ and $T \neq 0$
- Difficulties and how to solve them
- Polyakov loop + DSE
- Results
- Summary and outlook

Gap and in RLA truncation

$$S_f(p)^{-1} = i\gamma \cdot p + \widetilde{m}_f + \frac{4}{3}\int \frac{d^4q}{(2\pi)^4}\, g^2 D^{\text{eff}}_{\mu\nu}(p-q)\gamma_\mu S_f(q)\gamma_\nu$$

$$S_f(p)^{-1} = i\not{p}\, A_f(p^2) + B_f(p^2)$$



- Euclidean space: $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, $\gamma_\mu^\dagger = \gamma_\mu$, $a \cdot b = \sum_{i=1}^4 a_i b_i$
- $D^{\text{eff}}_{\mu\nu}(k)$ an "effective gluon propagator" - modeled !

Separable model

- To simplify calculations, take the separable form for $D_{\mu\nu}^{\text{eff}}$:

$$D_{\mu\nu}^{\text{eff}}(p-q) \to \delta_{\mu\nu}\, D(p^2, q^2, p\cdot q)$$

$$D(p^2, q^2, p\cdot q) = D_0\ f_0(p^2) f_0(q^2) + D_1\ f_1(p^2)(p\cdot q) f_1(q^2)$$

- two strength parameters D_0, D_1, and corresponding form factors $f_i(p^2)$. In the separable model, gap equation yields

$$\begin{aligned} B_f(p^2) &= \widetilde{m}_f + \frac{16}{3}\int \frac{d^4q}{(2\pi)^4} D(p^2, q^2, p\cdot q) \frac{B_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} \\ \left[A_f(p^2) - 1\right] p^2 &= \frac{8}{3}\int \frac{d^4q}{(2\pi)^4} D(p^2, q^2, p\cdot q) \frac{(p\cdot q) A_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)}\,. \end{aligned}$$

- This gives $B_f(p^2) = \widetilde{m}_f + b_f\ f_0(p^2)$ and $A_f(p^2) = 1 + a_f\ f_1(p^2)$, reducing to nonlinear equations for constants b_f and a_f.

A simple choice for 'interaction form factors' of the separable model:

- Interaction form factors:

$$f_0(p^2) = \exp(-p^2/\Lambda_0^2)$$

$$f_1(p^2) = \frac{1 + \exp(-p_0^2/\Lambda_1^2)}{1 + \exp((p^2 - p_0^2))/\Lambda_1^2}$$

- gives good description of pseudoscalar properties if the interaction is strong enough for realistic DChSB
- when $m_{u,d}(p^2 \sim small) \sim$ the typical constituent quark mass scale $\sim m_\rho/2 \sim m_N/3$.

Extension to $T \neq 0$

- At $T \neq 0$, the quark 4-momentum $p \longrightarrow p_n = (\omega_n, \vec{p})$, where $\omega_n = (2n+1)\pi T$ are the discrete ($n = 0, \pm 1, \pm 2, \pm 3, \ldots$) Matsubara frequencies, so that $p_n^2 = \omega_n^2 + \vec{p}^{\,2}$.
- Dressed quark propagator

$$S_f(p_n, T) = [i\vec{\gamma} \cdot \vec{p}\, A_f(p_n^2, T) + i\gamma_4 \omega_n\, C_f(p_n^2, T) + B_f(p_n^2, T)]^{-1}$$

$$= \frac{-i\vec{\gamma} \cdot \vec{p}\, A_f(p_n^2, T) - i\gamma_4 \omega_n\, C_f(p_n^2, T) + B_f(p_n^2, T)}{\vec{p}^{\,2} A_f^2(p_n^2, T) + \omega_n^2\, C_f^2(p_n^2, T) + B_f^2(p_n^2, T)} \; .$$

- There are now three amplitudes due to the loss of $O(4)$ symmetry, and at sufficiently high $T \geq T_d$ denominator can vanish. $\longrightarrow$ For $T \geq T_d$ quarks can be deconfined!

