$U_A(1)$ anomaly and η' mass from an infrared singular quark-gluon vertex

Reinhard Alkofer[1] Christian Fischer[2,3] Richard Williams[2]

[1]Uni Graz [2]TU Darmstadt [3]GSI Darmstadt

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TECHNISCHE UNIVERSITÄT DARMSTADT

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outline

1. Introduction
   - Motivation
   - The $U_A(1)$ Problem
2. Dyson-Schwinger Studies in Landau Gauge
   - Propagators
   - Higher-order Green's functions
   - Calculating anomalous mass
   - Results
3. Summary and Outlook

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Introduction

- Long accepted: strong interaction described by QCD
- non-Abelian
  - asymptotic freedom - perturbation theory
  - confinement - no coloured asymptotic states

Despite accedence to this:

- no satisfactory understanding of confinement mechanism

  $\rightarrow$ expected to manifest in the IR dynamics

Introduction

Develop non-perturbative tools:

- Lattice QCD - ab initio
- Functional methods (ERG, DSE) -truncations

  Effective Theories (symmetry based):

- NJL, χPT, Quark Models

All successful at describing range of meson observables

- No insight into confinement mechanism itself.

  $\rightarrow$ dynamical symmetry breaking more important here than confinement.

Introduction

Symmetries:

- Powerful tool in physics.
- No explicit dynamical calculations.

Group theory applied to quarks $\rightarrow$ multiplets.

- QCD lagrangian in chiral limit. u, d light, extend to include s.

  $SU_L(3) \times SU_R(3)$ chiral symmetry

Octet of axial vector currents $\implies$ 8 Goldstone bosons from

$$\partial_\mu \overline{\psi} \partial^\mu \gamma_5 \lambda^i \psi = 2\overline{\psi}\gamma_5 \left\{\lambda^i, M\right\} \psi$$

$\rightarrow$ immediate puzzle:
Predicts **ninth** isosinglet Goldstone boson ($\lambda^i \rightarrow \mathbf{1}$), $U_A(1)$ symmetry.

Only candidate pseudoscalar is the η'
Chiral picture: Squares of Goldstone boson masses are linear in the quark mass.

- 2 light quarks: $m_{\eta'} < \sqrt{3} m_\pi \simeq 240 MeV$
- 3 light quarks: $s\bar{s}$ heaviest meson
  $\rightarrow m_{\eta'} < \sqrt{2} m_K \simeq 700 MeV$

- Mass of $\sim$ 958 MeV.

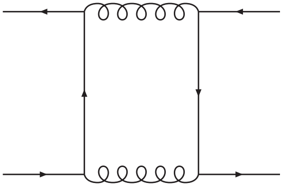
Conclude: η' not a Goldstone boson.
$U_A(1)$ symmetry must be broken – by anomaly.

Solution to everything - instantons?!?

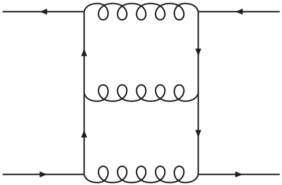
- classical solution of Euclidean Yang-Mills equations of motion

Diagrammatically, $U_A(1)$ anomaly enters through

Annihilation and recombination: $q\bar{q} \rightarrow \text{gluons} \rightarrow q\bar{q}$ in the isosinglet channel



$J^P = 0^-$
Pseudoscalar



$J^P = 1^-$
Vector

Amplitude contributes towards singlet mass.

η_0 and η_8 not mass eigenstates.
Working in singlet-octet basis:

$$\begin{aligned}
|\pi\rangle &= \left(|u\bar{u}\rangle - \left|d\bar{d}\right\rangle\right)/\sqrt{2} \\
|\eta_8\rangle &= \left(|u\bar{u}\rangle + \left|d\bar{d}\right\rangle - 2\,|s\bar{s}\rangle\right)/\sqrt{6} \\
|\eta_0\rangle &= \left(|u\bar{u}\rangle + \left|d\bar{d}\right\rangle + |s\bar{s}\rangle\right)/\sqrt{3}
\end{aligned}$$

Mass squared mixing matrix:

$$M^2 = \begin{pmatrix} M_\pi^2 & 0 & 0 \\ 0 & M_{88}^2 & M_{80}^2 \\ 0 & M_{08}^2 & M_{00}^2 + m_A^2 \end{pmatrix}$$

with m_A^2 contribution from annihilation channel.

$\rightarrow$ Diagonalise matrix:

π decoupled from η octet-singlet.

