Bethe-Salpeter equation studies of mesons: recent progress and challenges

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Work with:

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Work performed at/supported by/in collaboration with:

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Argonne National Laboratory

University of Graz

Outline

- QCD and hadrons
- DSE-BSE
- Solution strategies
- Mesons and their properties
- Symmetries $\leftrightarrow$ exact results
- Example (sophisticated) Ansatz
- Example results
- Conclusions and outlook

Motivation

- How can we describe nature?

Motivation

- How can we describe nature?

With the Force!

Motivation

- 

The Force?

Motivation

- 

Well, the Force is what gives a Jedi his power.
It's an energy field created by all living things.
It surrounds us and penetrates us.
It binds the galaxy together.

Motivation

- Dyson Schwinger Equations:
  a modern method in relativistic QFT

  P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12** (2003) 297
  R. Alkofer and L. von Smekal, Phys. Rept. **353** (2001) 281
  C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. 45 (2000) S1
  A. Holl, C. D. Roberts, S. V. Wright, nucl-th/0601071
  C. S. Fischer, J. Phys. G **32** (2006) R253

- Dyson Schwinger Equations:
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- Study hadrons as composites of quarks and gluons . . .
- . . . including:
  - Chiral symmetry and $D\chi SB$
  - correct perturbative limit (via $\alpha_p(Q^2)$)
  - quark and gluon confinement
  - Poincaré covariance
- Propagators and Bethe-Salpeter amplitudes $\rightarrow$ observables

- Solutions: Schwinger functions
  (Euclidean Green functions)
  (also calculated on the lattice)

DSEs

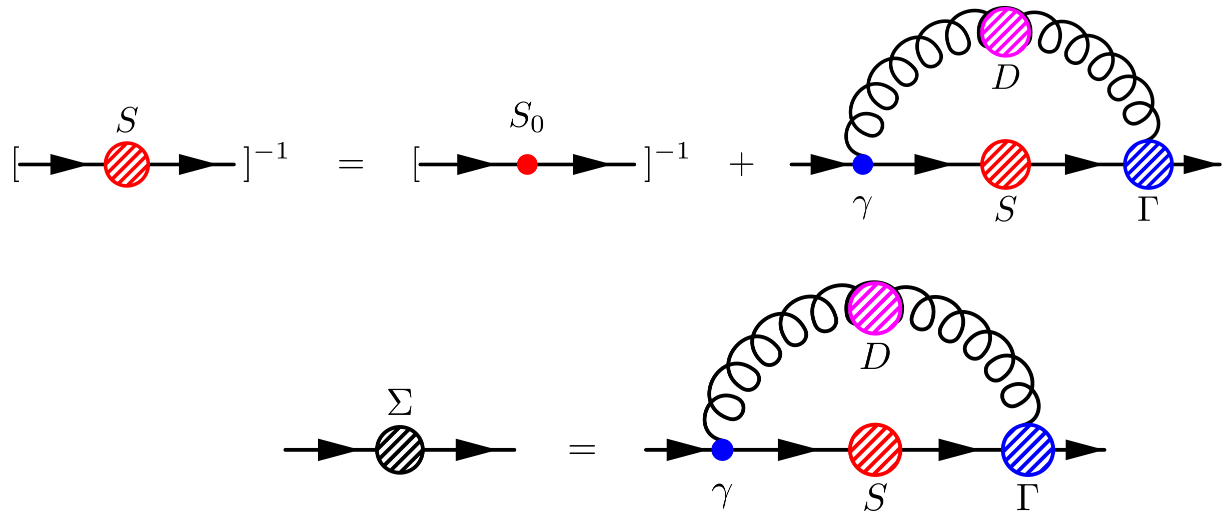
- Solutions: Schwinger functions
  (Euclidean Green functions)
  (also calculated on the lattice)
- Each function satisfies integral equation involving other functions $\Rightarrow$
- Infinite set of coupled integral equations
- Truncation scheme necessary $\Rightarrow$
- Generating tool for perturbation theory

DSEs

- Solutions: Schwinger functions
  (Euclidean Green functions)
  (also calculated on the lattice)
- Each function satisfies integral equation involving other functions $\Rightarrow$
- Infinite set of coupled integral equations
- Truncation scheme necessary $\Rightarrow$
- Nonperturbative truncation scheme
- Respect symmetries
- Prove exact (model independent) results
- Devise (sophisticated) models to illustrate them

Gap Equation

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

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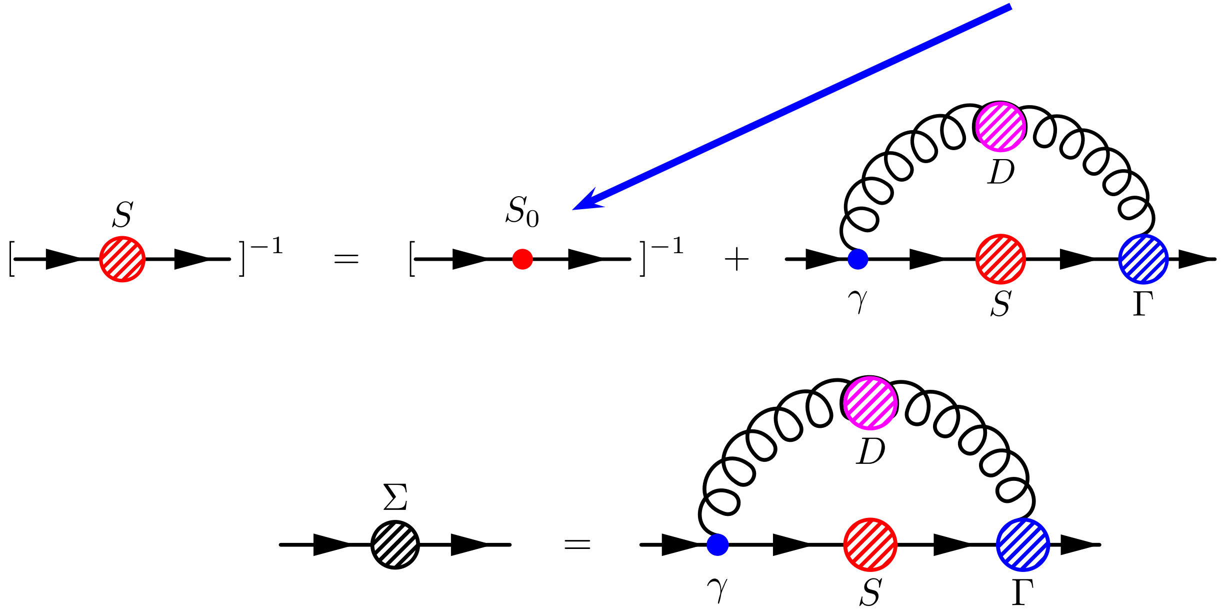
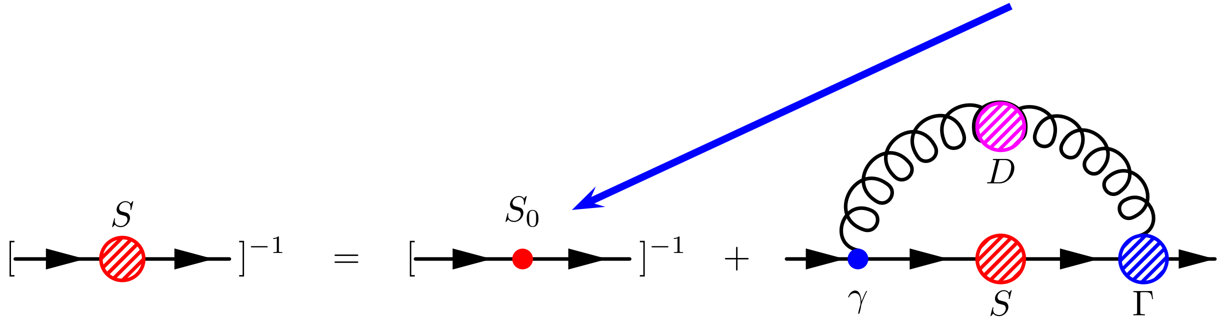
current quark mass m_ζ

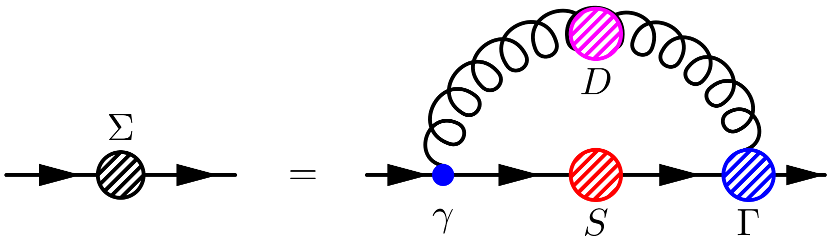


Gap Equation

$$\color{red}{S(p)} = \frac{Z(p^2)}{i\gamma \cdot p + \color{green}{M(p^2)}}$$

current quark mass m_ζ





- Weak coupling expansion reproduces every diagram in perturbation theory, but:
- Perturbation theory: $m_\zeta = 0 \;\Rightarrow\; \color{red}{M(p^2) \equiv 0}$

Quark Mass Function

Solution of gap equation: $$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