Extension to $T \neq 0$

- The solutions have the form $B_f = \widetilde{m}_f + b_f(T) f_0(p_n^2)$, $A_f = 1 + a_f(T) f_1(p_n^2)$, and $C_f = 1 + c_f(T) f_1(p_n^2)$

$$
\begin{aligned}
a_f(T) &= \frac{8D_1}{9}\, T \sum_n \int \frac{d^3p}{(2\pi)^3}\, f_1(p_n^2)\, \vec{p}^{\,2} \left[1 + a_f(T) f_1(p_n^2)\right] d_f^{-1}(p_n^2, T) \\
c_f(T) &= \frac{8D_1}{3}\, T \sum_n \int \frac{d^3p}{(2\pi)^3}\, f_1(p_n^2)\, \omega_n^2 \left[1 + c_f(T) f_1(p_n^2)\right] d_f^{-1}(p_n^2, T) \\
b_f(T) &= \frac{16D_0}{3}\, T \sum_n \int \frac{d^3p}{(2\pi)^3}\, f_0(p_n^2) \left[\widetilde{m}_f + b_f(T) f_0(p_n^2)\right] d_f^{-1}(p_n^2, T)
\end{aligned}
$$

- where $d_f(p_n^2, T)$ is given by

$$
d_f(p_n^2, T) = \vec{p}^{\,2} A_f^2(p_n^2, T) + \omega_n^2 C_f^2(p_n^2, T) + B_f^2(p_n^2, T)
$$

Deconfinement and chiral restoration temperature

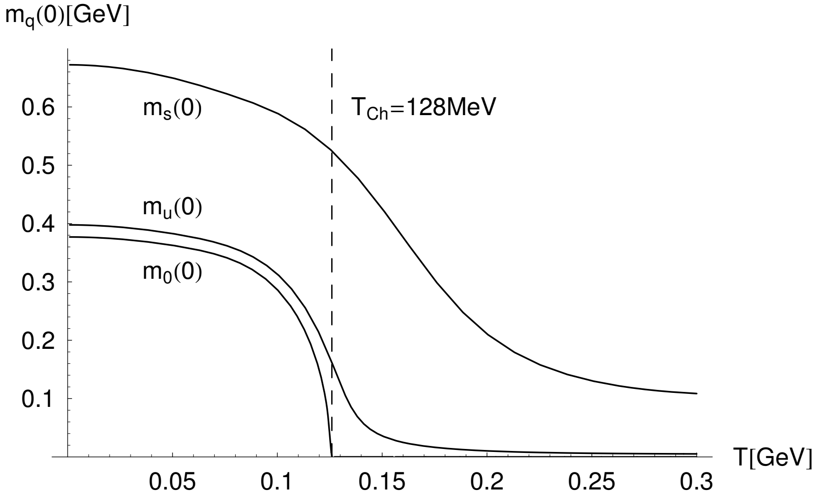
- Chiral restoration critical temperature T_{Ch} is

$$T_{\mathrm{Ch}} = 128 \ \mathrm{MeV}$$

- Deconfinement temperature T_d

$\widetilde{m}_q$ [MeV]	T_d[MeV]
0	97
5.5	107
115	194

Chiral symmetry restoration at $T = T_{Ch}$



DSE + Polyakov loop

- Consider DSE model for quark dynamics and generalize it by coupling to the Polyakov loop potential
- Use rank-2 separable form of the effective gluon propagator with the same form factors fixed by phenomenology
- Central quantity for the analysis of the thermodynamical behavior is the thermodynamical potential

$$
\begin{aligned}
\Omega(T) &= \mathcal{U}(\Phi, \bar{\Phi}) - \\
&- T\ \mathrm{Tr}_{\vec{p},n,\alpha,f,D} \left[\ln\{S_f^{-1}(p_n^\alpha, T)\} - \frac{1}{2}\Sigma_f(p_n^\alpha, T) \cdot S_f(p_n^\alpha, T) \right]
\end{aligned}
$$