Introduce mixing matrix

$$(\eta \quad \eta') = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} . \qquad (1)$$

Matrix elements $M_{00}, M_{08} = M_{80}, M_{88}$ related to masses of K, π and anomaly.

- Compute physical eigenstates for η, η' and mixing matrix.
- Determine mixing angle $\simeq -20^\circ$.

$\rightarrow$ Diagonalise matrix:

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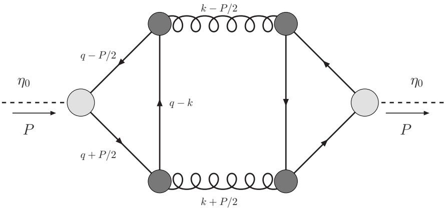
Are instantons the *only* solution?

Kogut and Susskind Mechanism

- $U_A(1)$ inextricably tied to the non-Abelian anomaly.

Kogut and Susskind identified the lowest order annihiliation graph containing the anomaly:



- simple consideration of the dimensions
  $\rightarrow$ concluded $D(p^2) \sim 1/p^4$ give non-zero χ^2

[J. B. Kogut and L. Susskind, Phys. Rev. D **10**(1974) 3468.]

Kogut and Susskind Mechanism

Explored in simple NJL-like model (no running masses):

$$g^2 D_{\mu\nu}(k) = T_{\mu\nu}(k) \left(\frac{8\pi\sigma}{k^4} + \frac{16\pi^2/9}{k^2 \ln\left(e + k^2/\Lambda^2_{\rm QCD}\right)} \right)$$

IR singular gluon, σ string tension.

- Calculated lowest order graph contributing to topological susceptibility

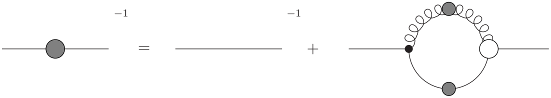
  All inputs bare, $P^2 \to 0$ limit, $\Pi \neq 0$

Found:

$$\begin{aligned} m_A^2 &\simeq \frac{3N_f}{f_0^2}\frac{\sigma}{\pi^4} \simeq 0.346\ \mathrm{GeV}^2 \\ m_\eta, m_{\eta'} &\simeq 430, 810\ \mathrm{MeV} \\ \theta &\simeq -30^\circ \end{aligned}$$

[L. von Smekal, A. Mecke and R. Alkofer, arXiv:hep-ph/9707210.]

Diagrammatically, DSE for quark:



Quark propagator $S_F(p^2)$:

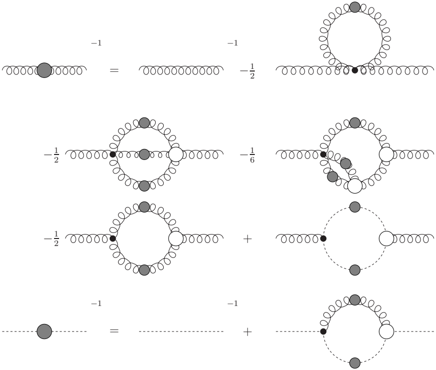
$$S_F^{-1}(p^2) = \mathcal{A}(p^2, \mu^2)\left(\not{p} + \mathcal{M}(p^2)\right)$$

- Depends:
  - Quark-gluon vertex.
  - Gluon propagatator.
  - Quark propagator.

$\rightarrow$ Understand YM sector before consider quarks.

Yang-Mills sector
in Landau gauge

Dyson-Schwinger equations for ghost, gluon:



- Infinite tower
  - Truncation scheme preserving UV anomalous dimensions
  - Analytic solutions obtainable in IR
  - IR singular ghost
  - IR vanishing gluon

[C.S.Fischer and R.Alkofer, PRD **67**, 0904020 (2003)]

Yang-Mills sector
in Landau gauge

Pure YM-sector: ghosts and gluons (no quarks)

- Propagators in Euclidean space:

$$D^G(p^2) = -\frac{G(p^2)}{p^2}\,, \quad D^{ab}_{\mu\nu} = \delta^{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}\,.$$

$G(p^2), Z(p^2)$: ghost, gluon dressing functions respectively.

- Described by power laws in IR:

$$G(p^2) \sim \left(p^2\right)^{-\kappa}\,, \qquad Z(p^2) \sim (p^2)^{2\kappa} \quad \kappa \simeq 0.595353$$

$\rightarrow$ diverging ghost propagator and vanishing gluon.

