A mechanism for Quark-Confinement in Quenched Landau gauge QCD

Reinhard Alkofer, Christian Fischer,
Felipe Llanes-Estrada &
Kai Schwenzer, University of Graz

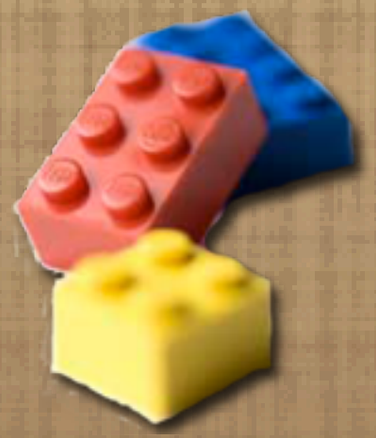
FWF-funded
project

UNI GRAZ

Description of matter

- The Physical degrees of freedom of matter at low scales ...

... are quite different from the underlying building blocks



- It would be very desirable to have a direct connection of the physical observables to the dynamics of the fundamental local constituents

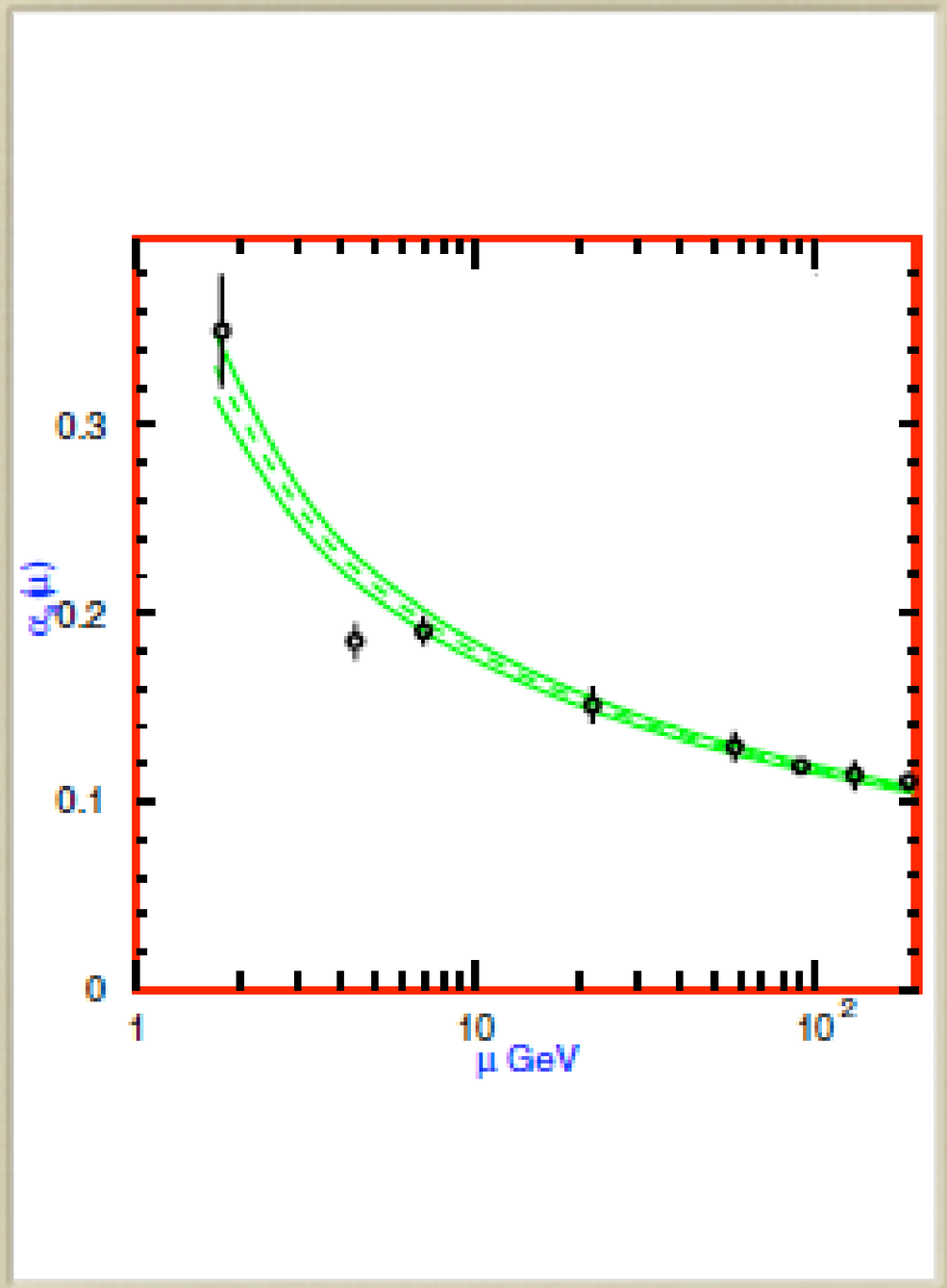
- Some kind of Construction manual ...

--> Diana Nicmorus´ talk

Degrees of freedom

- Pert. RG breaks down
- Scale generation
- description in terms of initial local degrees of freedom could fail ...
  - ... and it does for short range forces
  - but non-perturbative dynamics prevents this for long range gauge interactions

H. Gies, Phys. Rev. D 66 (2002) 025006



Gauge Fixing

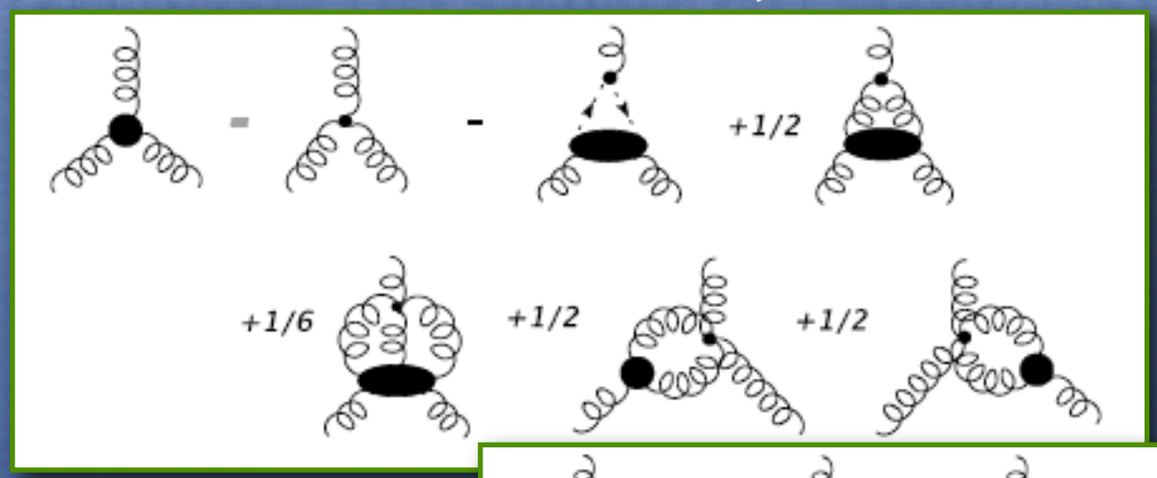
- Continuum description requires to fix a gauge
- Non-covariant gauge (Coulomb)
  - Static confining potential in simple truncation
  - but already perturbation theory is hard!