DSE + Polyakov loop

- Polyakov loop in a case of a constant background field:

$$\Phi = \frac{1}{N_c}\mathrm{Tr}_c T_\tau e^{i\int_0^\beta d\tau \lambda_a A_4^a(\vec{x},\tau)} = \frac{1}{N_c}\mathrm{Tr}_c e^{i\phi/T}$$

$$\phi = \phi_3\lambda_3 + \phi_8\lambda_8$$

- Here ϕ is related to Euclidean background field and diagonal in color space
- We require $\Phi = \bar{\Phi}$ to be real which sets $\phi_8 = 0$

$$\Phi = \bar{\Phi} = \frac{1}{N_c}\left(1 + e^{i\frac{\phi_3}{T}} + e^{-i\frac{\phi_3}{T}}\right) = \frac{1}{N_c}\left(1 + 2\cos\left(\frac{\phi_3}{T}\right)\right).$$

DSE + Polyakov loop

- Polyakov-loop potential

$$\frac{\mathcal{U}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi^*\Phi + b(T)\ln\left[1-6+4(\Phi^{*3}+\Phi^3)-3(\Phi^*\Phi)^2\right]$$

with

$$a(t) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2, \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3.$$

Corresponding parameters are

$$a_0 = 3.51, \quad a_1 = -2.47, \quad a_2 = 15.22, \quad b_3 = -1.75.$$

- Due to the coupling to the Polyakov loop the fermionic Matsubara frequencies ω_n are shifted

$$(p_n^\alpha)^2 = (\omega_n^\alpha)^2 + \vec{p}^2, \quad \omega_n^\alpha = \omega_n + \alpha\phi_3, \quad \alpha = -1, 0, +1$$

DSE + Polyakov loop

- Thermodynamic potential with Polyakov loop

$$\begin{aligned}\Omega(T) &= \mathcal{U}(\Phi,\bar{\Phi}) + \frac{a_u(T)^2}{C_1^u} + \frac{c_u(T)^2}{C_2^u} + \frac{b_u(T)^2}{C_3^u} + \frac{a_s(T)^2}{C_1^s} + \frac{c_s(T)^2}{C_2^s} + \frac{b_s(T)^2}{C_3^s} \\ &- 4T\sum_n \sum_{\alpha=0,\pm} \int \frac{d^3p}{(2\pi)^3} \ln\left[\vec{p}^{\,2} A_u^2((p_n^\alpha)^2,T) + (\omega_n^\alpha)^2 C_u^2((p_n^\alpha)^2,T) + B_u^2((p_n^\alpha)^2,T)\right] \\ &- 2T\sum_n \sum_{\alpha=0,\pm} \int \frac{d^3p}{(2\pi)^3} \ln\left[\vec{p}^{\,2} A_s^2((p_n^\alpha)^2,T) + (\omega_n^\alpha)^2 C_s^2((p_n^\alpha)^2,T) + B_s^2((p_n^\alpha)^2,T)\right]\end{aligned}$$

- Where C_f^i $(f = u, d, s$ and $i = 1, 2, 3)$ are:

$$C_1^u = \frac{2D_1}{27}, \quad C_2^u = \frac{2D_1}{9}, \quad C_3^u = \frac{4D_0}{9}$$

$$C_1^s = \frac{4D_1}{27}, \quad C_2^s = \frac{4D_1}{9}, \quad C_3^s = \frac{8D_0}{9}$$

- Thermodynamic potential still suffers from the divergency due to the zero-point energy. We employ a substraction scheme.