P. Maris, C. D. Roberts, Phys. Rev. **C56**, 3369 (1997)

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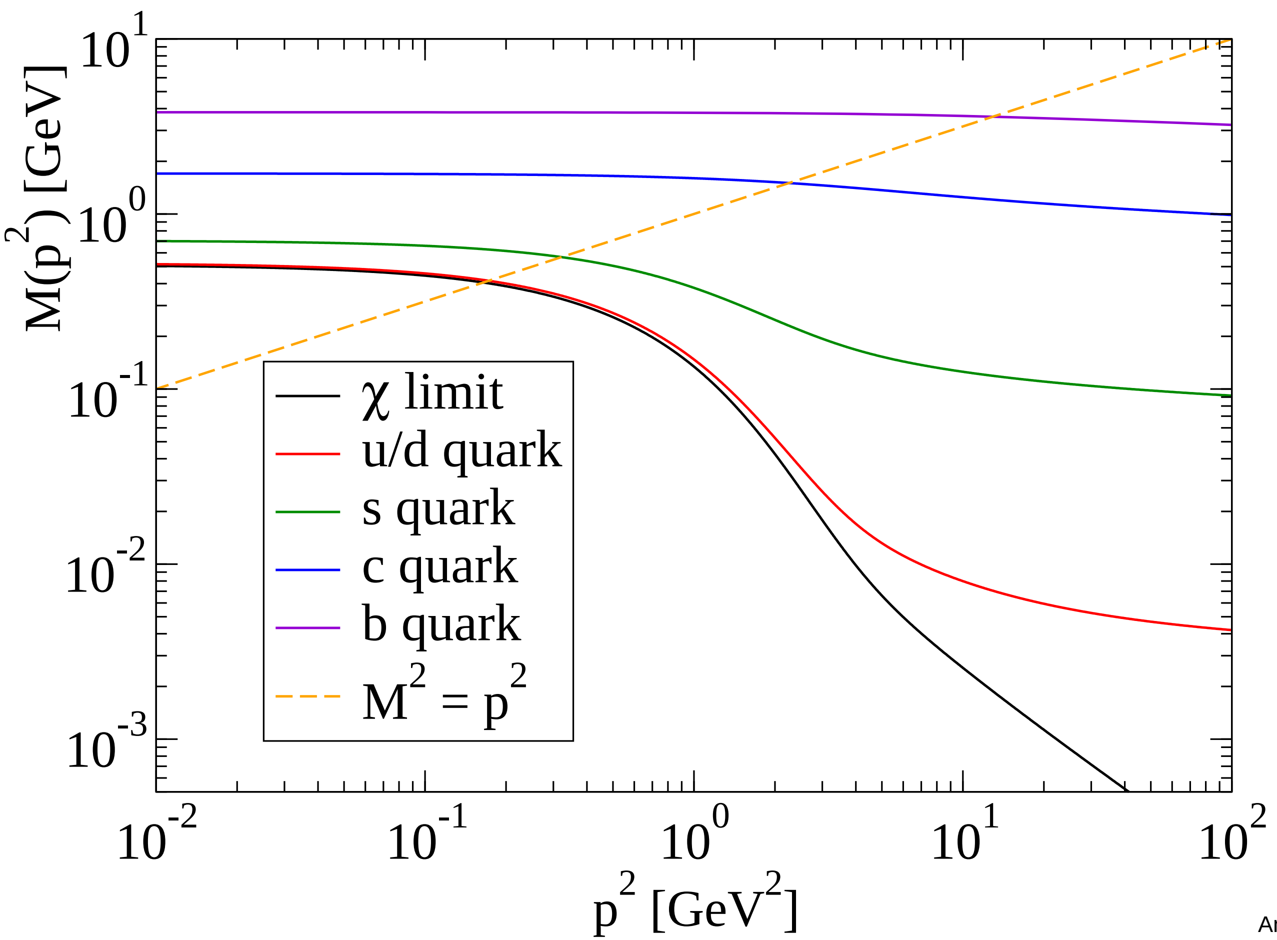

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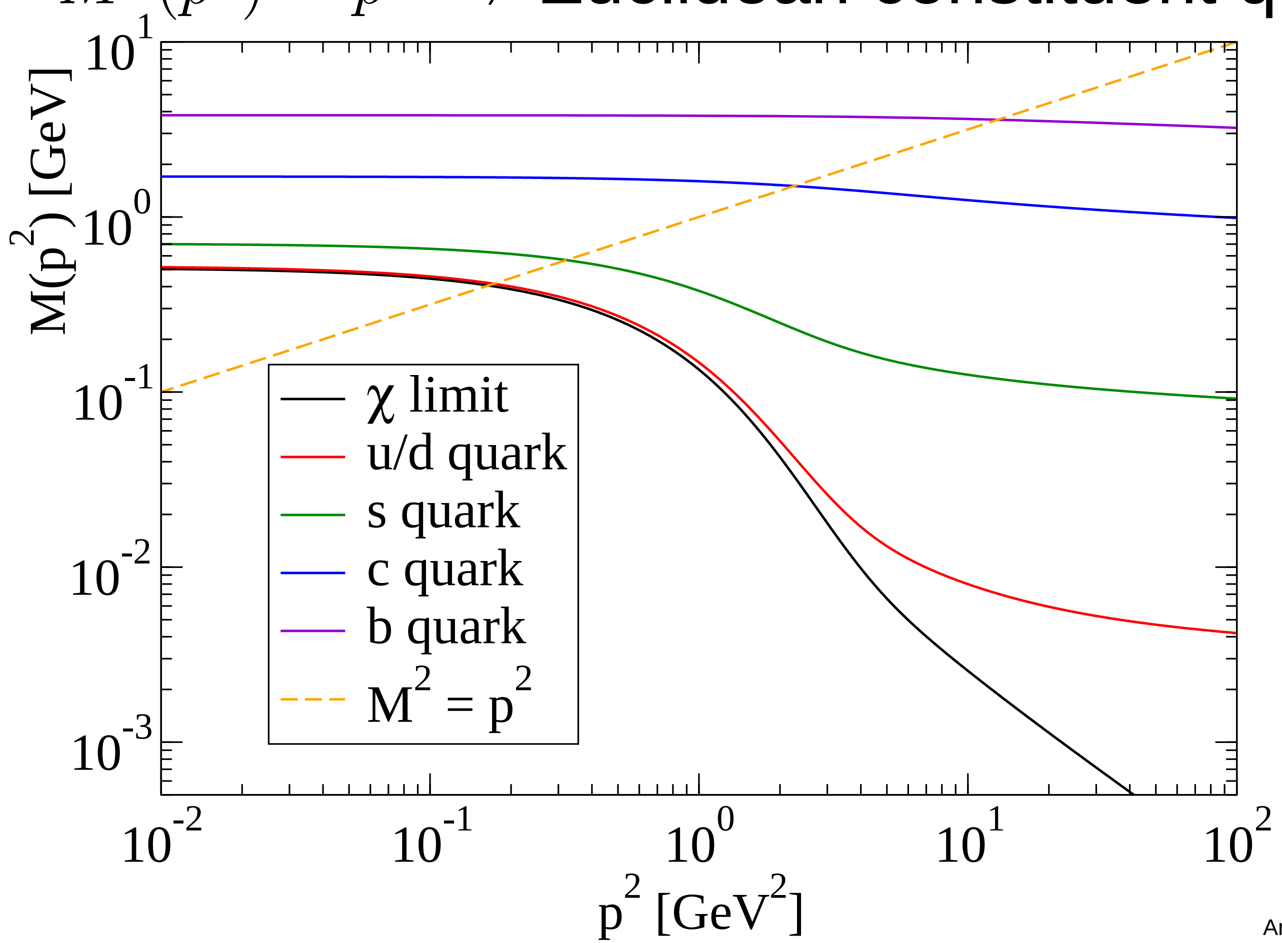
$M^2(p^2) = p^2 \;\Rightarrow$ Euclidean constituent quark mass M_E



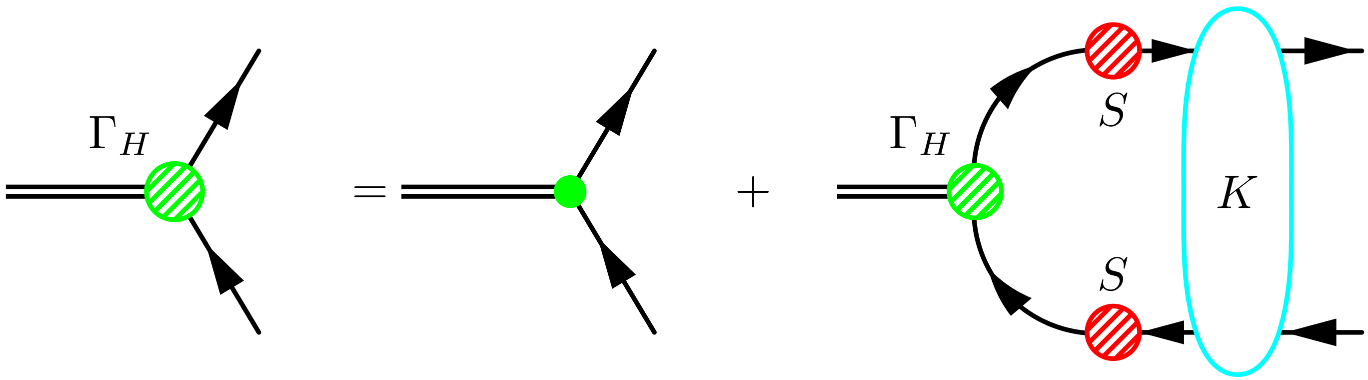
q	M_E/m_ζ
χ	∞
u/d	100
s	7
c	1.7
b	1.2

$\rightarrow \; D\chi SB$

Inhomogeneous BSE

- BSE for $q\bar{q}$ or qq bound states ($\chi = S\,\Gamma_H\,S$)