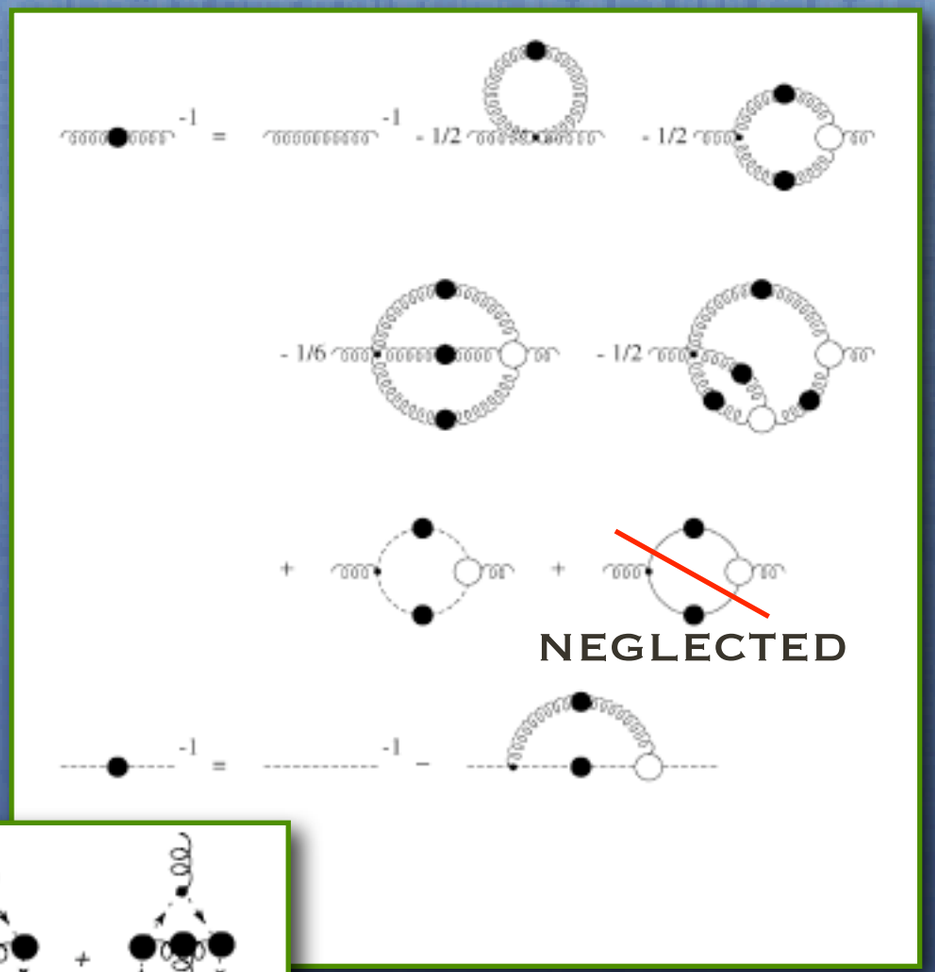
    P. Watson, H. reinhard, arXiv:0709.0140 [hep-th]
- Covariant gauge (Landau) - involves ghosts
  - UV regime and spontaneous chiral symmetry breaking obtained rather directly
  - challenge: quark confinement
- Eventually one would like to understand how they are connected (Interpolating gauges)

Gauge sector

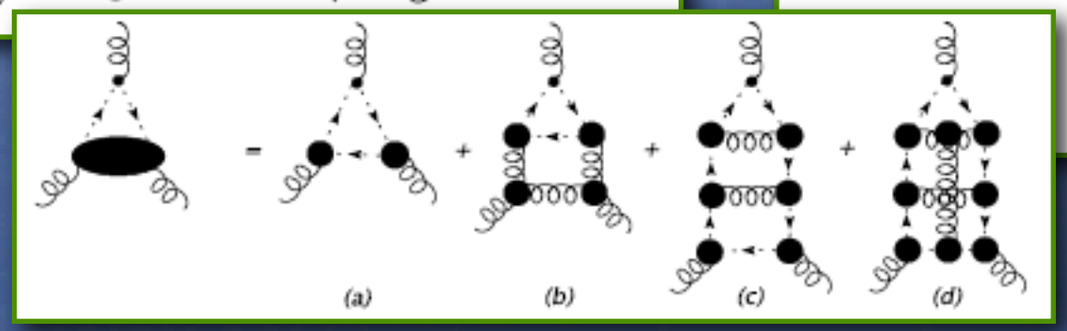
Yang-Mills DSEs

- In quenched approximation the matter part does not influence the gauge sector
  - No closed quark loops
  - Propagator DSEs coupled to vertex DSEs, e.g.





- Skeleton expansion



IR-Analysis

- Confinement is a long range / IR phenomenon
- Classical Yang-Mills theory is "conformal" but quantum Fluctuations induce a scale Λ_{QCD}
- Renormalization group:
  - far below this scale Greens functions should be described by some scaling solution

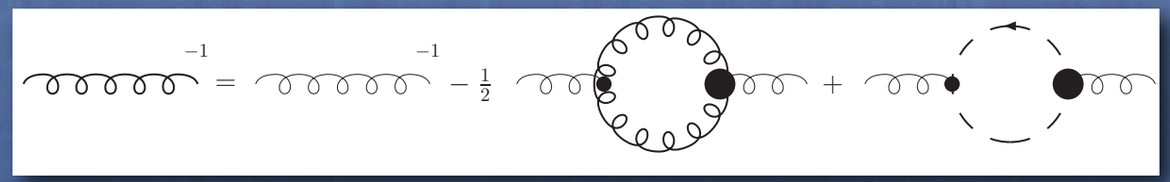
$$\Gamma \sim c \cdot \left(\frac{p^2}{\Lambda_{QCD}^2} \right)^{\delta}$$

- After the tensor decomposition the Integrals within the DSEs are dominated by the poles of the integrands

$$I_3(p,q) \equiv \int \frac{d^d k}{(2\pi)^d} \frac{1}{((k+p)^2)^{\nu_1}} \frac{1}{((k-q)^2)^{\nu_2}} \frac{1}{(k^2)^{\nu_3}}$$

Power counting

- The parametric IR-dependence of the integrals on the external scale can be obtained via a power counting analysis
  - Without numerically solving the DSEs
- Leading loop correction & leading tensor structure dominates and determines scaling of the vertex --> algebraic equations for exponents
- E.g. Gluon DSE

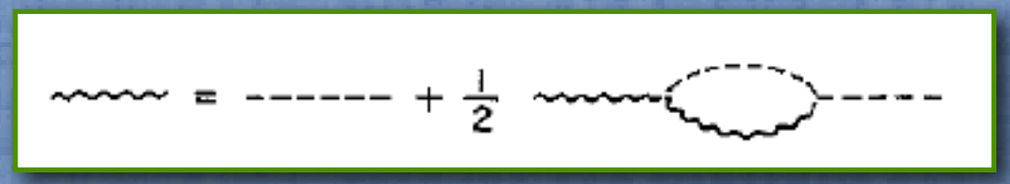


$$-\delta_{gl}+1=\min\left(1,\left(\delta_{3g}+\frac{1}{2}\right)+\frac{1}{2}+2\left(\delta_{gl}-1\right)+1,2+\left(\delta_{gg}+\frac{1}{2}\right)+\frac{1}{2}+2\left(\delta_{gh}-1\right),\cdots\right)$$

- Solvable system of such algebraic equations

Mandelstam solution

- Simplest self-consistent DSE truncation in Landau gauge S. Mandelstam, Phys. Rev. D 20 (1979) 3223
  - Only gluon DSE solved



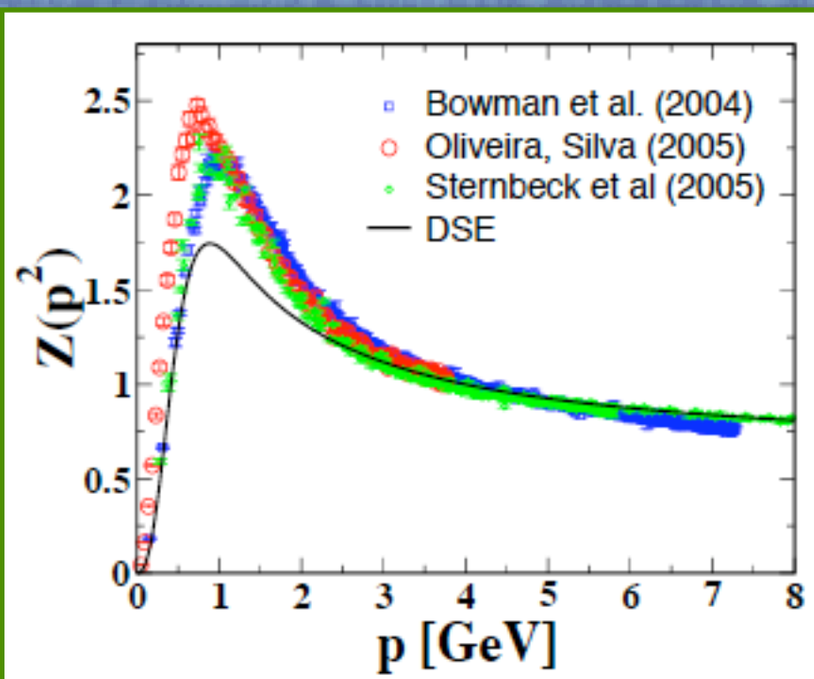
  - Ghost dynamics neglected
- Strong gluon interaction
  - IR-enhanced Gluon-propagator $1/q^4$
- Confining Forces - "infrared slavery"