DSE + Polyakov loop

$$\Omega^{\text{reg}}(T) = \Omega(T) - \Omega_{\text{free}}(T) + \Omega^{\text{reg}}_{\text{free}}(T) + \Omega(0)$$

- where

$$\begin{aligned}\Omega_{\text{free}}(T) &= -4T\sum_n \sum_{\alpha=0,\pm} \int \frac{d^3p}{(2\pi)^3} \ln\left[\vec{p}^2 + (\omega_n^\alpha)^2 + \widetilde{m}_u^2\right] - \\ &\quad -2T\sum_n \sum_{\alpha=0,\pm} \int \frac{d^3p}{(2\pi)^3} \ln\left[\vec{p}^2 + (\omega_n^\alpha)^2 + \widetilde{m}_s^2\right],\end{aligned}$$

- which afer Matsubara sumation and substraction of the zero-point energy is

$$\Omega^{\text{reg}}_{\text{free}}(T) = -4T \sum_{f=u,d,s} \sum_{\alpha=0,\pm} \frac{d^3p}{(2\pi)^3} \ln\left[1 + \exp\left(-\frac{E_f - \alpha\phi_3}{T}\right)\right]$$

DSE + Polyakov loop

- Obtain gap equations by minimizing the thermodynamic potential with respect to A_q, B_q, C_q and ϕ_3
- by minimizing only $\mathcal{U}(\Phi, \bar{\Phi})$ one obtains simple solution for gluon sector

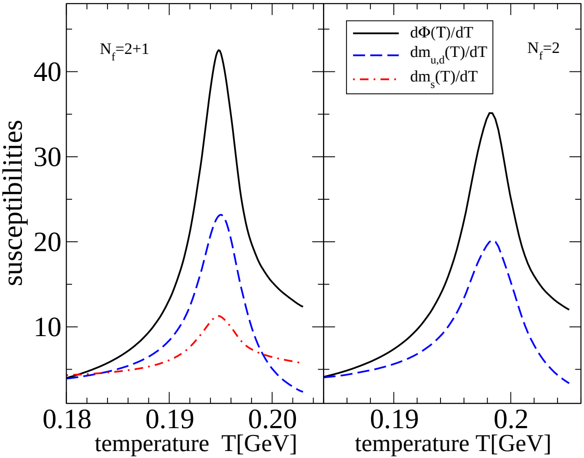
$$\frac{d\mathcal{U}(\Phi, \bar{\Phi})}{d\phi_3} = 0 \quad \Rightarrow \quad \Phi = \frac{a(T) + 2\sqrt{a(T)^2 + 9b(T)a(T)}}{3a(T)}$$

- and from that deconfinement temperature of 270 MeV
- For N_f flavors we have $3N_f + 1$ coupled gap equations
- Now results for quark and gluon sectors coupled

Results

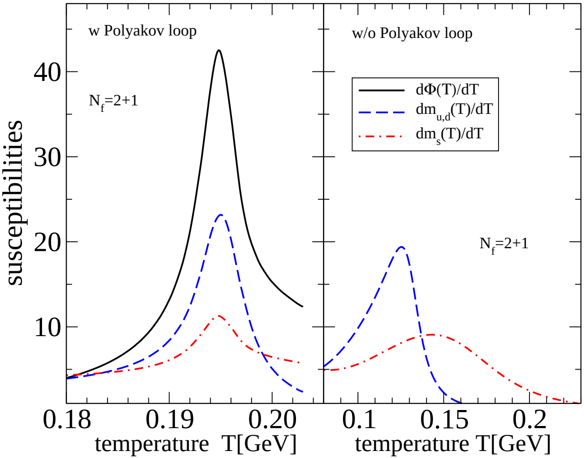
- The critical temperatures for chiral symmetry restoration (T_χ) and deconfinement (T_d) which differ by a factor of two when the quark and gluon sectors are considered separately get synchronized and become coincident when the coupling is switched on $T_c = T_\chi = T_d$.
- The critical temperatures $T_c = 194.8$ MeV for $N_f = 2+1$ and $T_c = 198.2$ MeV for $N_f = 2$ are in agreement with recent lattice QCD simulations.
- Varying the current quark mass of the model results in a dependence of T_c on the light pseudoscalar meson mass: $T_c = a + b\ M_{\rm ps}^d$ in fair agreement with lattice QCD. Plus ...