[L. von Smekal, R. Alkofer and A. Hauck Phys. Rev. Lett. **79** (1997) 3591]

[D. Zwanziger,Phys. Rev. D **65** (2002) 094039]

[C. Lerche and L. von Smekal, Phys. Rev. D **65** (2002) 125006]

Running coupling

Running coupling from the ghost-gluon vertex:

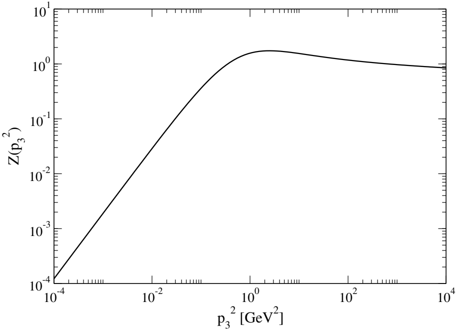
$$\alpha(p^2) = \alpha_\mu G^2(p^2) Z(p^2) \ ,$$

Numerical results well-reproduced by fit:

$$\begin{aligned} \alpha_{\rm fit}(p^2) &= \frac{\alpha_s(0)}{1+p^2/\Lambda^2_{\rm YM}} + \frac{4\pi}{\beta_0}\frac{p^2}{\Lambda^2_{\rm YM}+p^2} \\ &\times \left(\frac{1}{\ln(p^2/\Lambda^2_{\rm YM})} - \frac{1}{p^2/\Lambda^2_{\rm YM}-1}\right) \ . \end{aligned} \tag{2}$$

[C.S.Fischer and R.Alkofer, PRD **67**, 0904020 (2003)]

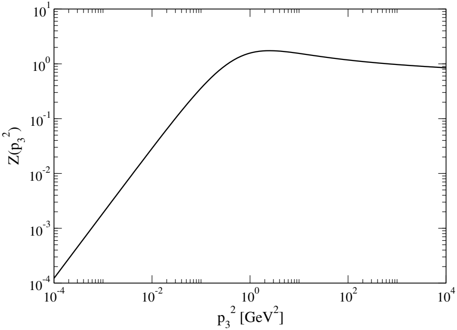
Gluon dressing function



$$Z\left(k^2\right) = \left(\frac{k^2}{k^2 + d_2}\right)^{2\kappa} \left(\frac{\alpha_{\text{fit}}\left(k^2\right)}{\alpha_\mu}\right)^{-\gamma}$$

- $Z(k^2)/k^2$ give IR exponent $2\kappa - 1 \neq -2$
  NB: IR vanishing ($\kappa \simeq 0.595\ldots$)

Gluon dressing function



$$Z\left(k^2\right) = \left(\frac{k^2}{k^2 + d_2}\right)^{2\kappa} \left(\frac{\alpha_{\mathrm{fit}}\left(k^2\right)}{\alpha_\mu}\right)^{-\gamma}$$

Look for singular behaviour in another structure.

Higher order Green's functions

Generalised for all scales vanishing symmetrically:

$$\Gamma^{n,m,l} \sim \left(p^2/\Lambda^2_{\mathrm{QCD}}\right)^{(n-m)\kappa - l/2}$$

	2n	:	external ghost legs
with	m	:	external gluon legs
	2l	:	external quark legs

- Skeleton expansion.

  $\rightarrow$ leading IR behaviour seen at lowest order

[R. Alkofer, C. S. Fischer and F. J. Llanes-Estrada, arXiv:hep-ph/0607293.]

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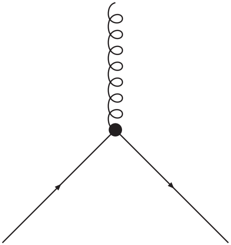


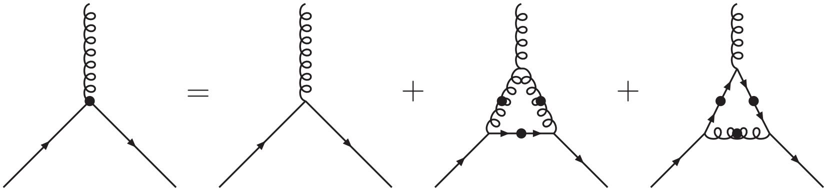
- Triangle anomaly - Q-G vertex
  IR exponents : $-\kappa - 1/2$

Infrared singular!

[R. Alkofer, C. S. Fischer and F. J. Llanes-Estrada, arXiv:hep-ph/0607293.]

