THE $U_A(1)$ ANOMALY AND THE η' MASS IN COULOMB GAUGE QCD

DIPLOMA THESIS

Gernot Lassnig

Institute for Physics, SIC!QFT Group
Universitity of Graz

Supervisor *Reinhard Alkofer & Diana Nicmorus*

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FUNDAMENTALS OF QCD

QCD LAGRANGIAN

$$\mathcal{L}_{QCD} = \sum_{f=1}^{N_f} \bar{\psi}^f (i\gamma_\mu D^\mu - m_f)\psi_f - \frac{1}{2} tr(F_{\mu\nu}F^{\mu\nu}) + \mathcal{L}_{ghost} + \mathcal{L}_{g.f}$$

Left and Right handed fields

$$\begin{aligned} \psi_R^f &= \frac{1}{2}(1+\gamma_5)\psi^f \\ \psi_L^f &= \frac{1}{2}(1-\gamma_5)\psi^f \end{aligned} \Rightarrow \bar{\psi}^f i\gamma_\mu D^\mu \psi^f = \bar{\psi}_R^f i\gamma_\mu D^\mu \psi_R^f + \bar{\psi}_L^f i\gamma_\mu D^\mu \psi_L^f$$

Fundamentals of QCD

Symmetry group of massless Lagrangian

$$U_L(N_f) \times U_R(N_F) \cong U_{L+R}(1) \times U_{L-R}(1) \times SU_L(N_F) \times SU_R(N_F)$$

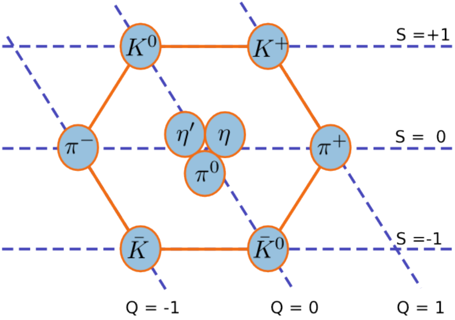
$U_{L+R}(1)$	Conservation of baryon number
$U_{L-R}(1)$	Axial symmetry
$SU_L(N_f) \times SU_R(N_f)$	Chiral symmetry, spontaneously broken

Hadronic vacuum only invariant under $SU(N_F)$

Goldstone theorem $\Rightarrow N_f^2 - 1$ massless Goldstone Bosons

$U_A(1)$ ANOMALY (SEE RICHARD WILLIAMS TALK)

- $U_A(1)$ not a symmetry of nature, so it's broken
- Upper limit for Goldstone Boson is $\sqrt{3}m_\pi$ [1]
- No particle known



[1] S.Weinberg

$U_A(1)$ Anomaly

- $U_A(1)$ Symmetry is “anomalously” broken
- Feynman graphs containing quark triangles create anomaly (ABJ)
- No conservation of axial vector current

Axial Current $\quad j^5_\mu = \bar{\psi}_f \gamma_\mu \gamma^5 \psi_f$

$$\partial^\mu J^5_\mu = \sum_f 2 i m_0^f \bar{q}_f \gamma^5 q_f + N_f \frac{g^2}{16\pi^2} F^{a\mu\nu} \tilde{F}^a_{\mu\nu}$$

Not conserved but . . .

$U_A(1)$ ANOMALY

$N_f \frac{g^2}{16\pi^2} F^{a\mu\nu} \tilde{F}^a_{\mu\nu}$ can be written as 4 divergence

$\Rightarrow$ new conserved current $\tilde{J}^5_\mu = J^5_\mu - \frac{N_f g^2}{2\pi^2} K_\mu$

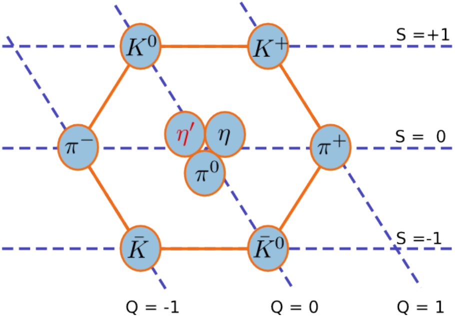
CONSERVED CHARGE

$$\frac{d\tilde{Q}_5}{dt} = \int d^3x \partial_0 \tilde{J}^5_0 = 0 \tag{1}$$

Looks like $U_A(1)$ problem persist

- General: K_μ not gauge invariant, so $\tilde{J}^5_\mu$ not gauge invariant
- t'Hofft: Equation (1) not correct in presence of instantons

η' Mass Generation

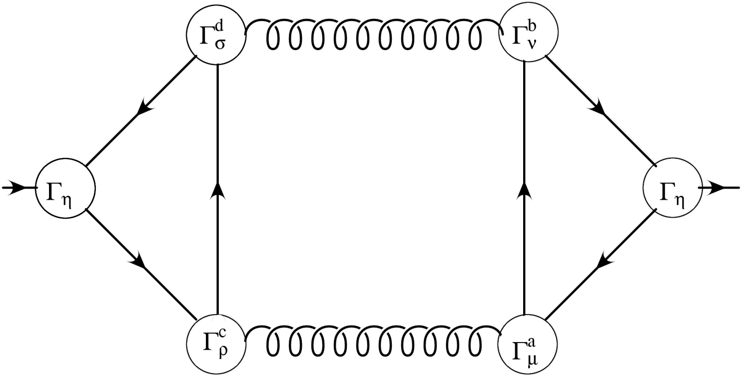


η' is $SU(3)$ singlet state $\Rightarrow$ mixes with 2 gluons
Diamond diagram can contribute to η' mass [1]