Results



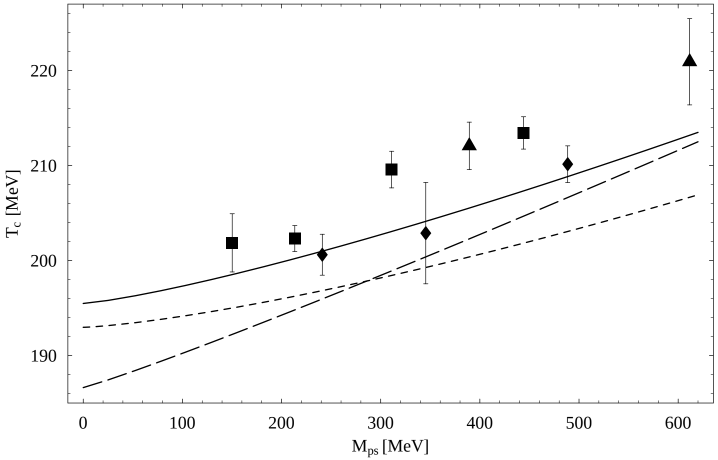
susceptibilities
40
30
20
10
0.18
0.19
0.20
temperature T[GeV]
N_f=2+1
dΦ(T)/dT
$dm_{u,d}(T)/dT$
$dm_s(T)/dT$
N_f=2
0.19
0.2
temperature T[GeV]

Results

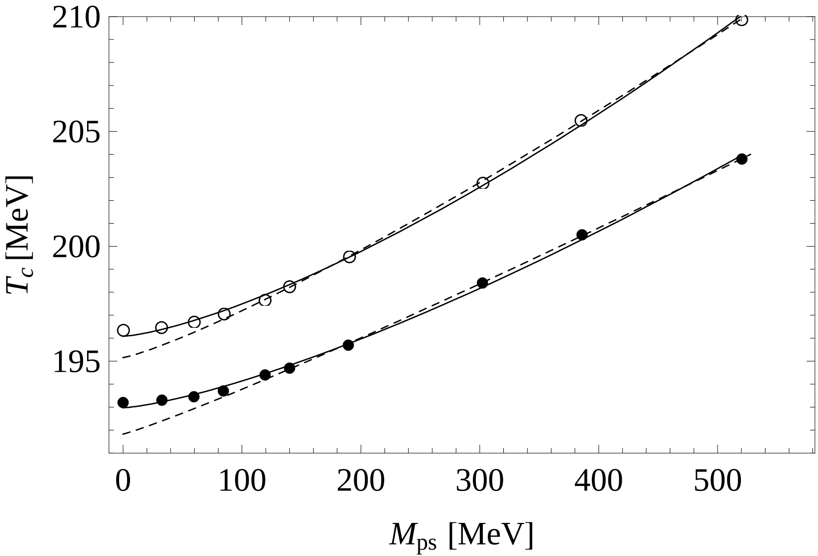

w Polyakov loop
w/o Polyakov loop
N_f=2+1
N_f=2+1
$d\Phi(T)/dT$
$dm_{u,d}(T)/dT$
$dm_s(T)/dT$
susceptibilities
40
30
20
10
0.18
0.19
0.20
0.1
0.15
0.2
temperature T[GeV]
temperature T[GeV]

Results

Data taken form P. Petreczky, `arXiv:0705.2175`



Results



Summary and outlook

- Sketched Dyson-Schwinger approach to quark-hadron physics & a convenient concrete model
- Coupled quark and gluon sector via Polyakov loop potential
- Few remarkable results
- Calculate full thermodynamics (mesonic contribution to thermodynamic potential ...)
- How to apply it to other DSE models ...