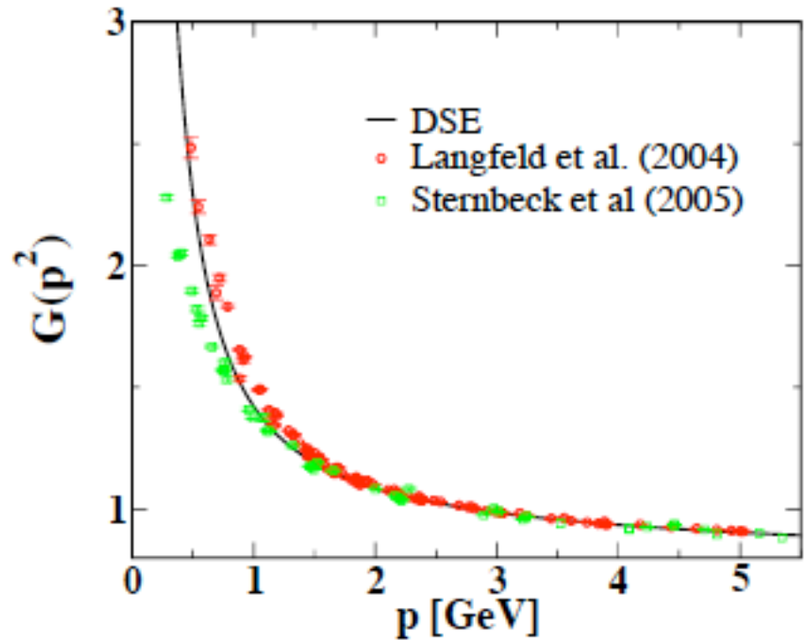
  G. West, Phys. Lett. B 115 (1983) 468
- Simple picture ... but does not seem to be realized

  R. Alkofer, M. Huber and K. Schwenzer, 0801.2762 [hep-th]

Including ghosts ...

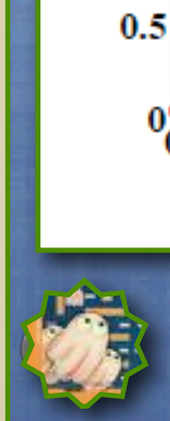
Results Compared to quenched lattice data





Reasonable qualitative agreement - but new lattice data available ...

C. S. Fischer and R. Alkofer, Phys. Lett. B 536, 177 (2002)

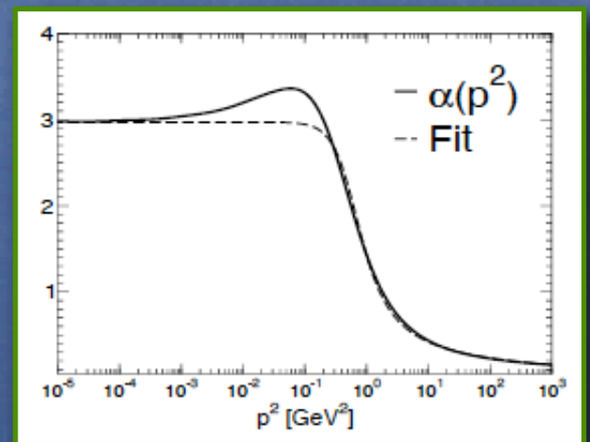


Ghost-dominance!?

$$Z(p^2) \sim (p^2)^{2\kappa}$$

$$G(p^2) \sim (p^2)^{-\kappa}$$

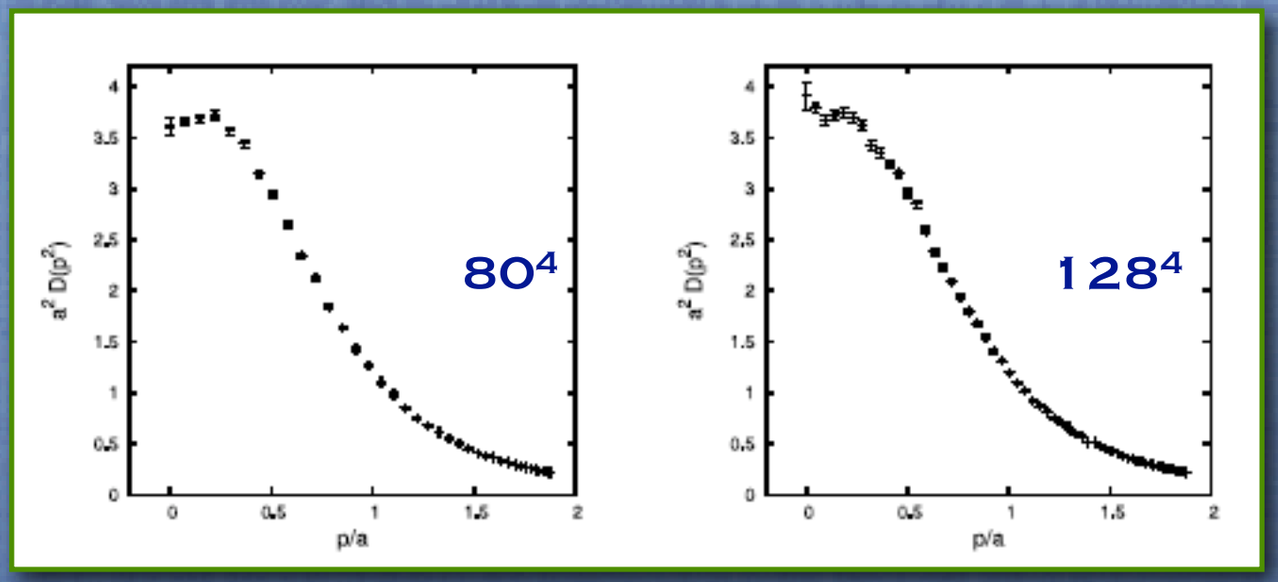
$$\kappa \approx 0.595$$



- Kugo-Ojima scenario
- Infrared limit for the coupling

New lattice data

- Challenging recent data on large lattices



A. Cucchieri, T. Mendes,
0710.0412 [hep-lat]

- point to a smaller IR-scaling parameter around 0.5 (gluon mass)
  - problems with Gribov copies?

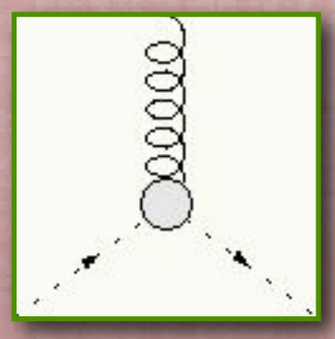
    A. Maas, in preparation
- General picture is not affected by this!

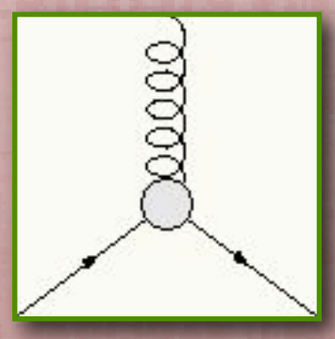
Yang-Mills vertices & Ghost dominance

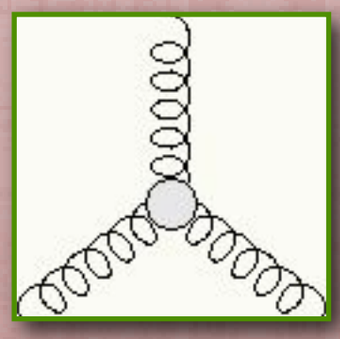
Dynamical vertices

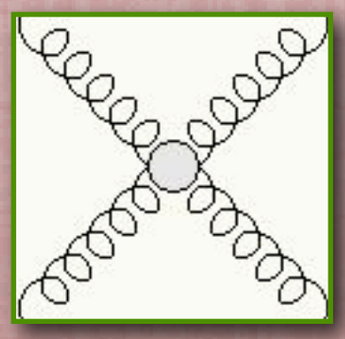
- Only **one** tensor structure per vertex is realized at the tree level