[1] Diploma Thesis, Almut Mecke, 1997 & R. Williams, C.S. Fischer, R. Alkofer to be published

Lowest order diagram

Diamond diagram



Why is Coulomb Gauge interesting

Working in Coulomb gauge ($\vec{\nabla}\vec{A} = 0$)

- advantages
  - Coulomb gauge is physical gauge
  - All degrees of freedom are physical
  - No confinement without Coulomb Confinement [1]
  - Working directly in Minkowski space
- disadvantages
  - No proof of renormalisability
  - Complicated calculations

[1] D.Zwanziger, Phys. Rev. Lett. 90, 102001 (2003)

DYSON-SCHWINGER AND BETHE-SALPETER EQUATIONS

Quark self-energy due to gluons

QCD GAP EQUATION

$$iS^{-1}(p) = \not{p} - m - \Sigma(p) \tag{2}$$

A $q\bar{q}$ bound state described by the Bethe-Salpeter equation

$$\Gamma(P,q) = \int d^4k K(q,k,P) S(k_+) \Gamma(P,K) S(k_-)$$

$\Gamma(P,q)$ Bethe-Salpeter Amplitude

Infinite coupled system $\Rightarrow$ truncation needed

DYSON-SCHWINGER AND BETHE-SALPETER EQUATIONS

$$\Sigma(p) = C_f 6\pi \int \frac{d^4 q}{(2\pi)^4} V_C(\vec{k})\gamma_0 S(q)\gamma_0 \tag{3}$$

Ansatz

$$S^{-1}(p) = -i\left[\gamma_0 p_0 - \vec{\gamma}\cdot\vec{p}C(p) - B(p)\right] \tag{4}$$

leads to two coupled integral equations

$$B(p) = m + \frac{1}{2\pi^2}\int d^3q V_c(k)\frac{M(q)}{\tilde{\omega}(q)}$$

$$C(p) = 1 + \frac{1}{2\pi^2}\int d^3q V_c(k)\hat{p}\cdot\hat{q}\frac{q}{p\tilde{\omega}(q)}$$

$M(p) := \frac{B(p)}{C(p)}$
$\tilde{\omega}(q) = \sqrt{M(q)^2 + p^2}$
$C_f = 4/3$

Quark Mass Function

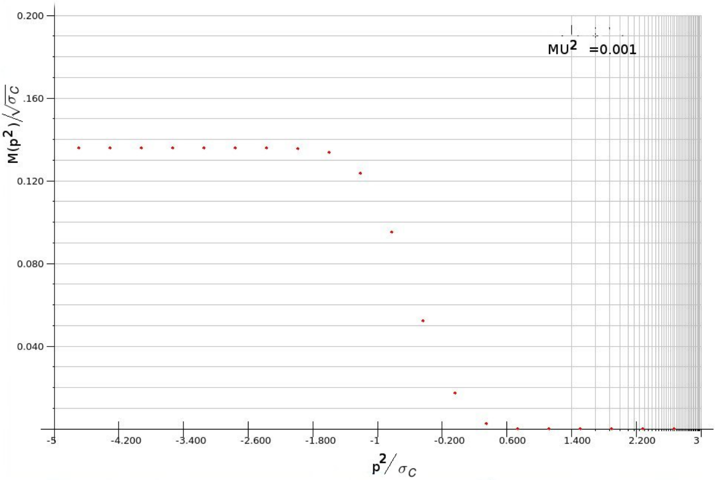
Highly singular non-linear coupled integral equation

- Iterating is the only known solution
- Gauß-Kronrod Integration system is used
- Coulomb Gluon Part $V_c = \frac{\sigma_c}{(k^2)^2}$

  $\Rightarrow$ Angular integrations can be done analytically

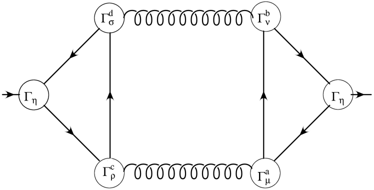
$$B(p) = m + \frac{1}{2\pi^2} \int d^3q V_c(k) \frac{M(q)}{\tilde{\omega}(q)}$$
$$C(p) = 1 + \frac{1}{2\pi^2} \int d^3q V_c(k) \hat{p} \cdot \hat{q} \frac{q}{p\tilde{\omega}(q)}$$

SOLUTION OF THE COUPLED INTEGRAL EQUATION



CURRENT WORK

Power counting



Chiral limit $\Rightarrow$ BSA results in $\gamma_5 B(q^2)/f_\pi$
Quark propagator known in terms of $B(q)$ and $C(q)$
Working directly in Minkowski space

Outlook

- Test of Quark-Gluon-Vertex: IR singularities needed?
- Using full Bethe-Salpeter Amplitude
- Higher order Diagrams?
- . . . lots of stuff to do