$$\Gamma_H(p;P) = \text{d.t.} + \int d^4q\;\chi(q;P)\,K(q,p;P)\,.$$

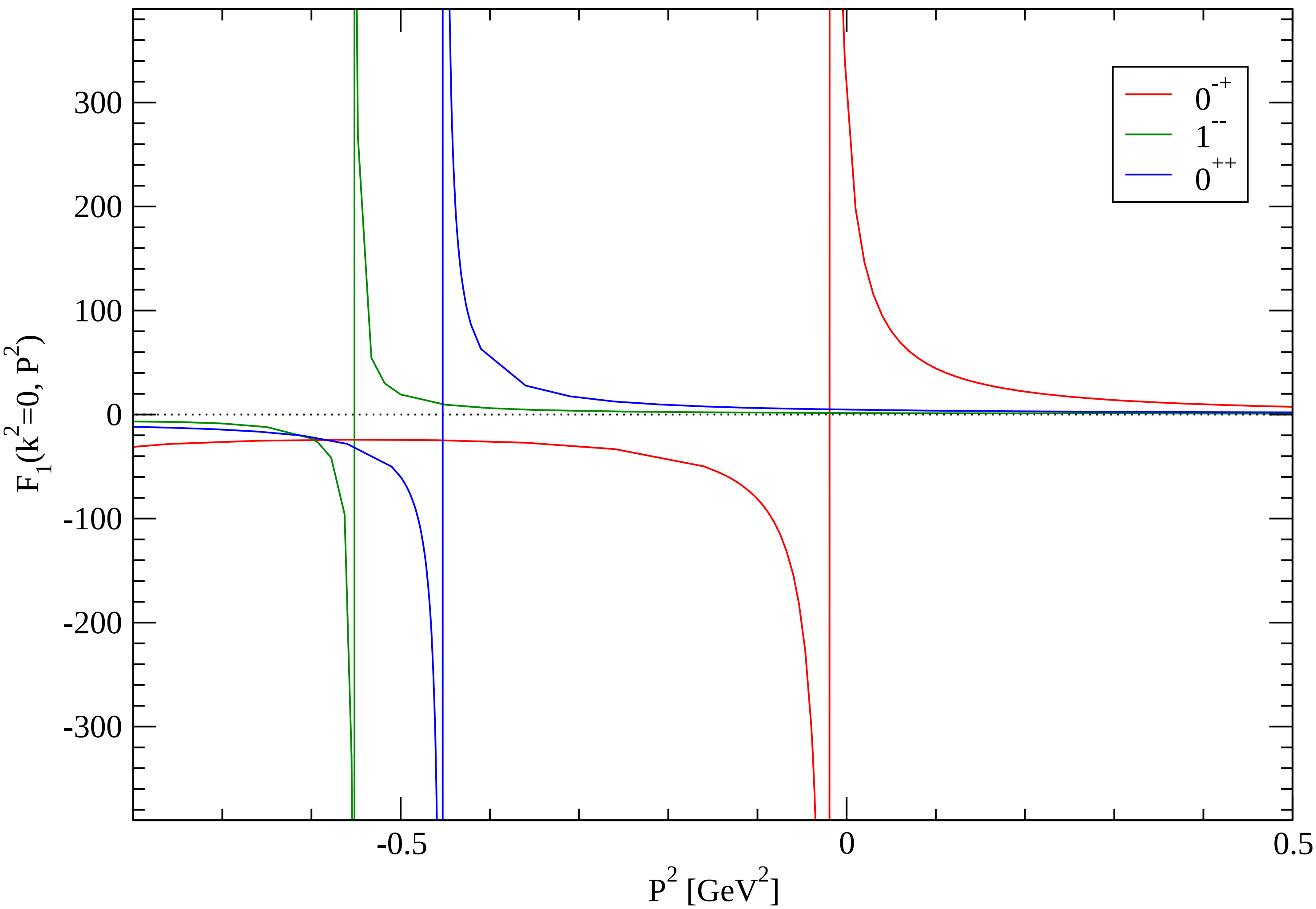


- Gap eq. output $\rightarrow$ BSE input
- Bound state at $P^2 = -m_H^2$:

$$\Gamma_H(q;P) = \frac{r_H\;\Gamma_h(q;P)}{P^2 + m_H^2} + \text{regular terms}$$

Inhomogeneous BSE

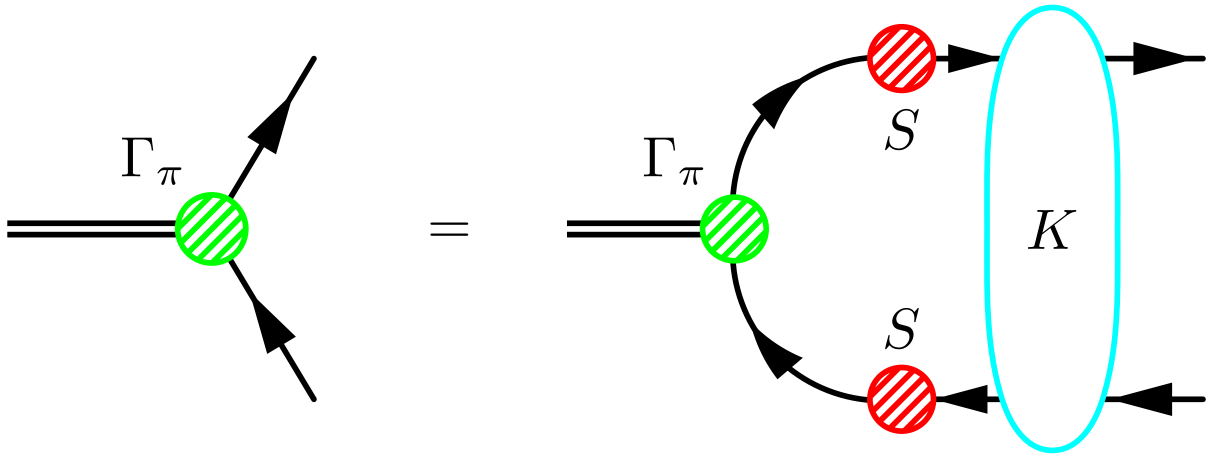
- 0^{-+}, 0^{++}, and 1^{--} meson amplitudes



Homogeneous BSE

- BSE for $q\bar{q}$ or qq bound states ($\chi = S\,\Gamma_h\,S$)

$$\Gamma_{h\;tu}(p;P) = \int d^4q\;[\chi(q;P)]_{sr}\,K^{tu}_{rs}(q,p;P)\,.$$



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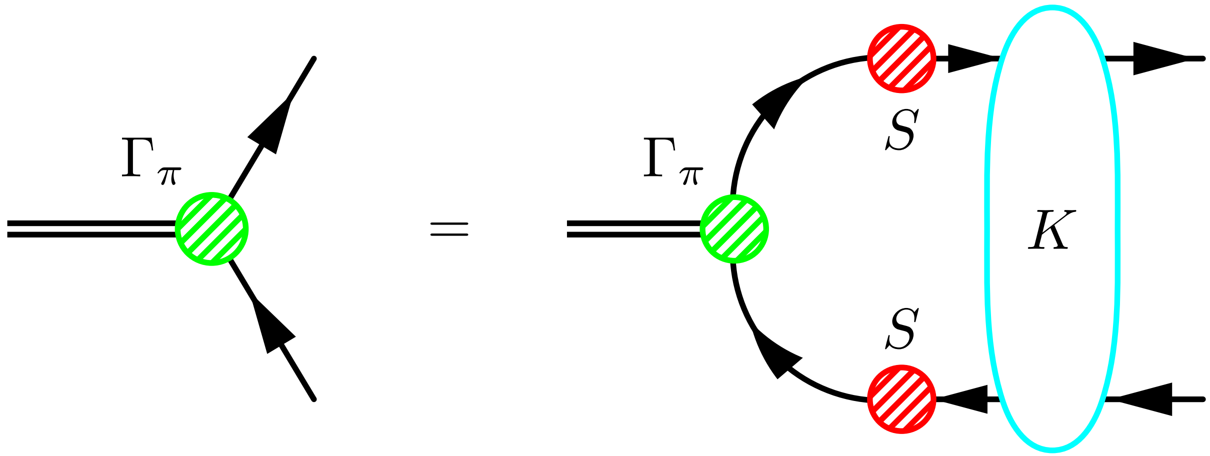
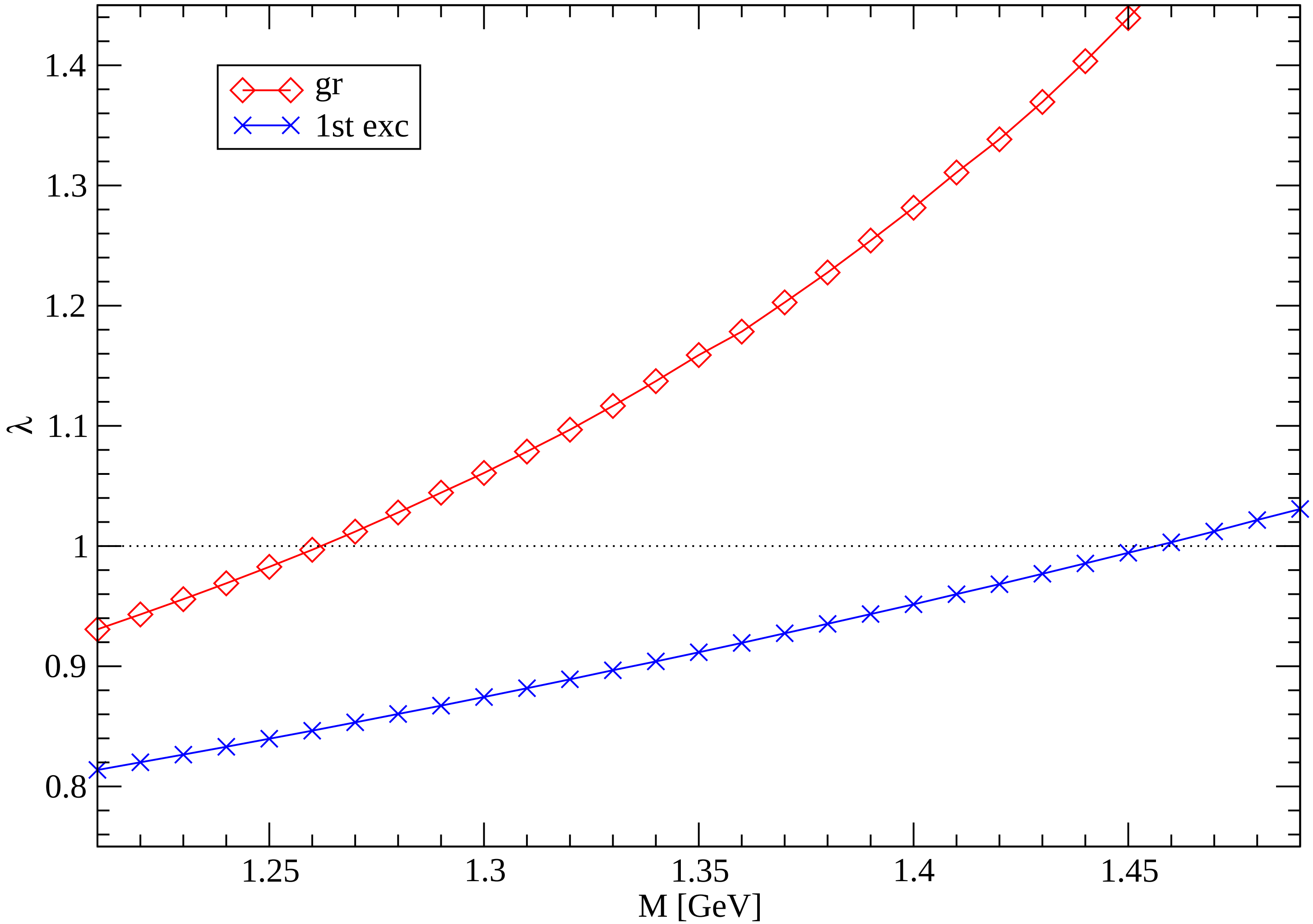
$$\Gamma_{h\,tu}(p;P)\;\lambda(P^2) \;=\int d^4q\;[\chi(q;P)]_{sr}\,K^{tu}_{rs}(q,p;P)\,.$$

