Electron-Positron Pair Creation in Impulse-Shaped Electric Fields

Diploma Thesis supervised by R. Alkofer

Florian Hebenstreit

Institute of Physics - Theoretical Physics
Graz University

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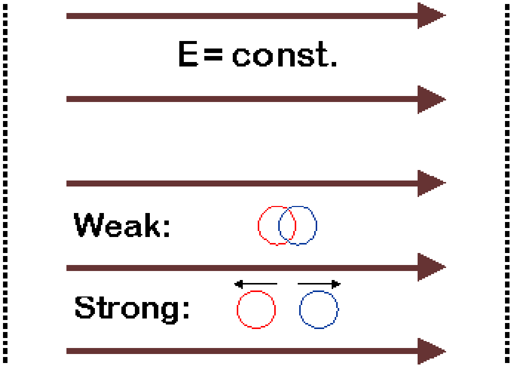
Outline

1. Introduction & Motivation
   - Electron-Positron Pair Creation in Electric Fields
   - Particle-Transport in Electric Fields
2. Quantum Kinetic Equation of Transport
   - Quantum Vlasov Equation with Source Term
   - Backreaction Mechanism
3. Numerical Results
   - Single Particle Distribution Function $f(\mathbf{k}, t)$
   - Particle Number Density $n(t)$
4. Summary & Outlook

Outline

Schwinger-Mechanism

Electron-postitron pair creation in spatially homogeneous, time-independent electric fields E_0:



Schwinger-Mechanism

- Pair creation probability per unit volume and time:

$$W[e^+e^-] = \frac{e^2E_0^2}{4\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{nm^2\pi}{eE_0}\right)$$

J. Schwinger, Phys. Rev. **82**, 664 (1951)

- Non-perturbative effect
- Strong electric fields needed → difficult to produce:

$$E_{\rm cr} = \frac{m^2}{e} \approx 1.3\cdot 10^{18}\,\mathrm{V/m}$$

- XFEL facilities at DESY and SLAC: $E_0 \approx 0.1E_{\rm cr}$
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Kinetic Equation of Particle-Transport

- Pair creation and transport is process far from equilibrium
- Time-dependet electric field $E(t) \rightarrow$ Vlasov equation

$$\frac{\mathrm{d}}{\mathrm{d}t} f(\mathbf{k},t) = \frac{\partial}{\partial t} f(\mathbf{k},t) + eE(t)\frac{\partial}{\partial k_3} f(\mathbf{k},t) = S(\mathbf{k},t)$$

- Phenomenological approach with Schwinger source term:

$$S(\mathbf{k},t) = -2eE(t)\ln\left[1 - \exp\left(-\frac{(m^2+\mathbf{k}_\perp^2)\pi}{eE(t)}\right)\right]\delta(k_3 - eA(t))$$

- Static field $E_0 \rightarrow$ time-dependent field $E(t)$
- Combination of quantum field theory and kinetic theory $\rightarrow$ quantum kinetic theory

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Starting point: QED

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- QED-Lagrangian: $\mathcal{L} = \bar{\Psi}\left[i\not{D} - m\right]\Psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$
- Quantize only matter field → Mean electric field
- Fourier-Transformation: Coordinate to momentum space

$$\Psi(\mathbf{x},t) = \int \frac{d^3k}{(2\pi)^3} \sum_{s=\pm} \left[u_{s,\mathbf{p}}(t)a_{s,\mathbf{k}} + v_{s,-\mathbf{p}}(t)b^{\dagger}_{s,-\mathbf{k}}\right] e^{i\mathbf{kx}}$$

- Ansatz for the spinors:

$$u_{s,\mathbf{p}}(t) = \left[i\gamma^0\partial_t - \vec{\gamma}\cdot\mathbf{p} + m\right] g_{\mathbf{p}}(t)R_s$$
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Time-Independent Particle Number

- Complex mode function $g_{\mathbf{p}}(t)$ satisfies:

$$\left[\partial_t^2 + \omega_{\mathbf{p}}^2(t) + ieE(t)\right] g_{\mathbf{p}}(t) = 0$$

$$\omega_{\mathbf{p}}^2(t) = [k_3 - eA(t)]^2 + \mathbf{k}_\perp^2 + m^2 = p_\parallel^2(t) + \epsilon_\perp^2$$