Quark-Gluon Vertex

- Leading diagrams in the IR

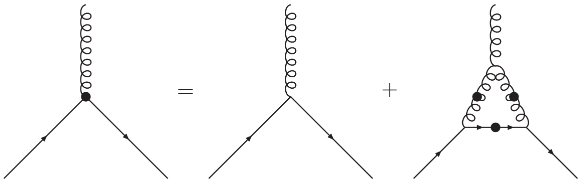


- Propagators from DS-equations - Yang-Mills.
- Ansatz for Γ_{3g}.
- Ansatz/fits for Γ_{qg}.

[Alkofer, Fischer, Llanes-Estrada, Schwenzer *unpublished*]

Quark-Gluon Vertex

- Leading diagrams in the IR
- Large N expansion



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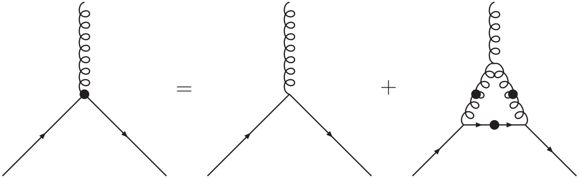
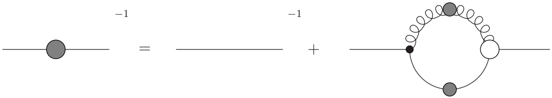
- Leading diagrams in the IR
- Large N expansion
- Coupling in Quark DSE





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Quark-Gluon Vertex

- Single out two Dirac structures: γ_μ and $-i\,(p_1+p_2)_\mu$
- Corresponding fits for dressing functions:

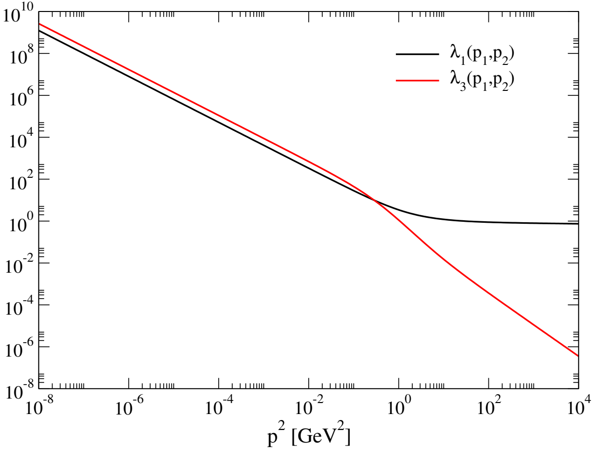
$$\lambda_1(p_1,p_2) \;=\; \left(\frac{x}{d_1+x}\right)^{-\kappa-1/2}\left(\frac{d_1}{d_1+x}+d_2\log\left(\frac{x}{d_1}+1\right)\right)^{-9/44}$$

$$\lambda_3(p_1,p_2) \;=\; \frac{1}{\sqrt{(p_1+p_2)^2}}\left(\frac{x}{d_1+x}\right)^{-\kappa-1/2}\left(\frac{d_2}{d_2+x}\right)^{n_2}\left(\frac{d_3}{d_3+x}\right)^{2}$$

- x is sum of all incoming momenta, $p_1^2+p_2^2+p_3^2$.
- λ_1: $d_1=2.0$, $d_2=0.5$
- λ_3: $d_1=4.0$, $d_2=0.5$, $d_3=0.5$, $n_1=1$, $n_2=-0.5$

Quark-Gluon Vertex

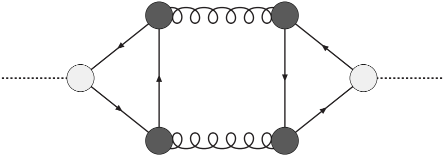
- Single out two Dirac structures: γ_μ and $-i\,(p_1 + p_2)_\mu$
- All 12 structures contribute to IR. L_1, L_3 dominant rôle.



Naïve application to Diamond Diagram

Look at infrared exponents. γ_μ part of vertex.

Contribution	:	IR exponent
Four $\lambda_1(p^2)$	:	$4 \cdot (-1/2 - \kappa)$
Two $Z(p^2)/p^2$	:	$2 \cdot (2\kappa - 1)$



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Two $Z(p^2)/p^2$	:	$2 \cdot (2\kappa - 1)$
Sum	=	-4

- Quick counting: Satisfies Kogut-Susskind
  Find $\Pi(P^2) \neq 0$ for $P^2 \to 0$?