- homogeneous $\rightarrow$ eigenvalue equation



Solution Strategy

- Solution strategy for homogeneous BSE



- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^j_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 \frac{\tau^j}{2} + i\gamma_5 \frac{\tau^j}{2} S^{-1}(k_-) - 2i\, m(\zeta)\, \Gamma^j_5(k;P),$$

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- Consequence (residues):

$$f_{\pi_n} m^2_{\pi_n} = 2\, m(\zeta)\, \rho_{\pi_n}(\zeta)\,;$$

AV WTI

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- Consequence (residues):

$$f_{\pi_n} m^2_{\pi_n} = 2\, m(\zeta)\, \rho_{\pi_n}(\zeta)\,;$$

- valid for every pseudoscalar meson
- valid for every current quark mass
- $\Rightarrow$ GMOR, PCAC

P. Maris, C. D. Roberts, Phys. Rev. **C56**, 3369 (1997)
A. Höll, A. K., and C. D. Roberts, Phys. Rev. C **70**, 042203 (2004)

Mass Formula

- Investigate the chiral limit of

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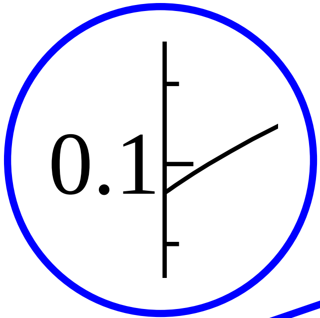
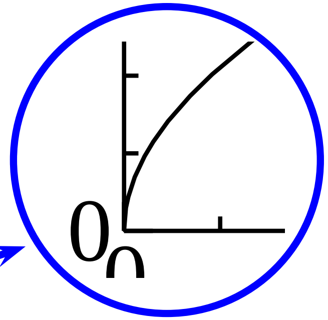
  - $f_{\pi_{gr}}$ finite
  - $m_{\pi_{gr}} \to 0$





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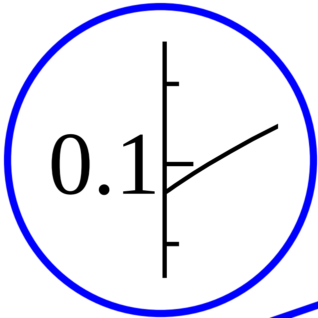
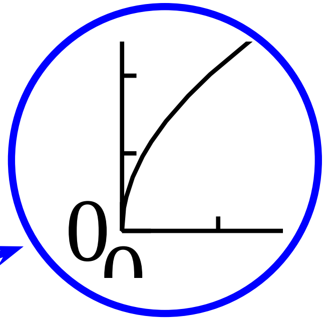
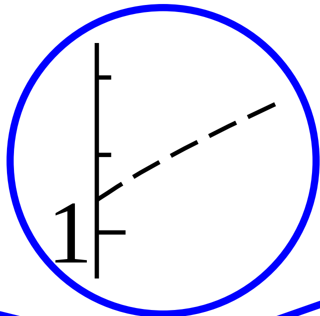
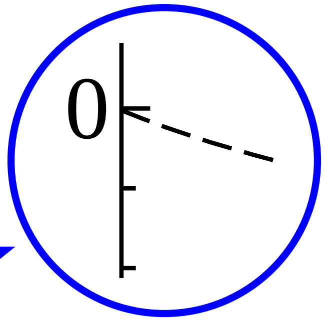




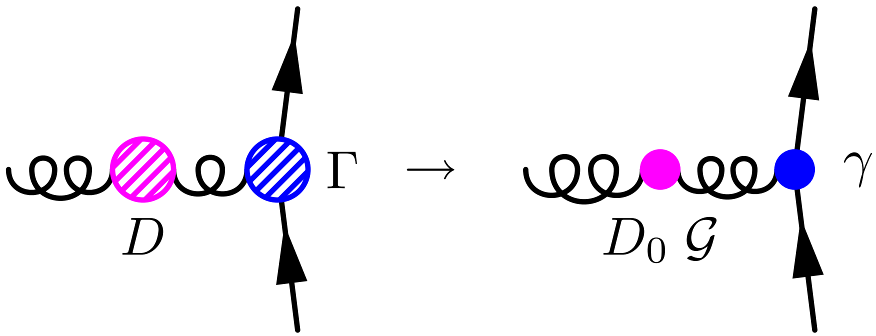
- Excited state pion:
  - $m(\zeta) \to 0$
  - $m_{\pi_{exc1}}$ finite
  - $f_{\pi_{exc1}} \to 0$





Rainbow-Ladder (RL) Truncation

- Rainbow approximation for gap equation
- Ladder approximation for BSE



- Effective coupling $\mathcal{G}$
- Bare quark-gluon vertex γ_ν
- Bare gluon propagator $D_{\mu\nu}^{\mathrm{free}}(p-q)$
- How good is this?

Model Building

Learn about the Force, Luke.

Model Building

- What do we know?
- Effective running coupling $\mathcal{G}(Q^2)$

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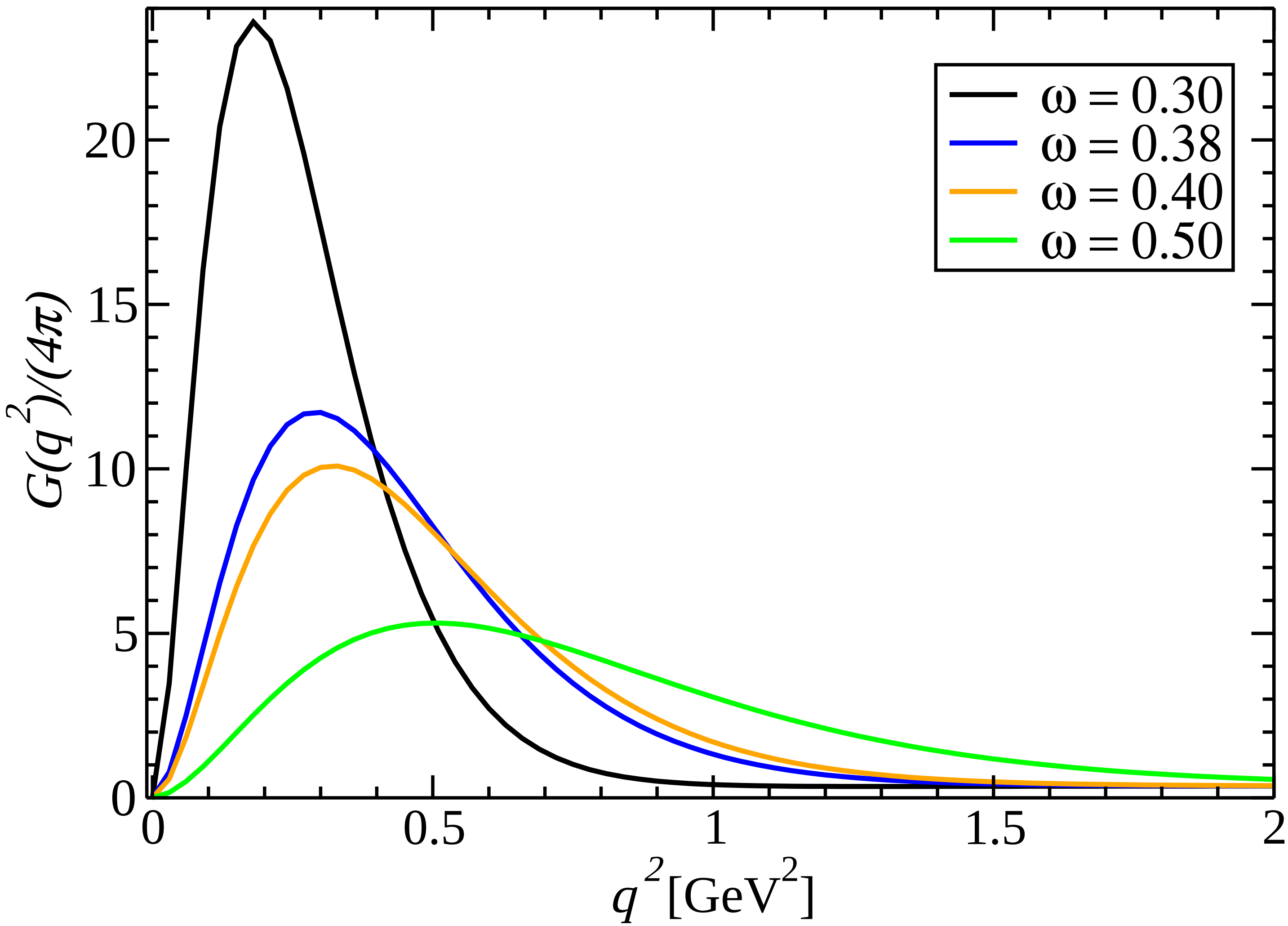
- IR enhancement necessary for dynamical breaking of chiral symmetry
- Integrated strength is essential
- Precise form at low $Q^2 \rightarrow$ model
- IR: two-parameters via Gaussian: strength D and width ω
- perturbative α in the UV region

Model Details

- Effective coupling $\mathcal{G}(Q^2)$: $\omega\, D =$const.