- Time-independent number of particles (antiparticles) with helicity s and canonical momentum $\mathbf{k}$ ($-\mathbf{k}$):

$$N_{s,\mathbf{k}}^+ = \left\langle a_{s,\mathbf{k}}^\dagger a_{s,\mathbf{k}} \right\rangle = \left\langle b_{s,-\mathbf{k}}^\dagger b_{s,-\mathbf{k}} \right\rangle = N_{s,-\mathbf{k}}^-$$

- Orthogonality relations for spinors do not hold anymore → no separation between positive & negative energy solution
- Hamiltonian operator achieves off-diagonal elements

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Bogoliubov Transformation

- Time-dependent operators → Bogoliubov transformation

$$\begin{aligned}\tilde{a}_{s,\mathbf{k}}(t) &= \alpha_{\mathbf{p}}(t)a_{s,\mathbf{k}} - \beta^*_{\mathbf{p}}(t)b^\dagger_{s,-\mathbf{k}}\\ \tilde{b}^\dagger_{s,-\mathbf{k}}(t) &= \beta_{\mathbf{p}}(t)a_{s,\mathbf{k}} + \alpha^*_{\mathbf{p}}(t)b^\dagger_{s,-\mathbf{k}}\end{aligned}$$

- For example: Remove $(e, s=+, \mathbf{k}) \to (0,0,0)$

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Adiabatic Spinor Functions

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$$\Psi(\mathbf{x},t) = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_{s=\pm} \left[\tilde{u}_{s,\mathbf{p}}(t)\tilde{a}_{s,\mathbf{k}}(t) + \tilde{v}_{s,-\mathbf{p}}(t)\tilde{b}^{\dagger}_{s,-\mathbf{k}}(t)\right] e^{i\mathbf{kx}}$$

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- Dynamical phase: $\Theta_{\mathbf{p}}(t_0,t) = \int_{t_0}^{t}\mathrm{d}\tau\,\omega_{\mathbf{p}}(\tau)$

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Quantum Kinetic Equation

- Orthogonality relations hold and Hamiltonian diagonal!
- Quantum Vlasov equation including a source term:

$$\dot{\mathcal{N}}_{s,\mathbf{k}}(t) = \frac{eE(t)\epsilon_\perp}{2\omega_{\mathbf{p}}^2(t)} \int_{t_0}^{t} \mathrm{d}t' \, \frac{eE(t')\epsilon_\perp}{\omega_{\mathbf{p}}^2(t')} \left[1 - 2\mathcal{N}_{s,\mathbf{k}}(t')\right] \times \cos\left(2\int_{t'}^{t} \mathrm{d}\tau \, \omega_{\mathbf{p}}(\tau)\right)$$

- Non-Markovian equation: Statistical factor & Cosine-term
- No spin preference → Replace $\mathcal{N}_{s,\mathbf{k}}(t) = \mathcal{N}_{\mathbf{k}}(t) = f(\mathbf{k},t)$
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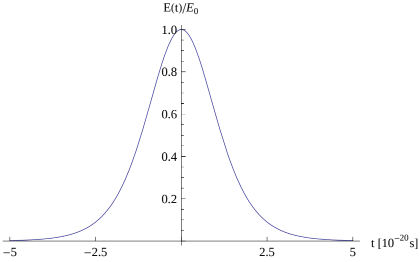
Outline

General Settings

- Impulse-shaped external electric field $E_{\text{ext}}(t)$:

$$E_{\text{ext}}(t) = \frac{E_0}{\cosh^2(t/\tau)}$$



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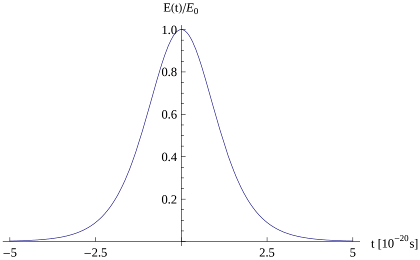