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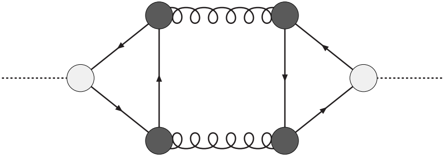
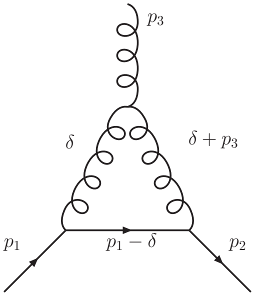


- IR exponents are of different momentum dependence.
- Loop integral *lowers* degree of divergence.

$\Pi(P^2) = 0$ in limit $P^2 \to 0$

IR Collinear Singularities



Choose kinematics so as to manifestly exhibit soft-divergence.

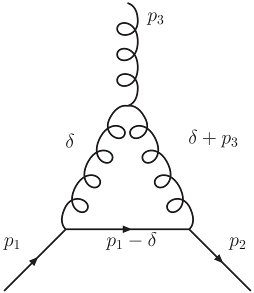
- Gluon momentum p_3 small.
- Loop dominated by small δ, $\delta + p_3$ internal gluon momenta.
- External quark mom $p_1 \simeq p_2$ can be large.

- 3-g vertex: all scales singularity, exponent -3κ.
- Internal QG-vertices: soft divergence, exponent β

  $\rightarrow$ Consistency with overall diagram

[Kai Schwenzer, *Priv. Comm.*]

IR Collinear Singularities



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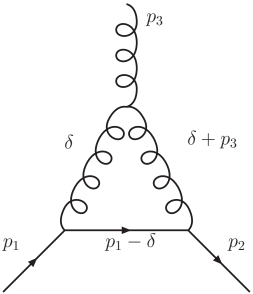
$$(p_3)^\beta \propto \int d^4\delta \left(\delta^2\right)^\beta \left(\delta^2\right)^{2\kappa-1} \left(\delta^2 + p_3^2 + (\delta + p_3)^2\right)^{-3\kappa} \Gamma^0_{3g} \left((\delta + p_3)^2\right)^{2\kappa-1} \left((\delta + p_3)^2\right)^\beta$$

[Kai Schwenzer, *Priv. Comm.*]

IR Collinear Singularities



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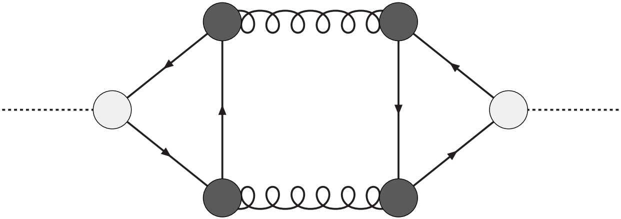
Consistency achieved with exponent:

$$\beta = -\kappa - 1/2$$

[Kai Schwenzer, *Priv. Comm.*]

Computing the Diamond diagram

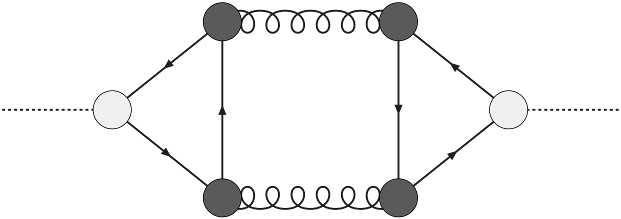


$$\Pi(P^2) = \int \frac{d^4k}{(2\pi)^4} G^{ac}_{\mu\rho}(P,k) D^{\mu\nu}_{ab}(k_+) D^{\rho\sigma}_{dc}(k_-) G^{db}_{\sigma\nu}(-P,k) \,,$$

The quark triangle can be factored as:

$$G^{ab}_{\mu\nu} = \frac{1}{2}\delta^{ab} 4_{\mu\nu\alpha\beta} P^\alpha k^\beta \; I\left(P^2, k^2, k\cdot P\right) \,,$$

with I a (complicated) scalar integral.



Required inputs:

- Quark propagator (solve GAP eqn)
- Gluon propagator (from Yang-Mills)
- Quark-gluon vertex (from fits)
- Pseudoscalar Bethe-Salpeter Amplitude for $q\overline{q}$.

$q\overline{q}$ bound state described by:

$$\Gamma_{tu}(p;P) = \int \frac{d^4k}{(2\pi)^4} K_{tu;rs}(p,k;P)\left[S(k_+)\Gamma(k;P)S(k_-)\right]_{sr}$$

- K quark-antiquark scattering kernel.
- Rest frame of meson: $P^2 = -M^2$ (Euclidean Space)

Pseudoscalar:

$$\Gamma(p,P) = \gamma_5 \left(E - i\not{P}\,F - i\not{p}\,p\cdot P G - \left[\gamma_\mu,\gamma_\nu\right] P_\mu p_\nu H\right)$$

Note E is leading component.