- fluctuations induce **many more structures**


2


12


14


>100

- DSE's form a huge coupled system of integral equations

--> Leo Fister´s talk

- The decomposition simplifies in various kinematic limits

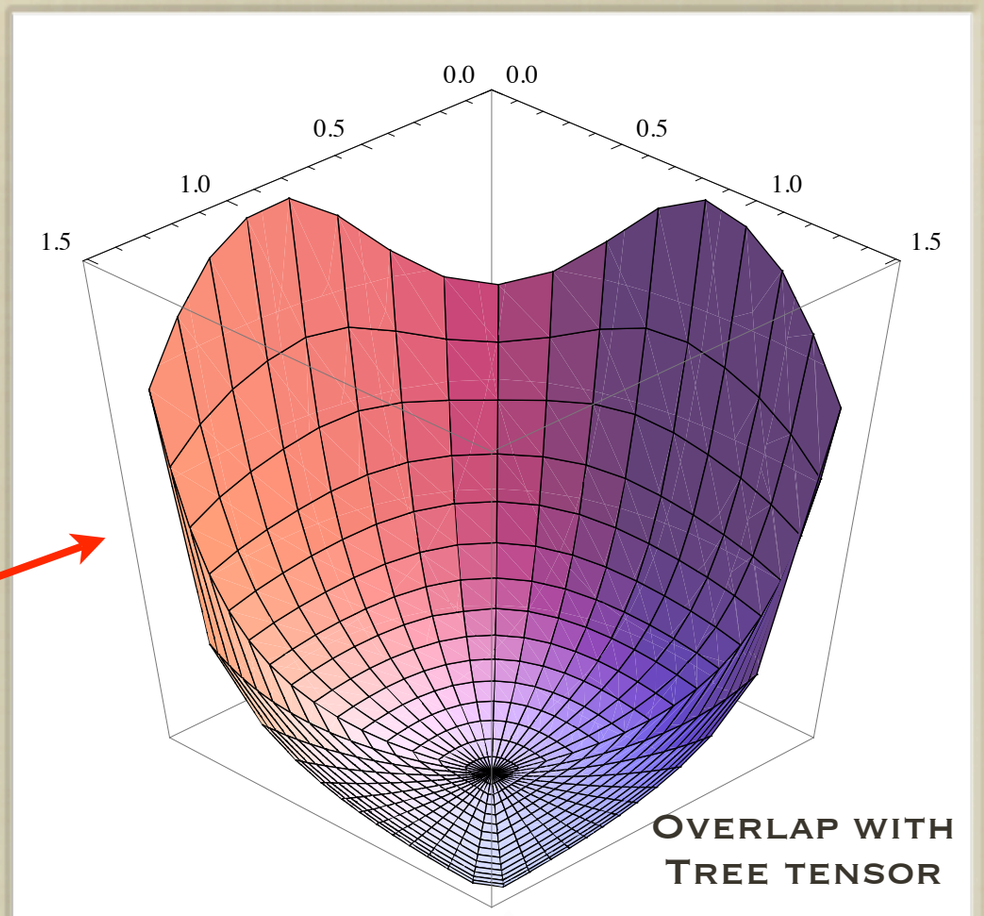
Soft singularities

- **Analytic** IR-result for the scalar vertex entering the 3-gluon vertex (agrees with numerical study)

- factorizes in a scaling part and a kinematic function

$$\Gamma_{\rm IR}(p^2,q^2,r^2) = \gamma\left(\frac{q^2}{p^2},\frac{r^2}{p^2}\right)(p^2)^{-3\kappa}$$

- **Additional kinematic ("soft") singularities** when a single gluon momentum vanishes



Soft Scaling

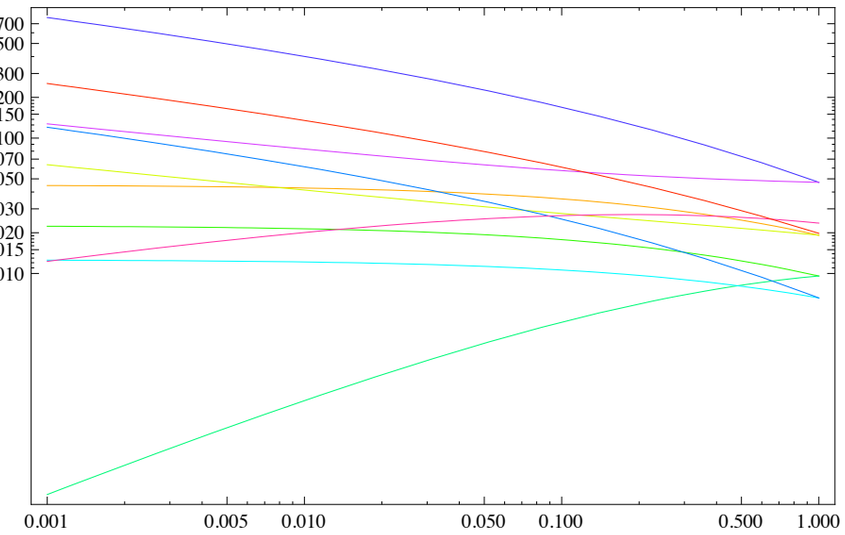
Unique non-perturbative IR-solution of the DSEs

δ_{gh}	δ_{gl}	δ^u_{gg}	δ^u_{3g}	δ^{gh}_{gg}	δ^{gl}_{gg}	δ^{gl}_{3g}	$\forall$
$-\kappa$	2κ	0	-3κ	0	$1-2\kappa$	$1-2\kappa$	$1/2 \leq \kappa \leq 3/4$

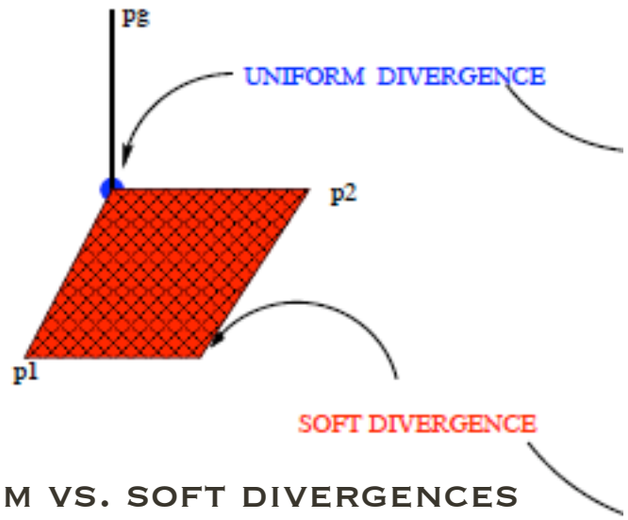
M. Huber, R. Alkofer and K. Schwenzer, 0801.2762 [hep-th]

- **Mild Soft singularities** $\sim (q^2)^{1-2\kappa}$ in form factors of both the 3-gluon vertex (and ghost-gluon) vertex whenever a **gluon momentum** vanishes
- Solves a problem!

  P. Boucaud, et. al. JHEP 0703 (2007) 76
  - No restriction from STI



3-gluon vertex

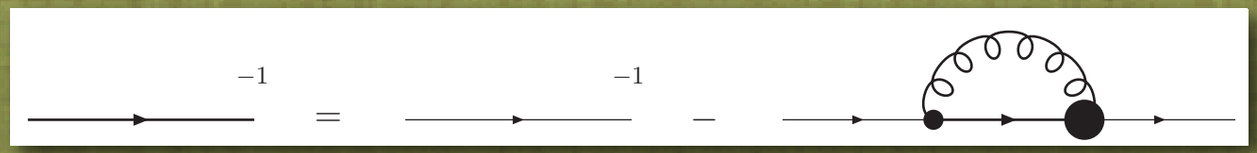


Uniform vs. soft divergences

Matter sector & Quark-gluon vertex

PROPAGATOR SOLUTION

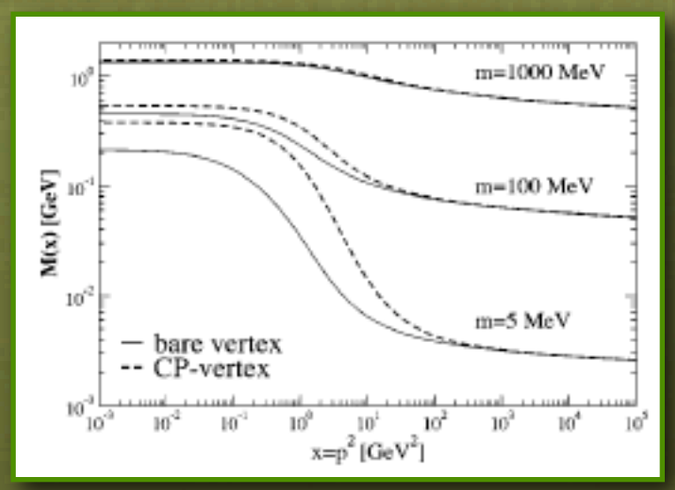
- DSE FOR THE QUARK PROPAGATOR



- TWO DIFFERENT TENSOR STRUCTURES IN THE IR REGIME
  - VECTOR PART $\sim \frac{\not{p}}{M^2}$ & SCALAR PART $\sim \frac{1}{M}$