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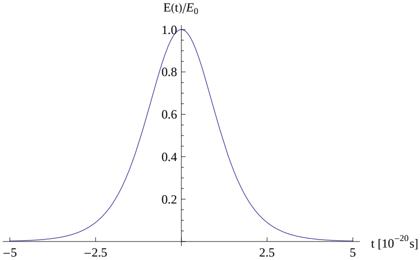

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Single Particle Distribution Function $f(\mathbf{k}, t)$ for $k_\perp = 0$

Solve full non-Markovian equation for the production rate:

$$\dot{f}_{\mathrm{nm}}(\mathbf{k}, t) = \frac{e\,E(t)\epsilon_\perp}{2\omega_{\mathbf{p}}^2(t)} \int_{t_0}^{t} \mathrm{d}t' \, \frac{e\,E(t')\epsilon_\perp}{\omega_{\mathbf{p}}^2(t')} \left[1 - 2f_{\mathrm{nm}}(\mathbf{k}, t')\right] \times \cos\left(2 \int_{t'}^{t} \mathrm{d}\tau \, \omega_{\mathbf{p}}(\tau)\right)$$

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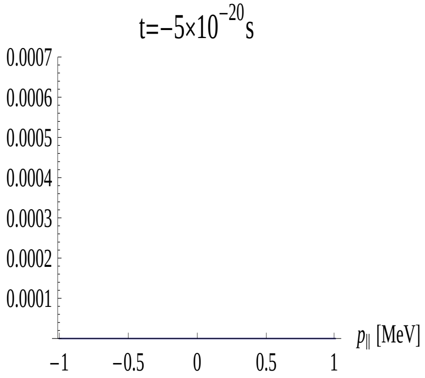

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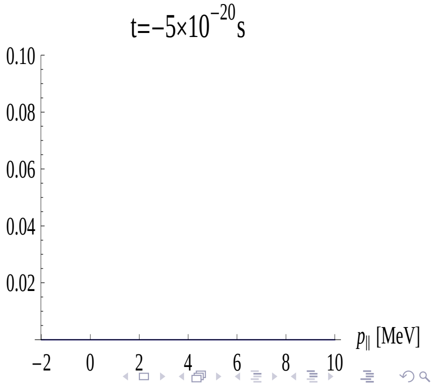

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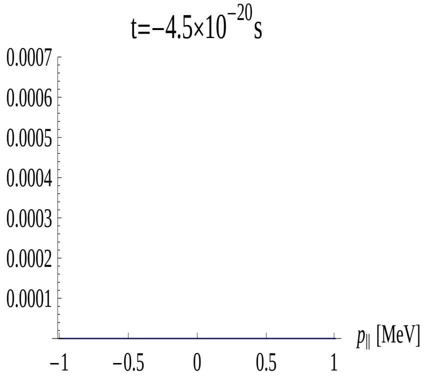

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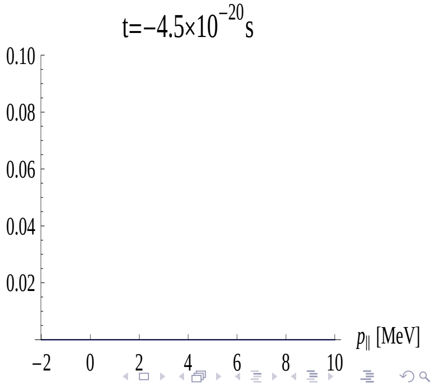

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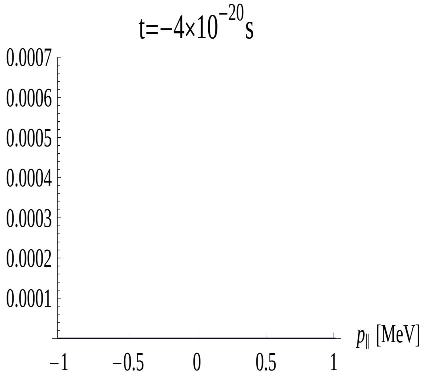