Computing the Diamond diagram

Bethe-Salpeter equation

$q\overline{q}$ bound state described by:

$$\Gamma_{tu}(p;P) = \int \frac{d^4k}{(2\pi)^4} K_{tu;rs}(p,k;P)\left[S(k_+)\Gamma(k;P)S(k_-)\right]_{sr}$$

ax-WGTI

$$\begin{aligned} P_\mu \Gamma^a_{5\mu}(k;P) &= S^{-1}(k_+)\frac{1}{2}\lambda^a_f i\gamma_5 + \frac{1}{2}\lambda^a_f i\gamma_5 S^{-1}(k_-) \\ &\quad - M_\zeta i\Gamma^a_5(k;P) - i\Gamma^a_5(k;P)M_\zeta\ . \end{aligned}$$

- BSE on left, DSE on right.
  $\rightarrow$ intimate relationship between kernels.

Computing the Diamond diagram

Bethe-Salpeter equation

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$$\Gamma_{tu}(p;P) = \int \frac{d^4k}{(2\pi)^4} K_{tu;rs}(p,k;P)\left[S(k_+)\Gamma(k;P)S(k_-)\right]_{sr}$$

Only consistent known truncation
Rainbow-ladder

- Bare vertex - γ^μ
- Gluon ladder exchange.

Vertex allowed a dressing function:
but only with a dependence on the gluon momentum

- Typically absorb into an effective Gluon.

$q\overline{q}$ bound state described by:

$$\Gamma_{tu}(p;P) = \int \frac{d^4k}{(2\pi)^4} K_{tu;rs}(p,k;P) \left[S(k_+)\Gamma(k;P)S(k_-)\right]_{sr}$$

Only consistent known truncation
Rainbow-ladder

Create a model:

- Take qualitative features of soft-singular quark-gluon vertex.
- Neglect L_3 tensor structure.
- Fit scales to meson phenomenology.

Effective Gluon

Compose from $g^2 \times$ Gluon $\times$ Vertex Dressing.

$$\alpha_{eff}(z) = \alpha_\mu Z(z)\,\lambda_1(z)$$

$$\begin{aligned}
\lambda_1(z) &= \left(\frac{z}{z+d_2}\right)^{-1/2-\kappa} \\
&\times \left(\frac{d_1}{1+z/d_2} + z\frac{d_3}{d_2^2+(z-d_2)^2}\right. \\
&+ \left.\frac{z}{d_2+z}\left(\frac{4\pi}{\beta_0\alpha_\mu}\left(\frac{1}{\log(z/d_2)} - \frac{1}{z/d_2-1}\right)\right)^{-2\delta}\right)
\end{aligned}$$

Effective Gluon

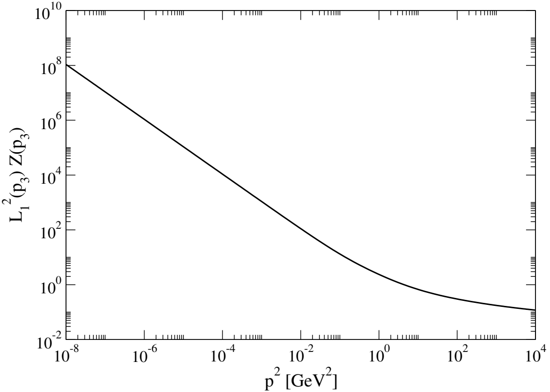
z	:	gluon momentum
d_1	:	IR strength.
d_2	:	soft scale.
d_3	:	added integrated strength.

$$
\begin{aligned}
\lambda_1(z) &= \left(\frac{z}{z+d_2}\right)^{-1/2-\kappa} \\
&\times \left(\frac{d_1}{1+z/d_2} + z\frac{d_3}{d_2^2+(z-d_2)^2}\right. \\
&+ \left.\frac{z}{d_2+z}\left(\frac{4\pi}{\beta_0\alpha_\mu}\left(\frac{1}{\log(z/d_2)} - \frac{1}{z/d_2-1}\right)\right)^{-2\delta}\right)
\end{aligned}
$$