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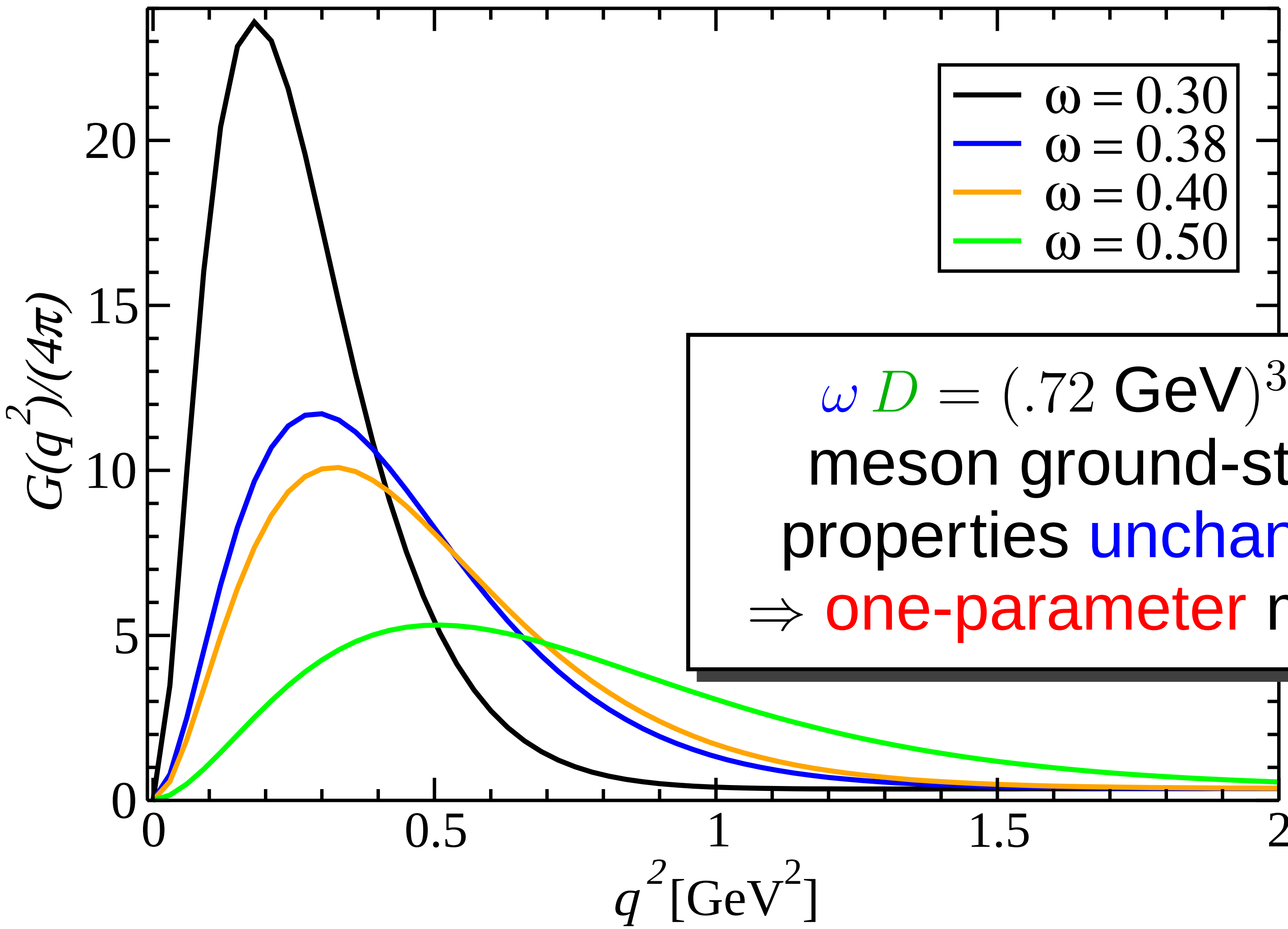

$\omega\, D = (.72\ \text{GeV})^3 \Rightarrow$
meson ground-state properties unchanged
$\Rightarrow$ one-parameter model

So Far – Ground States

- P. Maris, P. C. Tandy: series of papers following
  P. Maris and P. C. Tandy, Phys. Rev. C **60**, 055214 (1999).
- Successful description of light pseudoscalar and vector mesons

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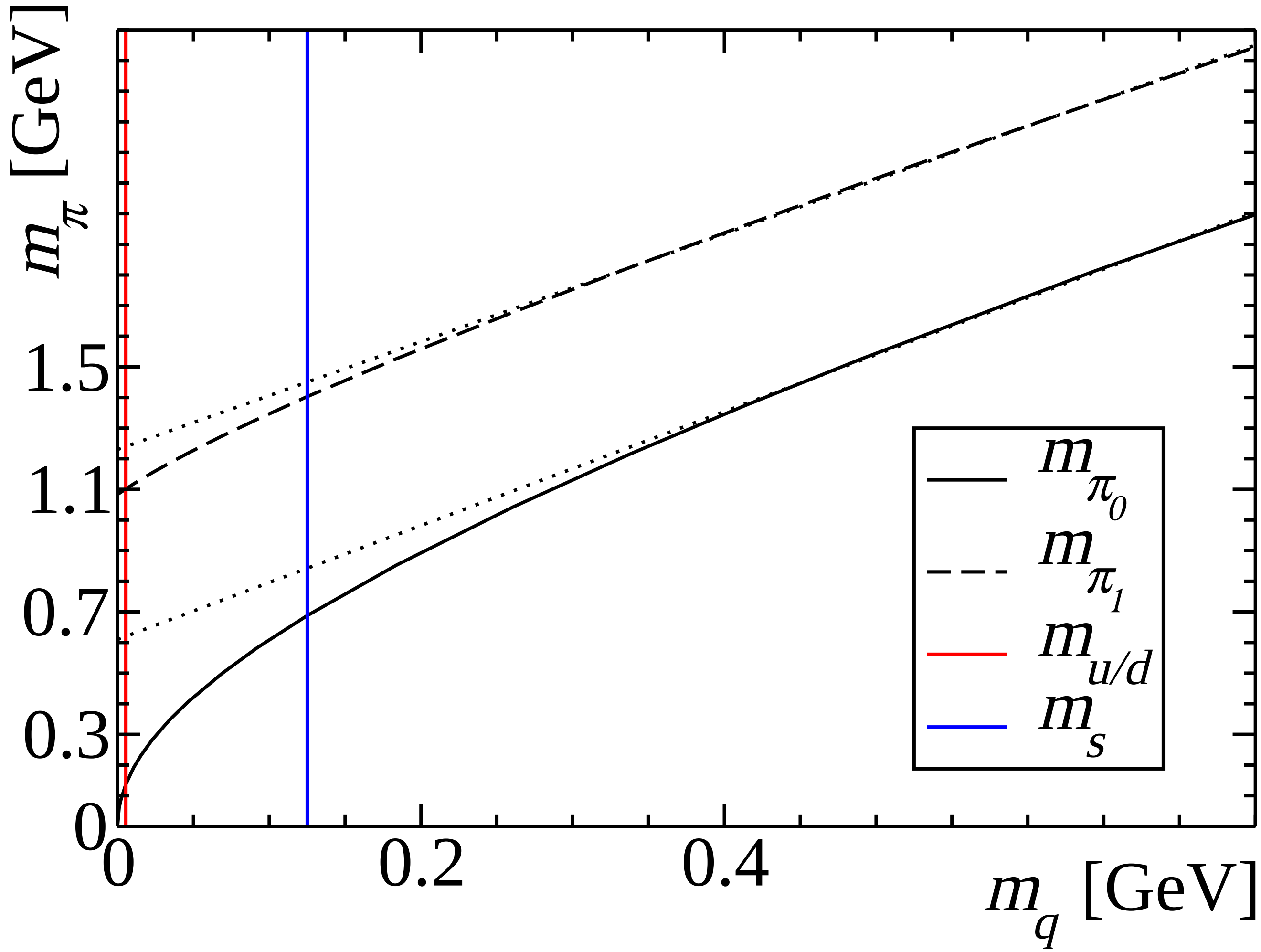
- Successful description of light pseudoscalar and vector mesons
- Now:
  - Radial excitations
  - Scalar mesons
  - Axial vector mesons
- Study long range part of the strong interaction between light quarks