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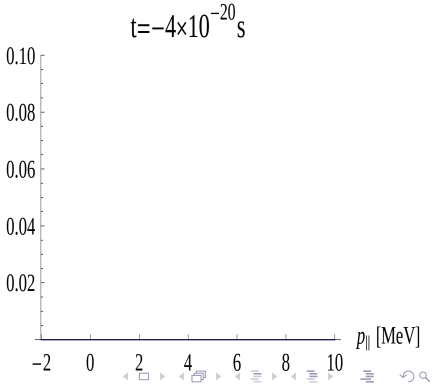

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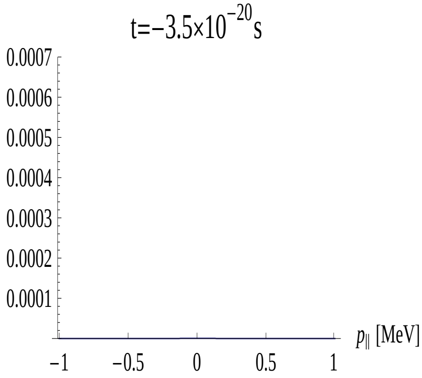

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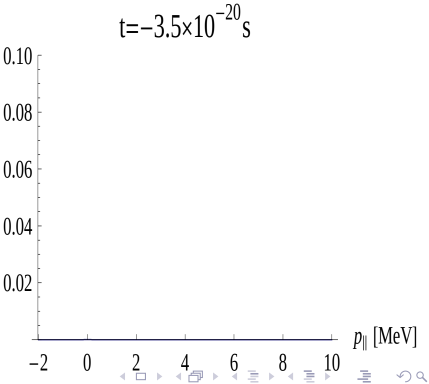

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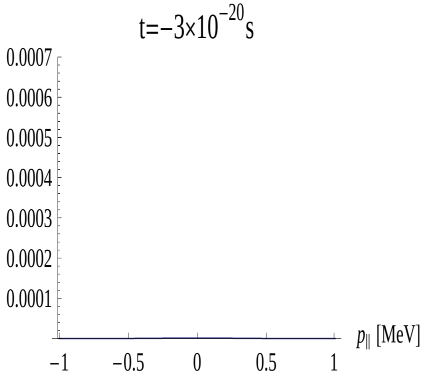

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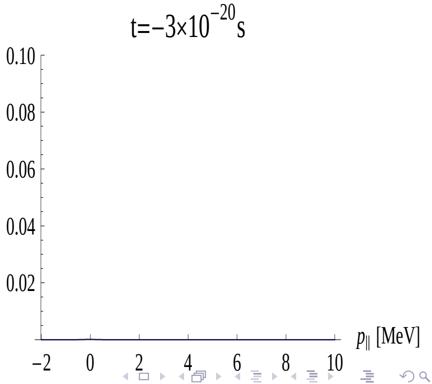

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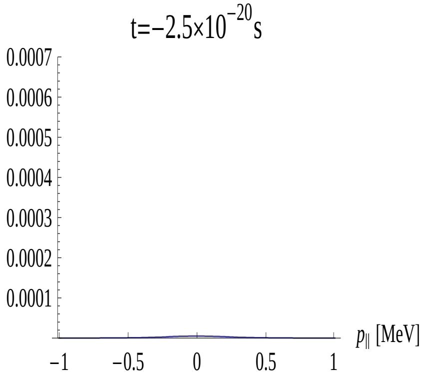

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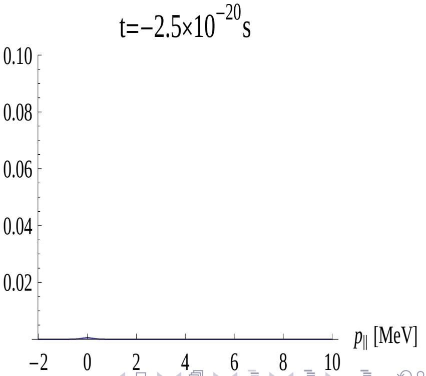