Effective Gluon dressing in Diamond Diagram

c.f. q^{-4} singular gluon of Mecke *et al.*

α_μVertex dressing^2 $\times$ Gluon

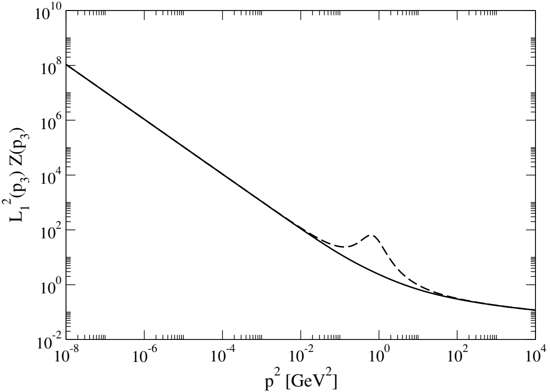


$d_1 = 1.67$, $d_2 = 0.5$, $d_3 = 0$

- No added integrated strength

Effective Gluon dressing in Diamond Diagram

c.f. q^{-4} singular gluon of Mecke *et al.*
α_μVertex dressing^2 $\times$ Gluon



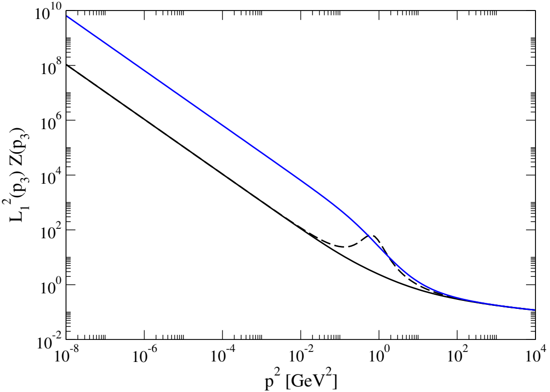
$d_1 = 1.67$, $d_2 = 0.5$, $d_3 = 2.6$

- Integrated strength added.
- Meson observables fix parameters $m_\pi = 138$, $f_\pi = 99$, $m_\rho = 747$.

Effective Gluon dressing in Diamond Diagram

c.f. q^{-4} singular gluon of Mecke *et al.*

$\alpha_\mu \text{Vertex dressing}^2 \times \text{Gluon}$



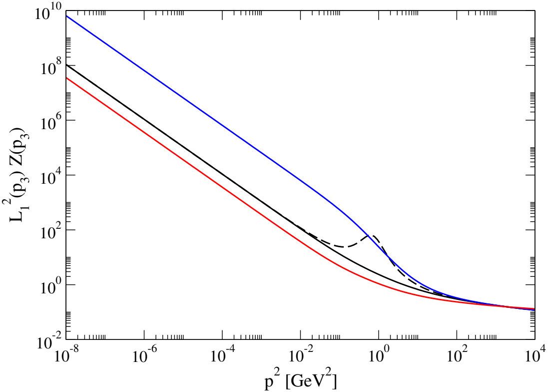
$d_1 = 13$ (!!), $d_2 = 0.5$, $d_3 = 0$

- No added integrated strength.
- Meson observables fix parameters. $m_\pi = 140$, $f_\pi = 95$, $m_\rho = 770$.

Effective Gluon dressing in Diamond Diagram

c.f. q^{-4} singular gluon of Mecke *et al.*

α_μVertex dressing^2 $\times$ Gluon



Effective gluon (bare quark-gluon vertex) of Mecke *et al.*

- String tension, $\sigma = 0.18$.
- Would not give 'right' pion mass.

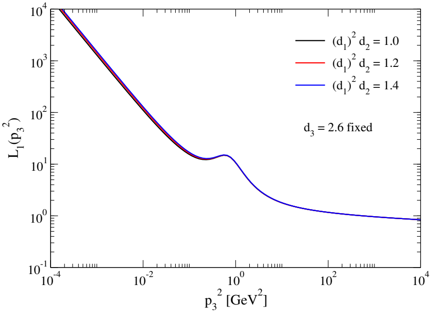
- Solve the Bethe-Salpeter equation for pion, rho

Fix current quark masses at 170 GeV^2
Tune parameters such that meson observables reproduced:

$$\begin{aligned} m_\pi &\sim 135\text{ MeV} \\ f_\pi &\sim 93\text{ MeV} \\ m_\rho &\sim 750\text{ MeV} \end{aligned}$$

scale/set	1	2	3	4	5	6	7
d_1	13	1.73	1.28	1.05	1.41	1.55	1.67
d_2	0.5	0.2	0.4	0.6	0.5	0.5	0.5
d_3	–	2.9	2.6	2.9	2.6	2.6	2.6