- SPONTANEOUS CHIRAL SYMMETRY BREAKING IN THE PROPAGATOR

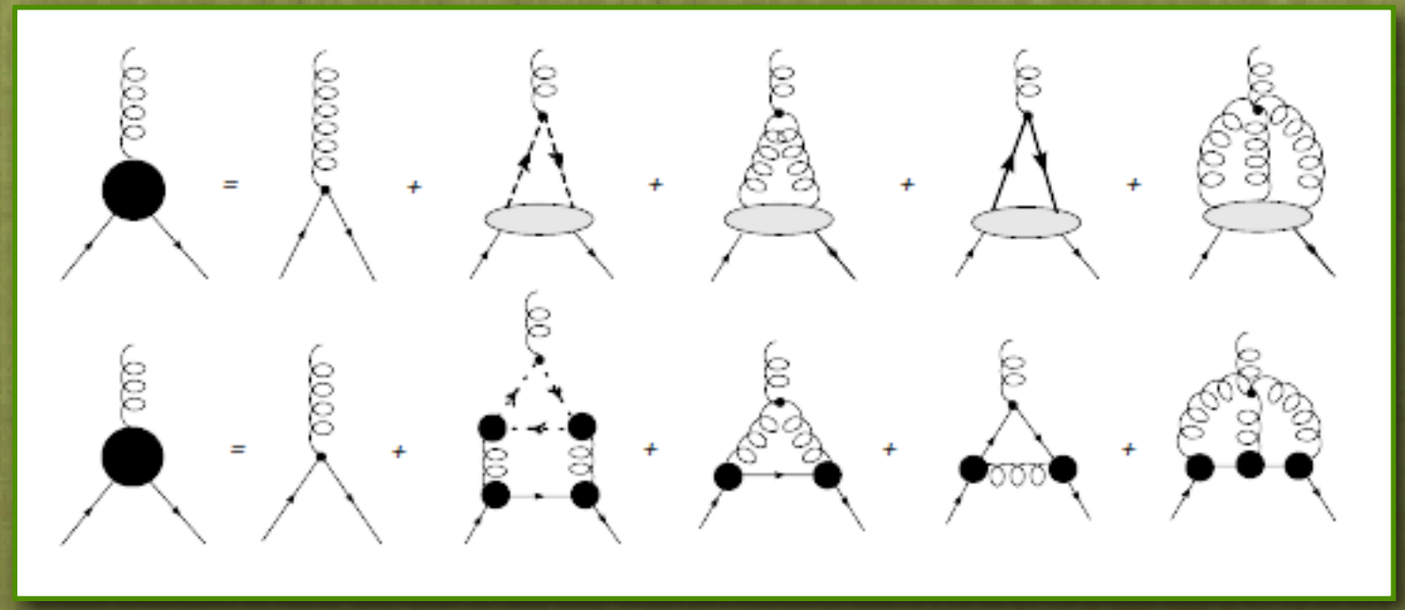
REINHARD ALKOFER
& CHRISTIAN FISCHER
PHYS. REV. D 67 (2003) 094020



- NO EXPLICIT POSITIVITY VIOLATIONS AND NO STRONG GLUONIC INTERACTIONS --> QUARKS ARE NOT CONFINED

Quark-gluon DSE

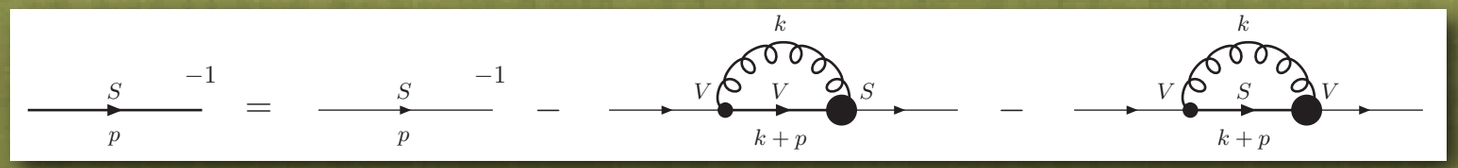
- Vertex DSE
  - skeleton expansion crucial
  - Abelian graph N_c -suppressed
  - Non-Abelian graph is "ghost enhanced"



- The 12 possible tensors fall into two classes according to their Dirac structure
  - Vector part (odd): γ^μ , $p_i^\alpha \gamma_\alpha p_j^\mu$, $p_{1,\alpha} p_{2,\beta} \sigma^{\alpha\beta} \gamma^\mu$
  - Scalar part (even): p_i^μ , $p_i^\alpha \gamma_\alpha \gamma^\mu$, $p_{1,\alpha} p_{2,\beta} \sigma^{\alpha\beta} p_i^\mu$

IR-Analysis

- The Dirac structure does not allow all possible scalar/vector combinations within a given graph
  - E.g. scalar quark DSE:



- Tensor basis that shows the all qualitative features of the full solution (complete in the kinematic limit $q = 2p$):
  - Propagator (heavy mass limit): $S(p) \to \frac{iZ_f \not{p}}{M^2} + \frac{Z_f}{M}$
  - Vertex :

$$\Gamma_\mu(p^2) = ig\left(\lambda_1(p^2)\gamma_\mu + \lambda_2(p^2)\hat{p}_\mu + \lambda_3(p^2)\not{\hat{p}}\hat{p}_\mu + \lambda_4(p^2)\not{\hat{p}}\gamma_\mu\right)$$

IR-Solution

- In general different IR-scaling for the different structures $\lambda_i(p^2) = c_i(p^2)^{\beta_i}$
- Solution of the lengthy system of equations for the uniform IR-exponents in the massive case

$$\beta_1 = \beta_2 = \beta_3 = -1/2 - \kappa \ , \ \beta_4 = 0$$

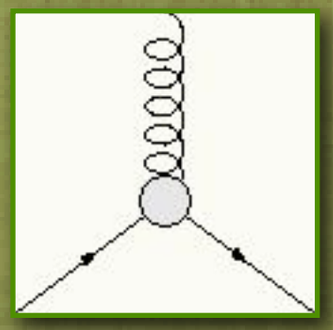
- Strongly IR-divergent vertex

R.Alkofer, C.S.Fischer & F.J.Llanes-Estrada, hep-ph/0607293

- Chiral symmetry breaking scalar parts present!
- Solution confirmed by numerical computations of the reduced DSE system
- Possibly different solution(s) for chiral quarks

Running Coupling

- Running coupling from the quark gluon vertex



$$\alpha^{q/g}(p^2) = \alpha_\mu(\Gamma^{q/g}(p^2))^2 Z_f^2(p^2) Z(p^2) \to \infty$$

- Enhanced vertex could overturn the suppression of the gluon propagator

- But only in the uniform limit $p_i^2 \to 0\ ,\ \forall i$

  - Corresponds to the light cone in Minkowski space

- NO total confinement for massive quarks

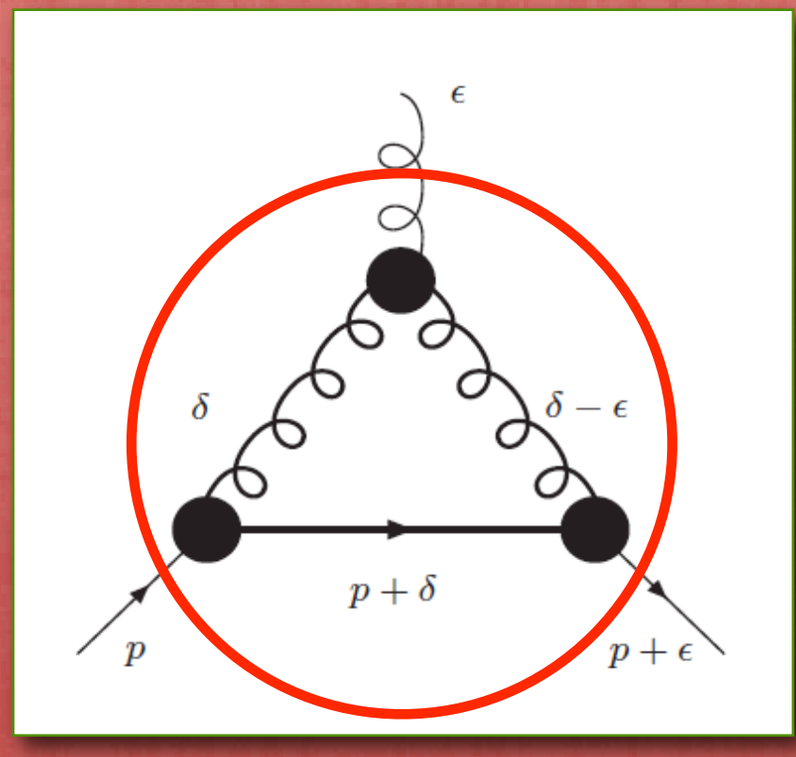
A mechanism for quark confinement

Soft IR-singularity

- The quark-gluon vertex has an additional soft divergence when only the gluon vanishes

- Mild soft divergence of the 3-gluon vertex induced in the Non-Abelian graph

- And even self-consistently enhanced!