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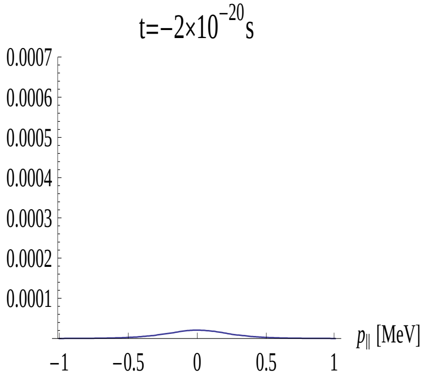

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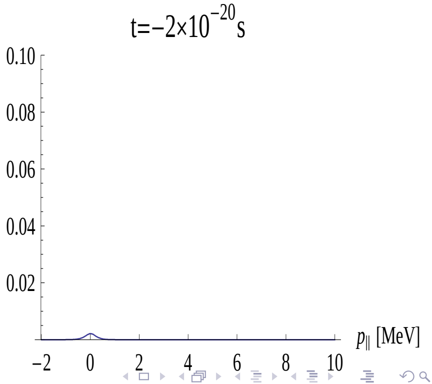

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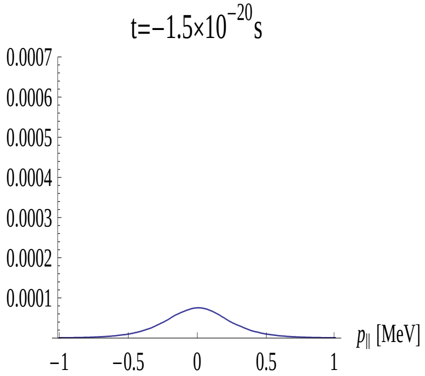

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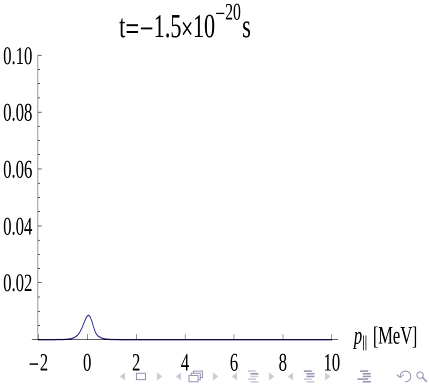

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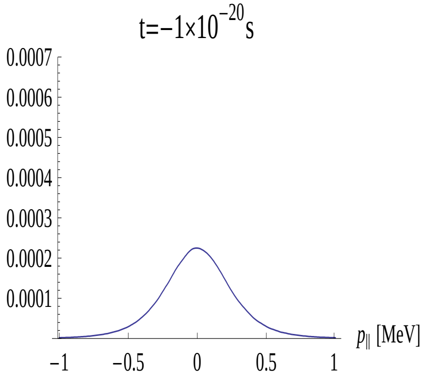

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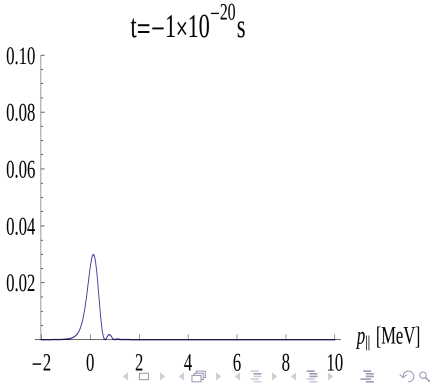

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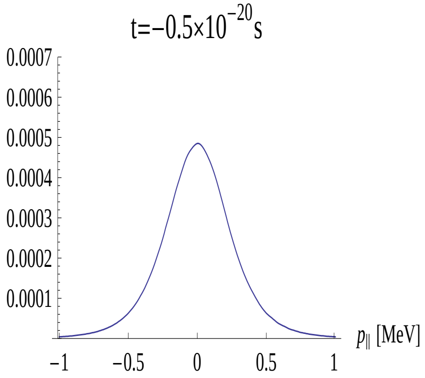

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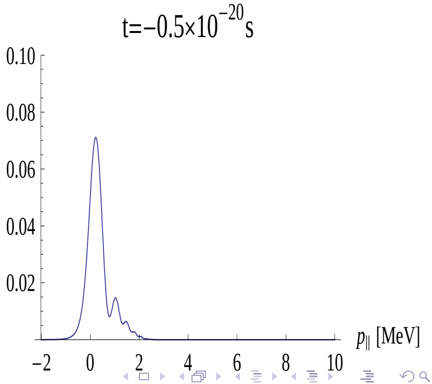