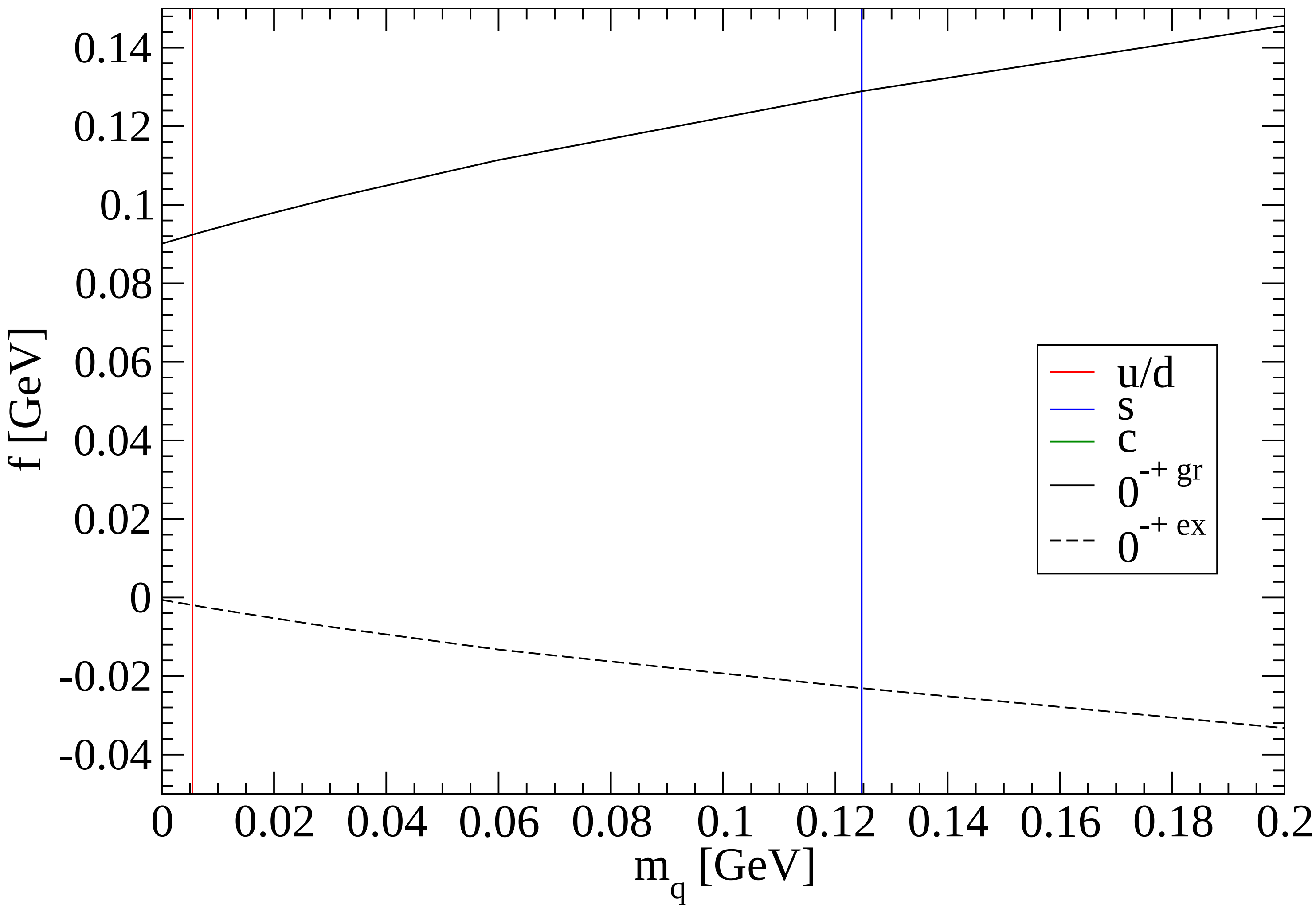
Masses

- $m_{0^{-+}_{gr}}$ and $m_{0^{-+}_{exc1}}$ as functions of current quark mass



Leptonic Decay Constants

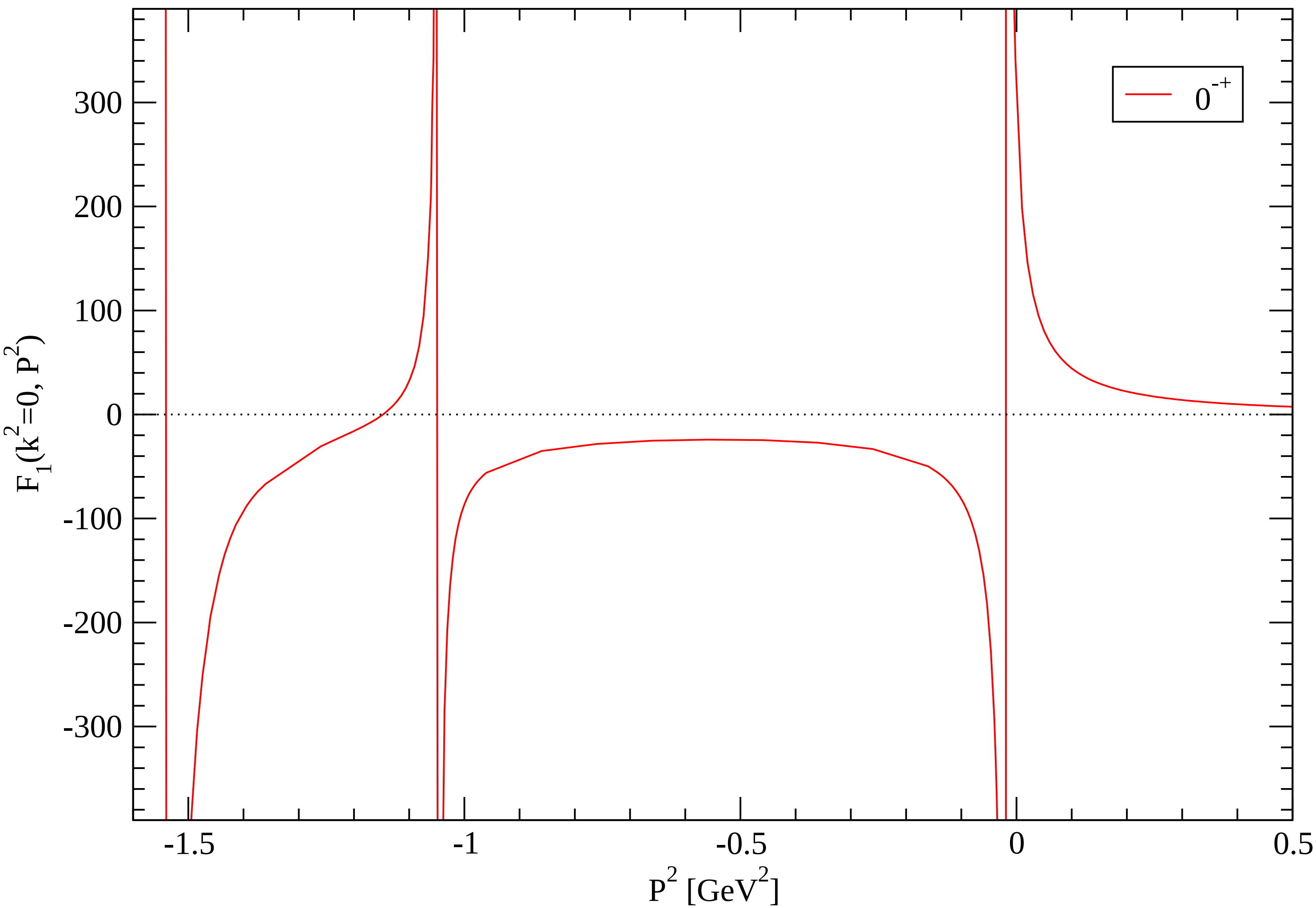
- $f_{0^{-+}_{gr}}$ and $f_{0^{-+}_{exc1}}$ as functions of current quark mass