- Same strong IR-scaling in the soft gluon limit for arbitrary quark kinematics

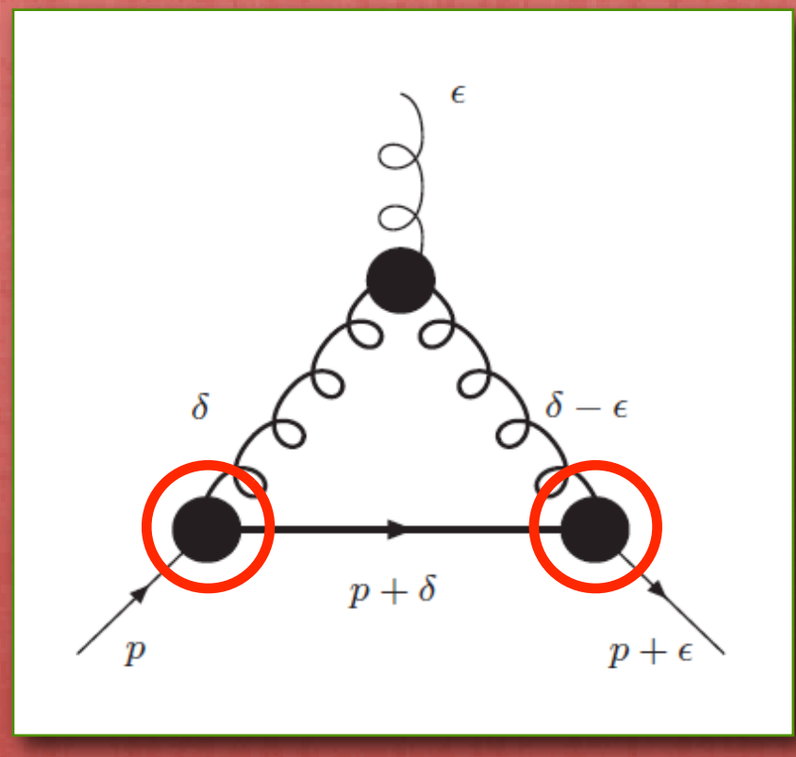
  Application --> Richard Williams´ talk

- Supported by first numerical results ...

Soft IR-singularity

- The quark-gluon vertex has an additional soft divergence when only the gluon vanishes

- Mild soft divergence of the 3-gluon vertex induced in the Non-Abelian graph



- And even self-consistently enhanced!

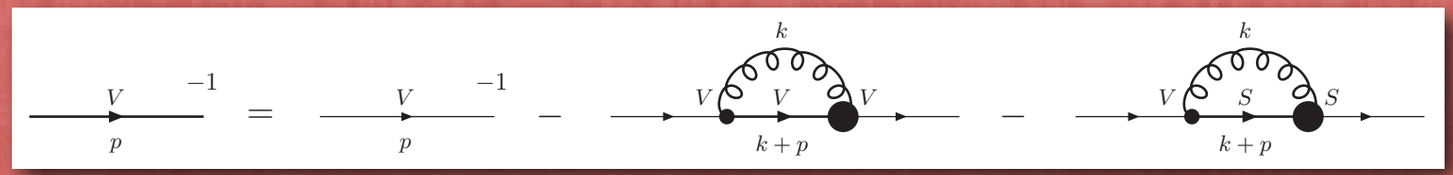
- Same strong IR-scaling in the soft gluon limit for arbitrary quark kinematics

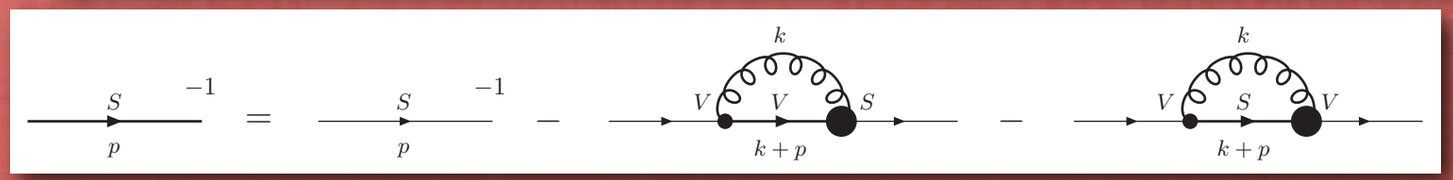
  Application --> Richard Williams´ talk

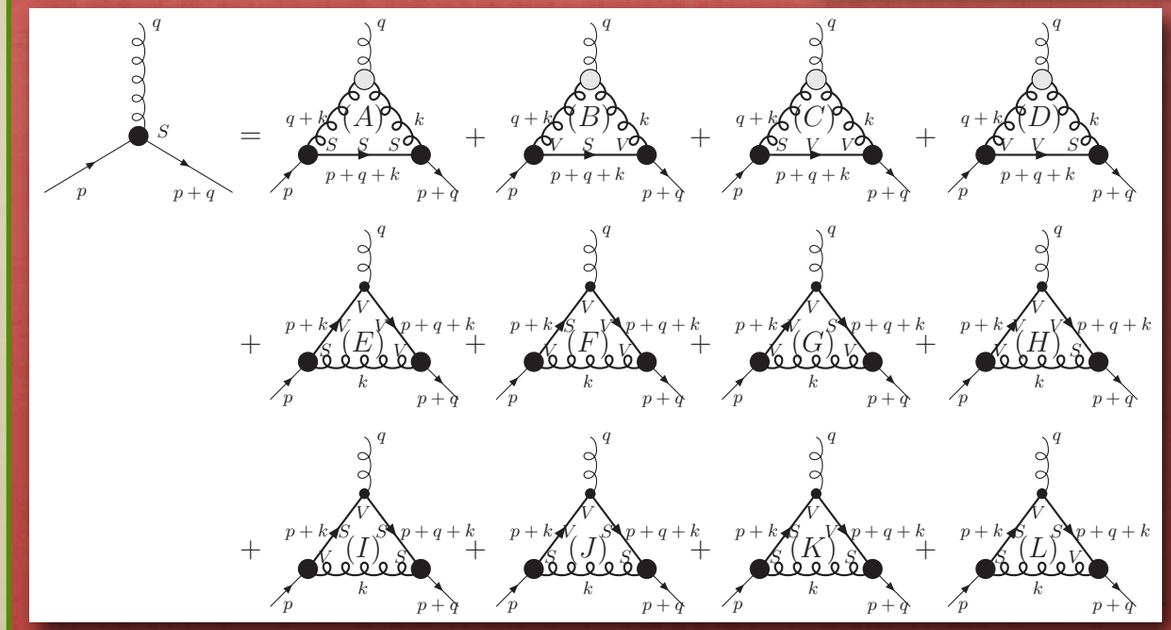
- Supported by first numerical results ...