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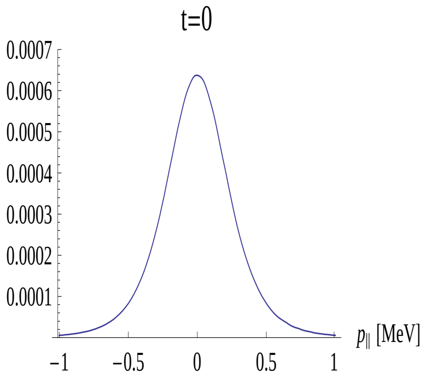

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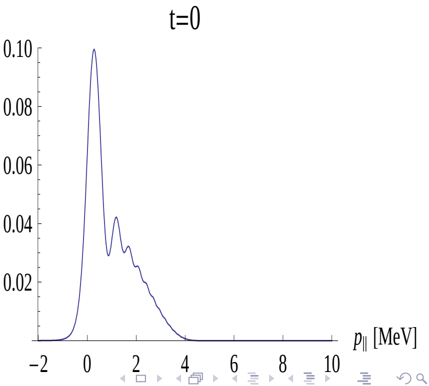

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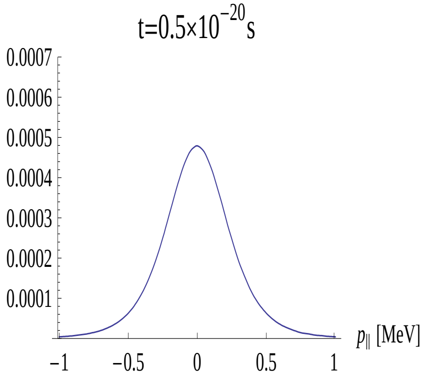

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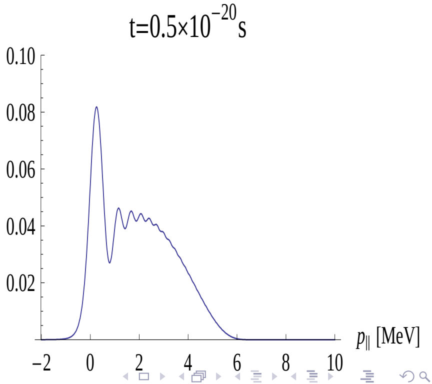

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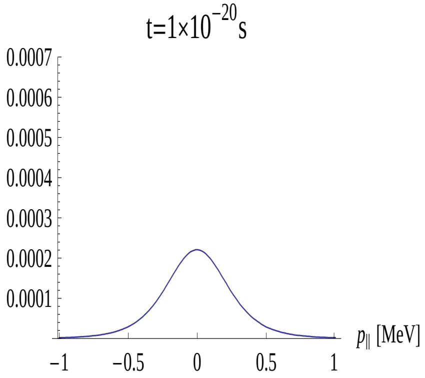

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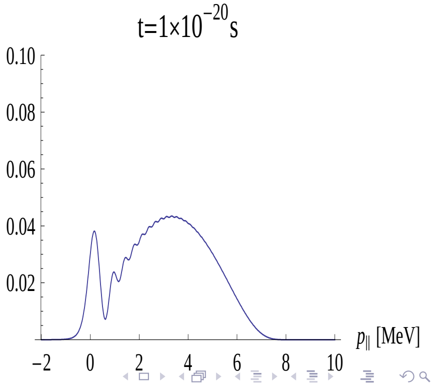