Inhomogeneous BSE

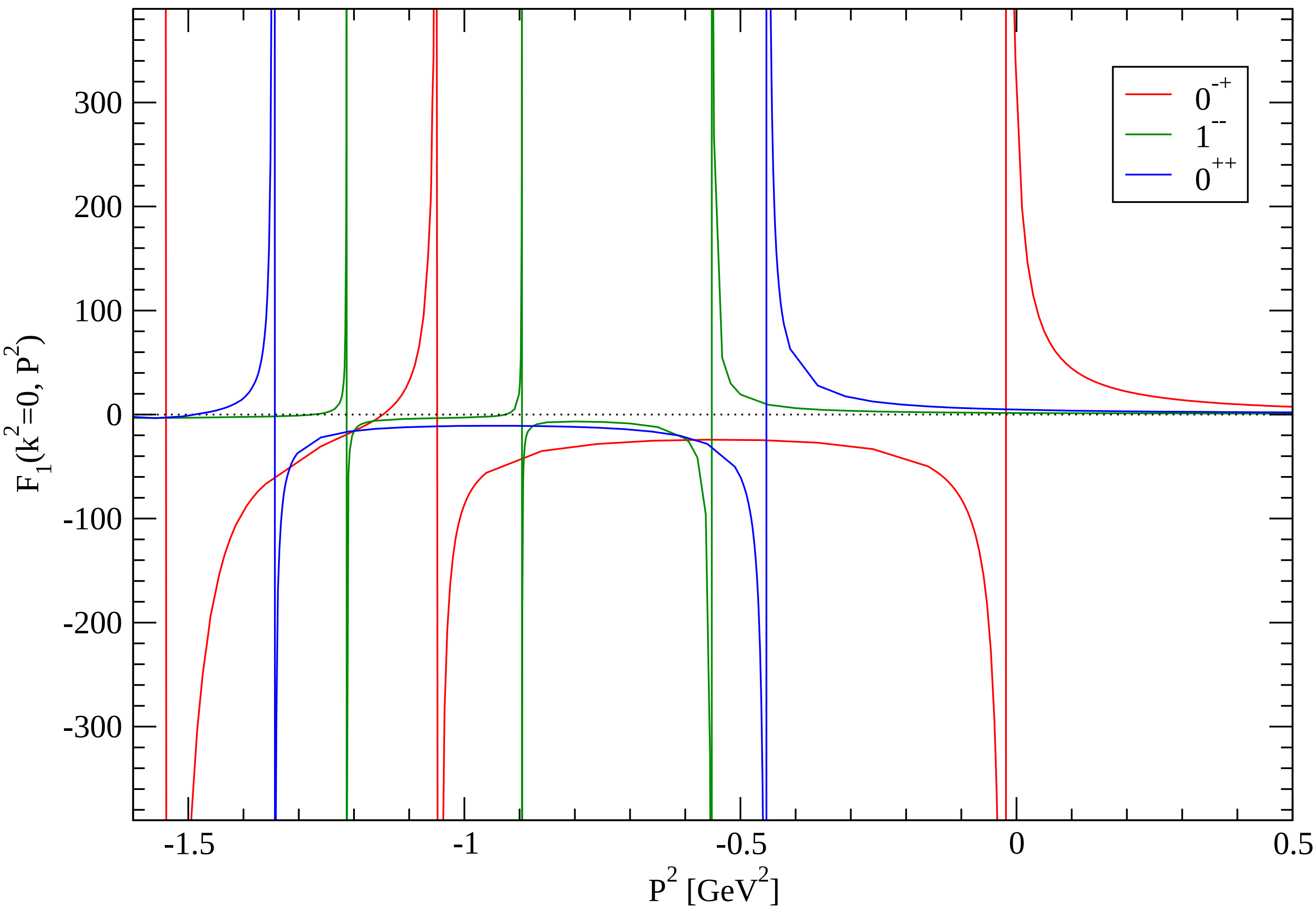
- 0^{-+} meson amplitude

M. Bhagwat, A. Höll, A. K., C. D. Roberts and S. V. Wright, arXiv:nucl-th/0701009



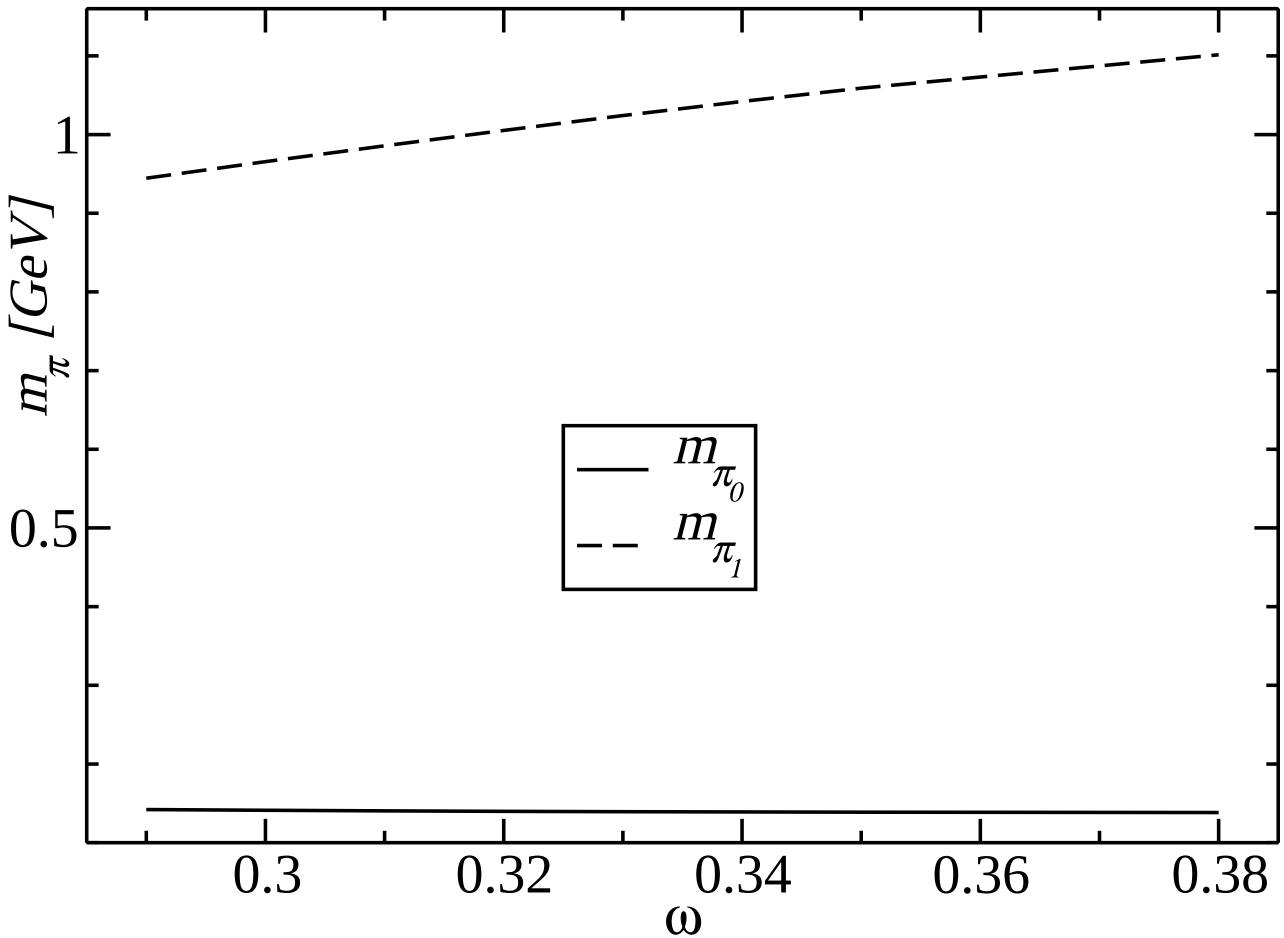
Inhomogeneous BSE

- 0^{-+}, 0^{++}, and 1^{--} meson amplitudes



Model parameter dependence

- $m_{\pi_{gr}}$ and $m_{\pi_{exc1}}$ as functions of ω



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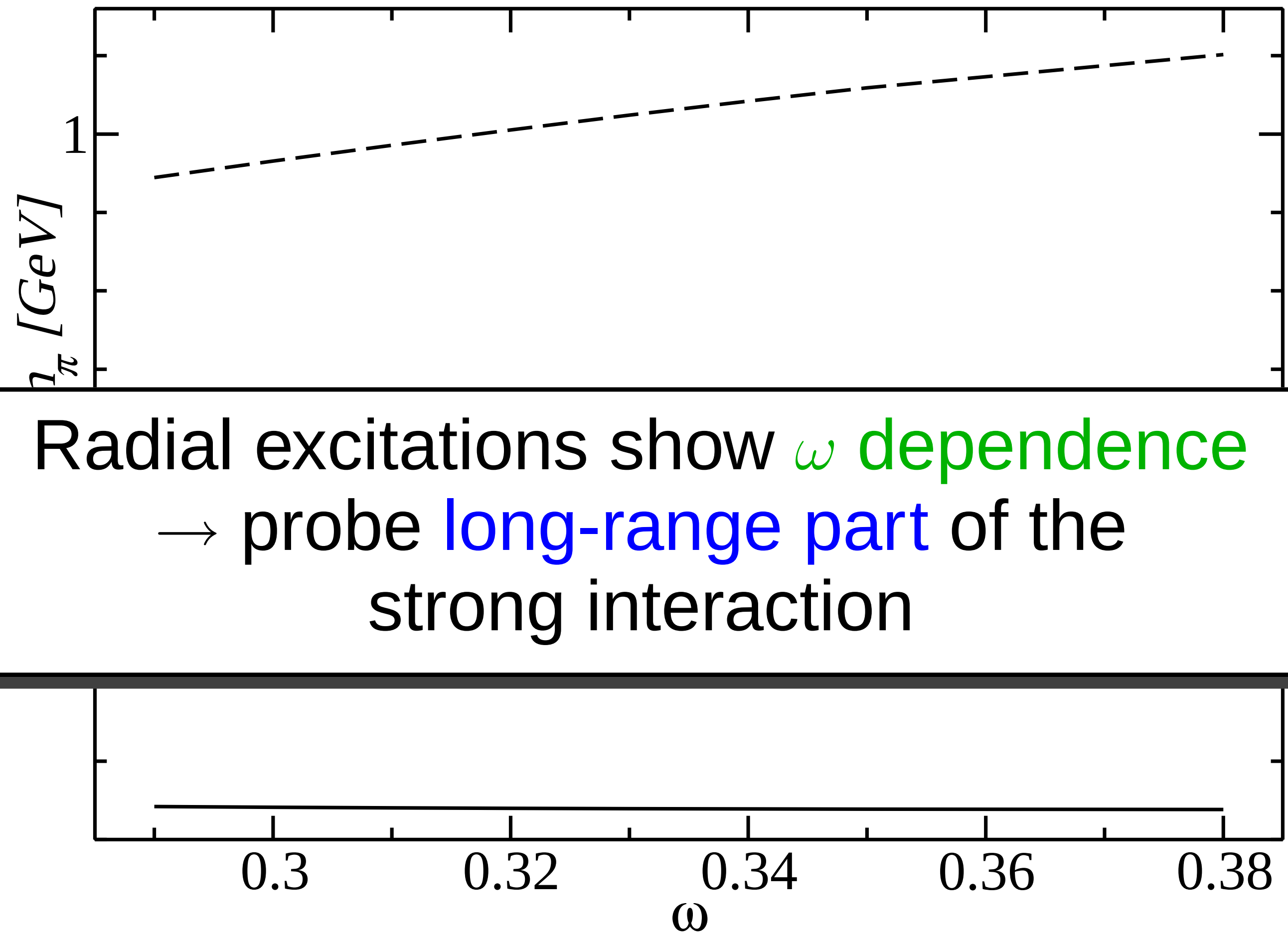
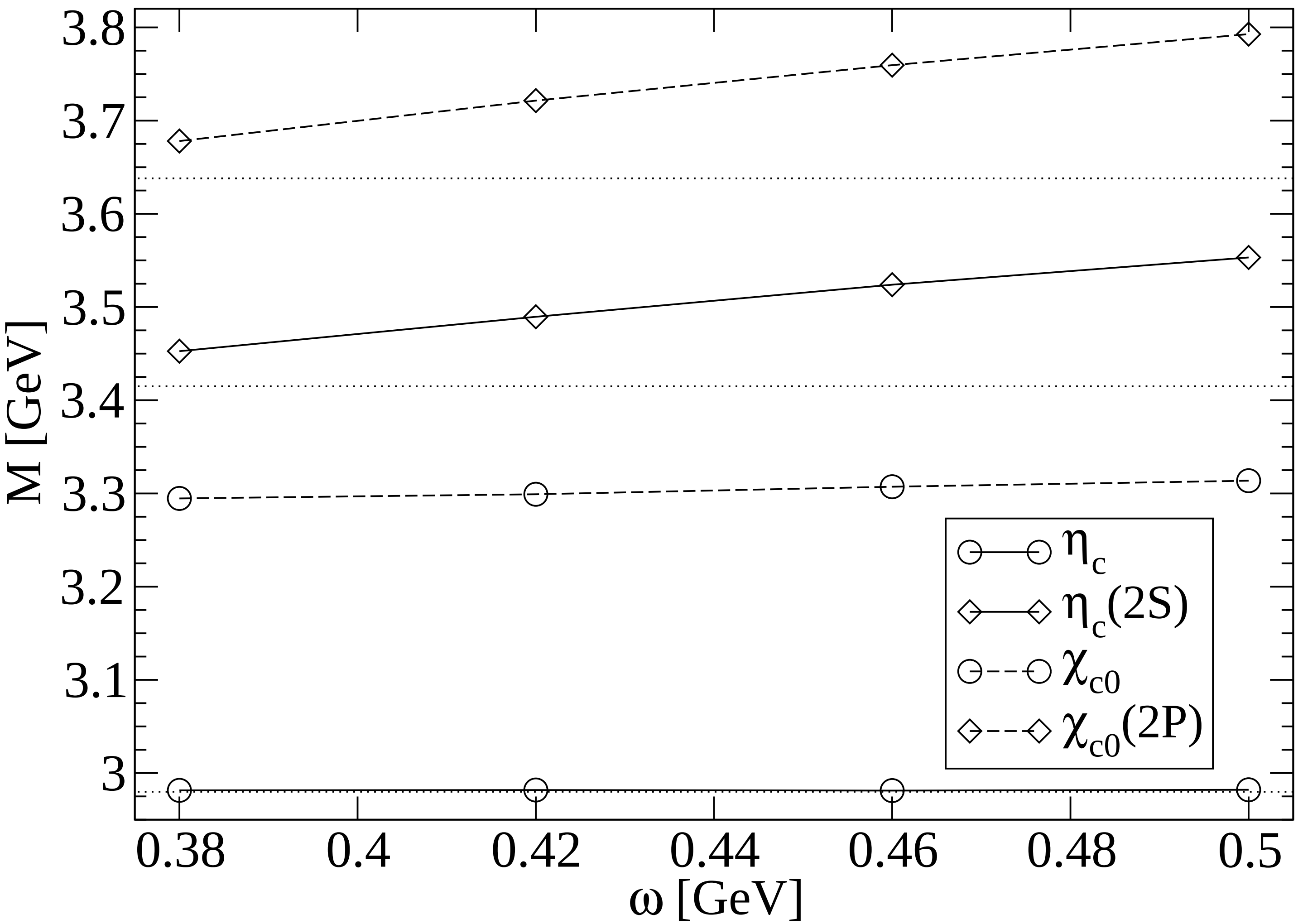