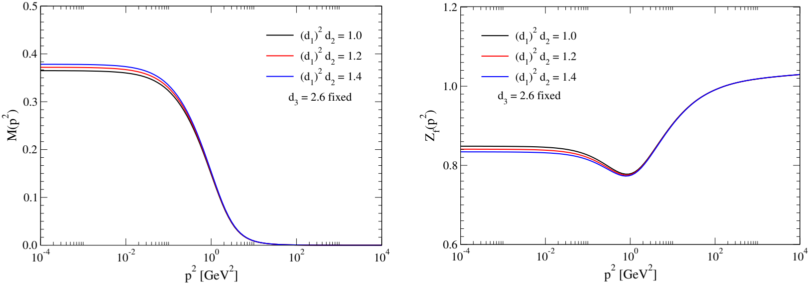
Last three sets with constant d_2 look like:



- Very similar (though note log-scale)

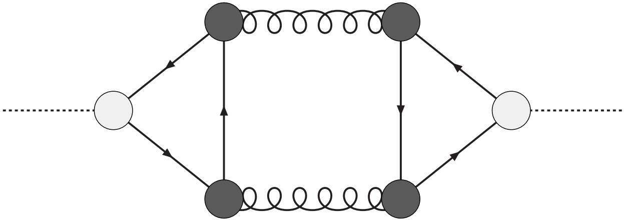
  $\rightarrow$ will see very sensitive to infrared strength d_1.
- Meson observables near independent of 20% change in d_1.

This parameter set yields quark dressing functions of the form:



- $M(0) \simeq 370\mathrm{MeV}$
- $\langle q\overline{q}\rangle \simeq 250$ MeV ($\overline{MS}$).

Turnover in Z_f attributable to artificial term yielding necessary integrated strength.



Integrals performed using Gauss-Konrod adaptive quadrature

Three loop integral!

- Neglect terms $\sim F(k + P/2) - F(k - P/2)$ in $P \to 0$ limit.
- Discard all but leading component of BSA.
  $\to F, G, H$ linear in P.

$\to$ demand modest accuracy (10^{-6}) due to computing time.

Results

- Soft scale chosen to be fixed from YM-sector.
- Vary IR strength parameter.

  . . . meson phenomenology relatively unchanged.

  but anomalous mass very sensitive.

Obtain:

d_1 GeV^2	d_2 GeV^2	d_3 GeV^2	m_π MeV	m_ρ MeV	m_A^2 GeV^2
1.41	0.5	2.6	135	735	0.302
1.55	0.5	2.6	135	741	0.417
1.67	0.5	2.6	135	747	0.558

Realistic η and η' masses eminently achievable with model.

$$M^2 = \begin{pmatrix} M_\pi^2 & 0 & 0 \\ 0 & M_{88}^2 & M_{80}^2 \\ 0 & M_{08}^2 & M_{00}^2 + m_A^2 \end{pmatrix}$$

Employ singlet-octet mass-squared mixing matrix.
Diagonalise to obtain physical mass eigenstates.

Obtain:

d_1 GeV^2	m_A^2 GeV^2	θ	m_η MeV	$m_{\eta'}$ MeV
1.41	0.302	-35.3	412	790
1.55	0.417	-29.1	450	840
1.67	0.558	-23.2	479	906

Realistic η and η' masses eminently achievable with model.

outline

Summary and Outlook

Summary

- Motivated interest in η, η' problem
- Brief overview of Landau Gauge studies in QCD.
- Recognised importance of soft singularities
- Formulated effective gluon interaction to exhibit:
  - qualitative IR features of QG-vertex study.
  - correct UV anomalous dimensions.
- Applied to problem of $U_A(1)$ anomaly:
  - Diamond diagram - Topological Susceptibility
  - Effect on physical mass eigenstates
  - Mixing angle.

Summary and Outlook

Outlook

- Fine tune parameters to wider range of observables
- Mixing in strange non-strange basis - exhibit flavour symmetry breaking. Calculable within model.
- $f_0 \simeq f_\pi \left(1 + \Pi'\left(P^2\right)\right)\Big|_{P^2 \to 0}$.
  - High accuracy determination of Π (Compute via numerical derivative).
- Take $P^2 < 0$ - continue to complex plane
- Higher order diagrams (large? re-summation $\to$ meson exchange)
- Full tensor structure. For $P^2 \to 0$ leading BSA sufficient - quark B/f_π
- Diamond diagram for ω, ϕ - three gluons.
  - Contribution should be small. Four loop.