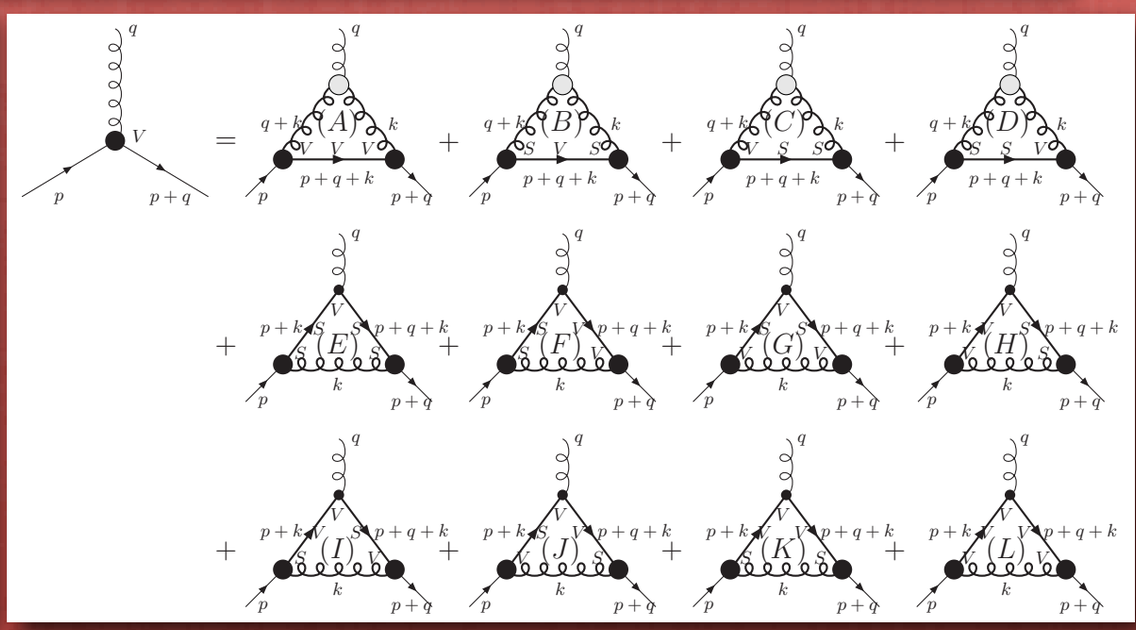
Quenched IR-system

- Gauge correlators as an input
- **Different** kinematic limits (uniform & soft)
- Scalar & vector tensors behave differently





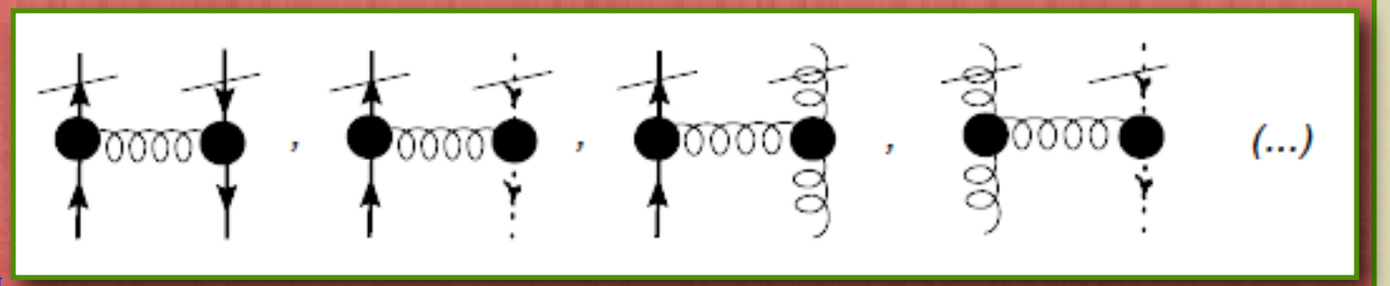




Quenched IR-solution

- **Different IR exponents** for uniform and kinematic singularities

- Employ necessary conditions for a **skeleton expansion**



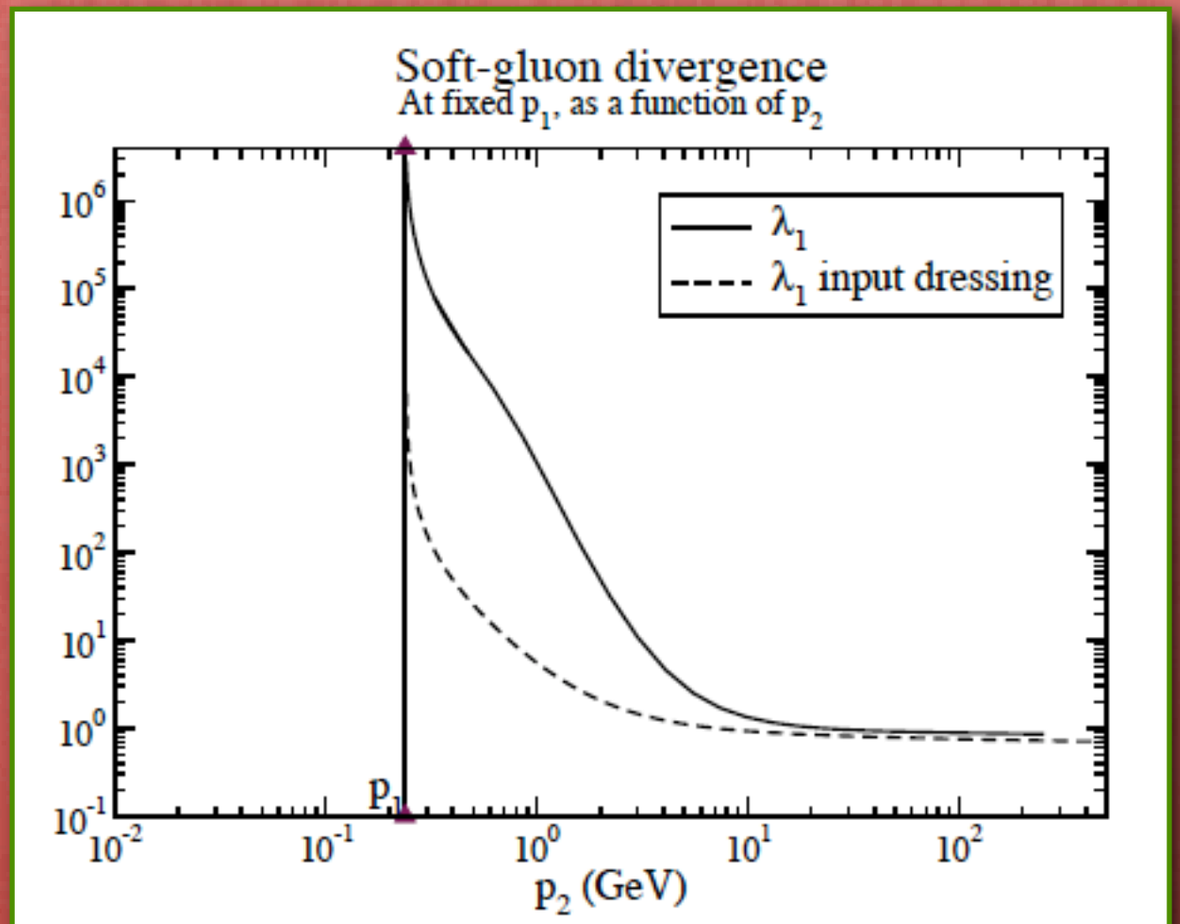
- System simplifies strongly
  - Uniform and soft-gluon sectors decouple
  - **"Nearly unique" solution:**

$$
\begin{aligned}
-\delta_{qv} &= \min\left(0, \delta^u_{qgv} + \delta_{qv} + \frac{3}{2} + 2\kappa, \delta^u_{qgs} + \delta_{qs} + 1 + 2\kappa\right)\\
-\delta_{qs} &= \min\left(0, \delta^u_{qgs} + \delta_{qv} + \frac{3}{2} + 2\kappa, \delta^u_{qgv} + \delta_{qs} + 1 + 2\kappa\right)\\
\delta^u_{qgv} &= \min\left(2\delta^u_{qgv} + \delta_{qv} + 1 + \kappa, 2\delta^u_{qgs} + \delta_{qv} + 1 + \kappa, 1 - 2\kappa\right)\\
\delta^u_{qgs} &= \min\left(2\delta^u_{qgs} + \delta_{qs} + \frac{1}{2} + \kappa, 2\delta^u_{qgv} + \delta_{qs} + \frac{1}{2} + \kappa, 1 - 2\kappa\right)\\
\delta^{gl}_{qgv} &= \min\left(2\delta^{gl}_{qgv} + \frac{1}{2} + \kappa, 2\delta^{gl}_{qgs} + \frac{1}{2} + \kappa, 1 - 2\kappa\right)\\
\delta^{gl}_{qgs} &= \min\left(2\delta^{gl}_{qgs} + \frac{1}{2} + \kappa, 2\delta^{gl}_{qgv} + \frac{1}{2} + \kappa, 1 - 2\kappa\right)
\end{aligned}
$$

δ_{gh}	δ_{gl}	δ_q	δ^u_{gg}	δ^u_{3g}	δ^u_{qg}	δ^{gh}_{gg}	δ^{gl}_{gg}	δ^{gl}_{3g}	δ^q_{qg}	δ^{gl}_{qg}	$\forall$
$-\kappa$	2κ	0	0	-3κ	$-\frac{1}{2} - \kappa \vee 1-2\kappa$	0	$1-2\kappa$	$1-2\kappa$	0	$-\frac{1}{2} - \kappa \vee 1-2\kappa$	$1/2 \le \kappa \le 3/4$