Radial excitations show ω dependence
$\rightarrow$ probe long-range part of the
strong interaction

Ratios, e.g. Charmonium

- A. K., C. D. Roberts, and S. V. Wright, arXiv:nucl-th/0608039

$m_{0^{-+}}$ and $m_{0^{++}}$ as functions of ω



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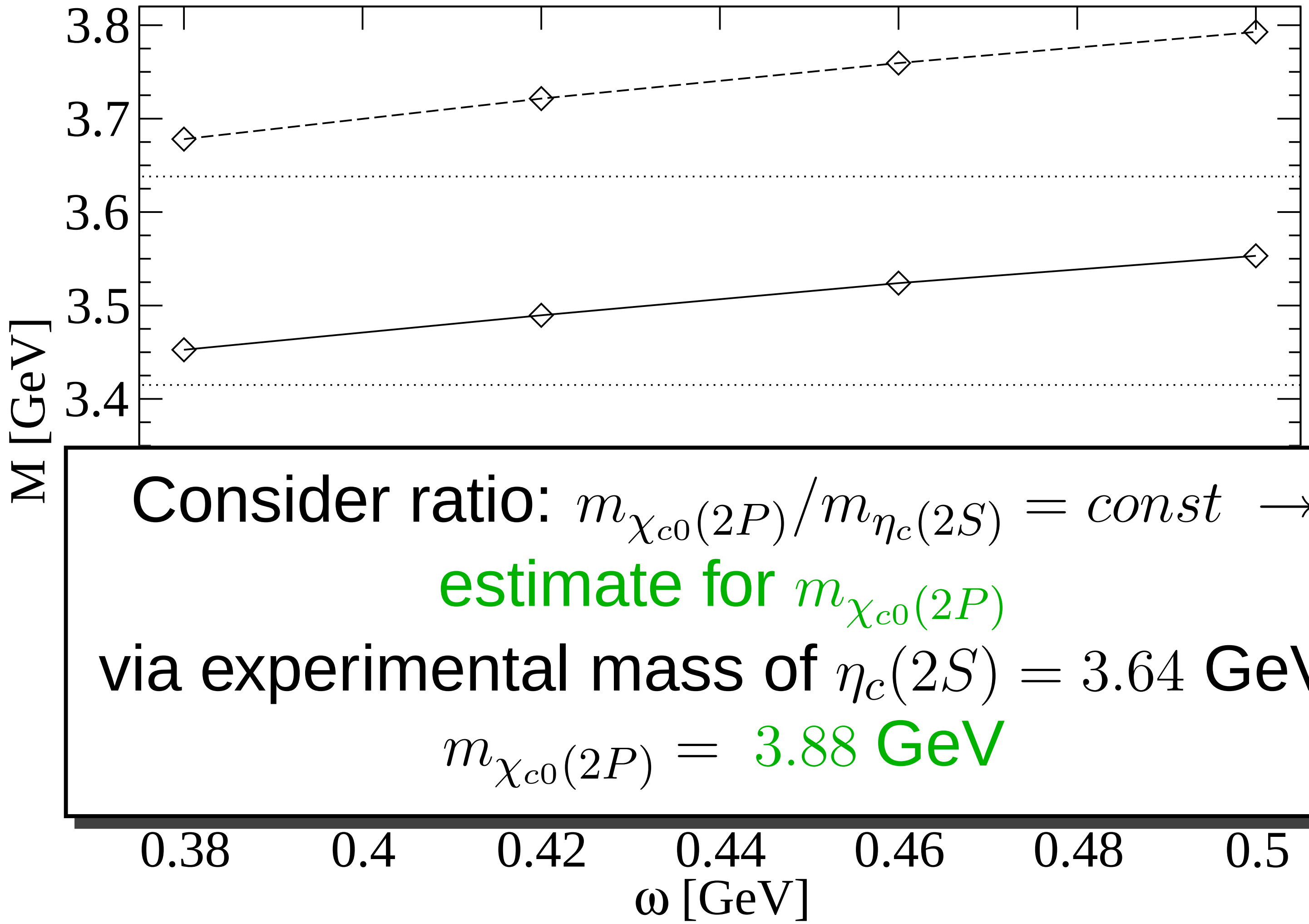

Consider ratio: $m_{\chi_{c0}(2P)}/m_{\eta_c(2S)} = const \rightarrow$

estimate for $m_{\chi_{c0}(2P)}$

via experimental mass of $\eta_c(2S) = 3.64$ GeV:

$m_{\chi_{c0}(2P)} =$ 3.88 GeV

Meson Masses

- I. C. Cloet, A. K. , C. D. Roberts, arXiv:0710.5746 [nucl-th]
- Calculated masses for all mesons with $J = 0, 1$ for (equal) light and strange quark masses (MeV)

J^{PC}	u/d	exp	s	exp
0^{-+}	139	140	695	–
0^{--*}	860	–	1170	–
0^{++}	670	??	1080	??
0^{+-*}	1040	–	1385	–
1^{--}	740	770	1065	1020
1^{-+*}	1000	1376?	1310	1600?
1^{++}	900	1260	1240	1426
1^{+-}	830	1235	1165	1386

$*$ $=$ exotic quantum numbers

More Meson Masses

- Calculated masses for all strange mesons with $J = 0, 1$(MeV)

J^P	gr	exp	exc	exp
0^-	497	497	1032	~ 1460
0^+	894	672	1239	1414
1^-	935	892	1230	1414
1^+	1014	1272	1107	1403

Other Things and Elsewhere

- Finite temperature and density
- Heavy-meson observables
- Diquark confinement (model-independent)
- Gluon propagator and quark-gluon vertex
- Comparison to lattice gauge QCD
- Baryon studies via quark-diquark Ansatz

WIP - Wish List

- Work in progress
  - Hadronic decays, e. g. $\pi_{exc1} \longrightarrow \varrho\,\pi_{gr}$
  - Higher J (tensor mesons)
  - Higher radial excitations
  - Heavy-light mesons and radial excitations
  - Nucleon properties (diquarks)

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- Wish list
  - Sophisticated meson model beyond RLT
  - Good description of axial-vector mesons
  - Study states with “exotic” quantum numbers
  - Include hadronic decay channels in BSE kernel
  - . . .

Conclusions

Don’t underestimate the power of the Force.

Summary and Conclusions

- Dyson-Schwinger equations provide a nonperturbative continuum approach to QCD
- Bethe-Salpeter equation used to describe bound states in a manifestly covariant way
- Symmetry-preserving truncation scheme enables proof of exact results and reliable studies of hadron properties

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- Dyson-Schwinger equations provide a nonperturbative continuum approach to QCD
- Bethe-Salpeter equation used to describe bound states in a manifestly covariant way
- Symmetry-preserving truncation scheme enables proof of exact results and reliable studies of hadron properties
- Step beyond Rainbow-Ladder truncation needed to go for axial vectors, scalars, exotics, radially excited states
- These provide means to study the long-range behavior of the strong interaction

The End

Thank you!