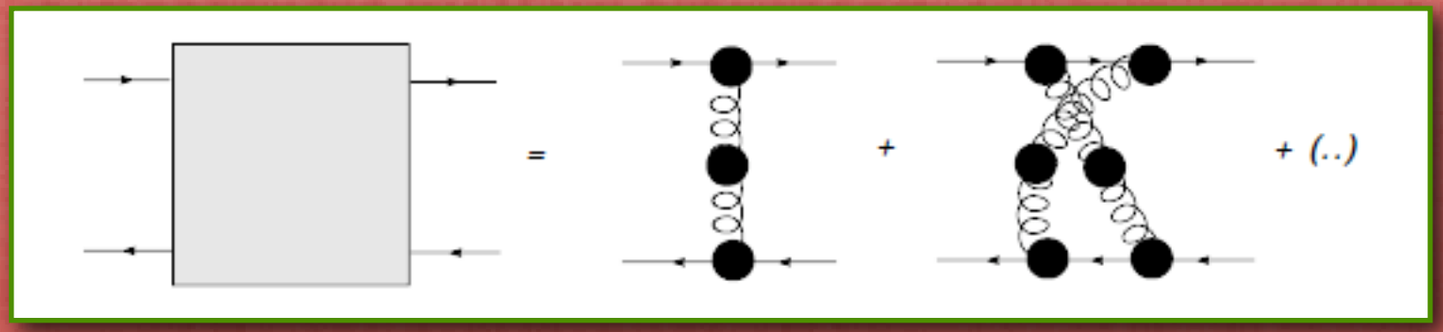
Numerical analysis

- Mild soft divergence from 3-gluon vertex as input
- Self-consistent solution of the quark propagator
- Single iteration of the vertex integral
- Singularity strongly enhanced
- Very likely that the solution is actually realized



Quark confinement

- Quark gluon vertex induces quark-quark interaction
- **Quark confinement** due to a IR-divergent quark-quark scattering kernel

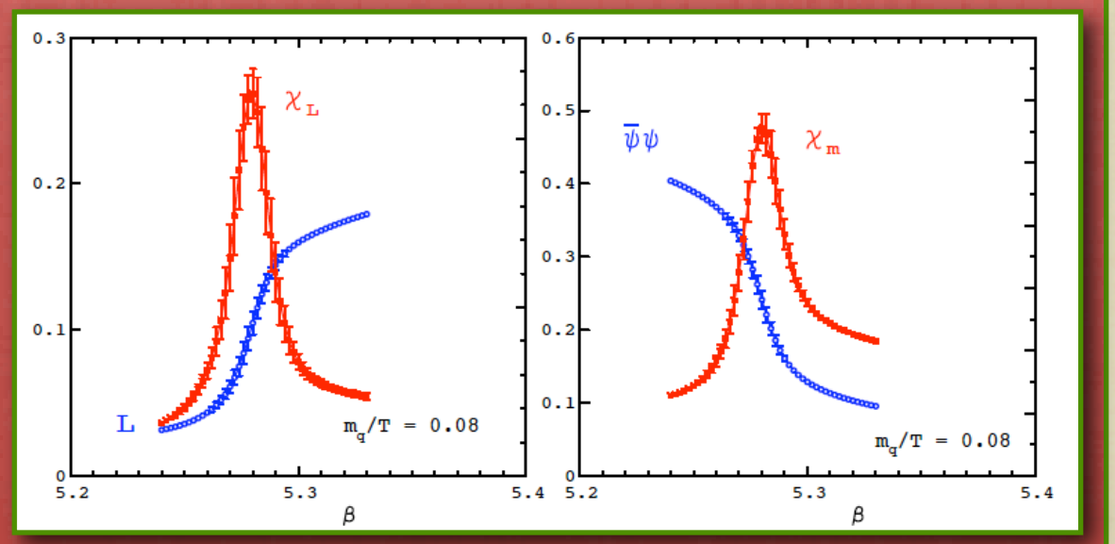


$$T^{4q} \sim 1/p^4$$

- Linear rising interaction whenever the quarks are far apart
- **Infrared slavery**
  - different mechanism than for the gluons
- Only possible if **chiral symmetry** is broken

Deconfinement

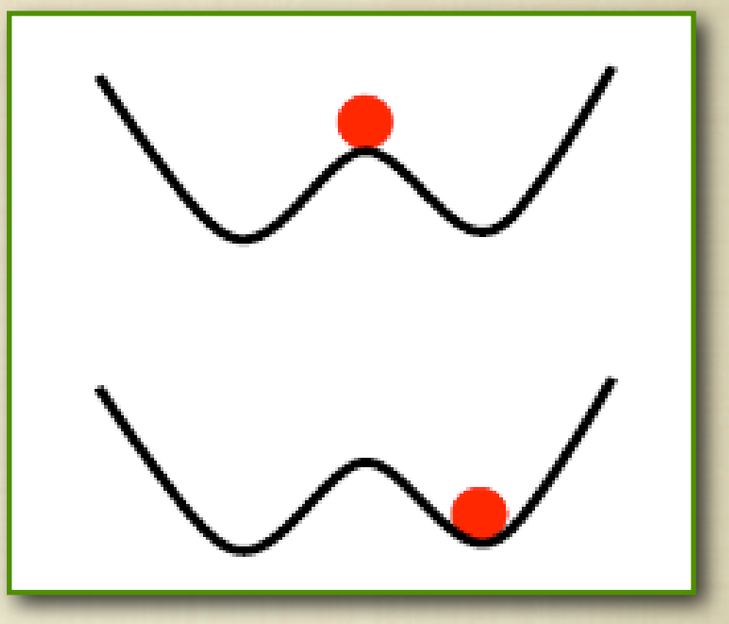
- In the chiral symmetric regime the symmetry-breaking tensors vanish
- Coupled (gauge/matter sector) analysis necessary
- Different solution, e.g. at most: $\beta_1 = \beta_3 = -\kappa$
- Too weak singularities
- chiral restoration and deconfinement transition simultaneous
- In agreement with (some) lattice results



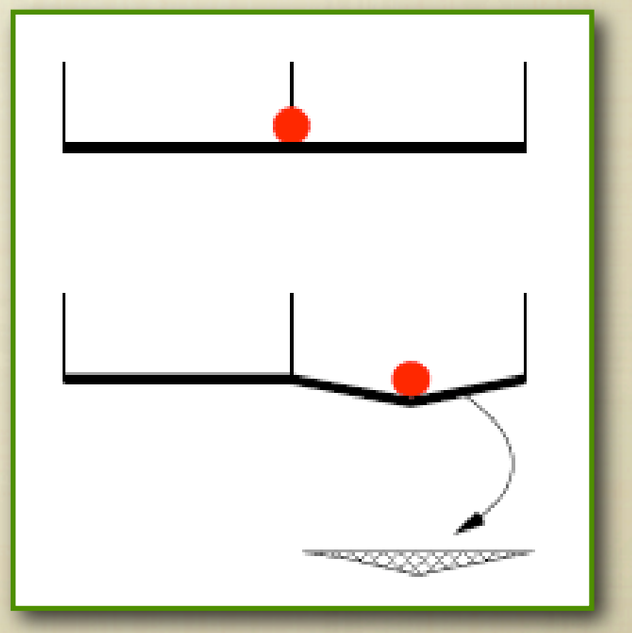
F. Karsch,
Lect. Notes Phys. 583 (2002) 209

A Mechanical Analog

- Simple picture for ordinary chiral symmetry breaking
  - Potential from bosoniza-tion of short ranged force
  - Minimum position determines the quark mass



- Vertex enhanced chiral symmetry breaking
  - Bending of the elastic band is analog to the size of the scalar vertex part
  - Both mechanism are linked



Conclusion

- Novel mechanism for total quark confinement in Landau gauge QCD
  - relies on the vertex and not on the propagator
  - Qualitatively Consistent with YM lattice data
    - valid for arbitrary quark kinematics due to soft-gluon singularities
- Old infrared slavery picture - Very Different from the confinement mechanisms for gluons
- Dynamical Connection between chiral symmetry breaking & confinement

Outlook

- Gauge invariant formulation via Wilson loop
- Improved Yang-Mills IR-fixpoint
- Unquenching effects & String breaking
  - DSEs are a Framework to study this
  - First hints that the IR-fixpoint indeed changes when quark loops are included
  - Becomes a problem of finite (hadronic) scales where which mechanism dominates
- ... Analysis of the hadron spectrum