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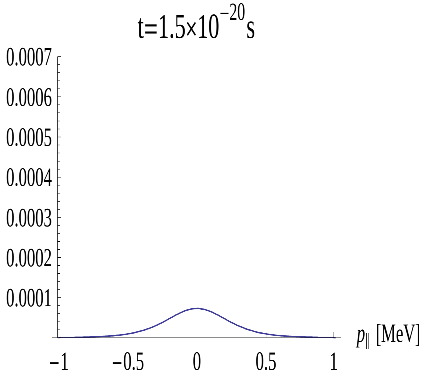
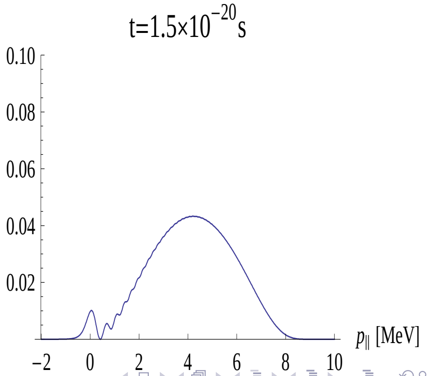
Weak field: $E_0 = 0.1E_{cr}$

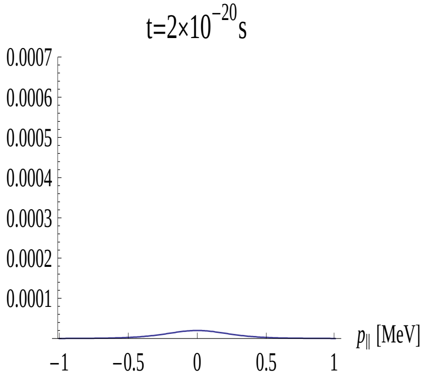


Strong field: $E_0 = E_{cr}$:

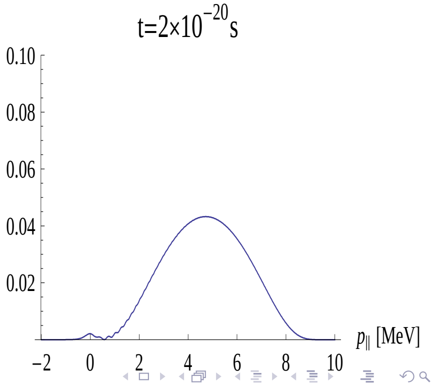


Single Particle Distribution Function $f(\mathbf{k}, t)$ for $k_\perp = 0$

Weak field: $E_0 = 0.1 E_{\mathrm{cr}}$

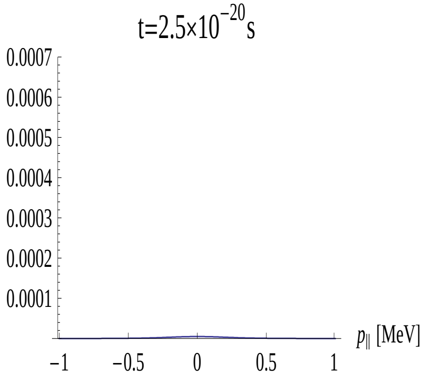


Strong field: $E_0 = E_{\mathrm{cr}}$:

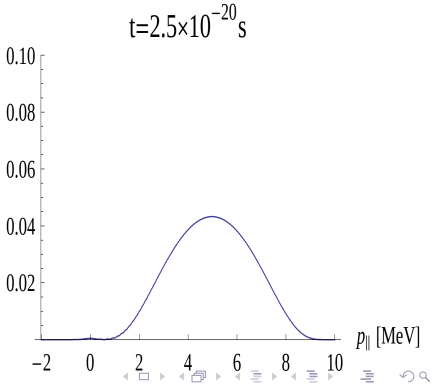


Single Particle Distribution Function $f(\mathbf{k}, t)$ for $k_\perp = 0$

Weak field: $E_0 = 0.1 E_{\mathrm{cr}}$

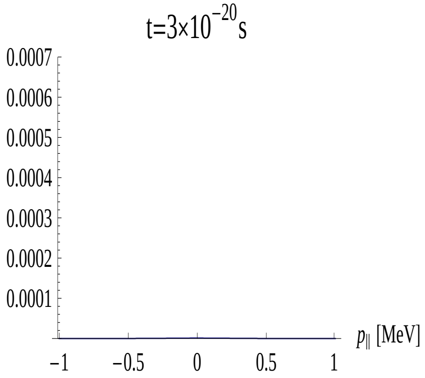


Strong field: $E_0 = E_{\mathrm{cr}}$:

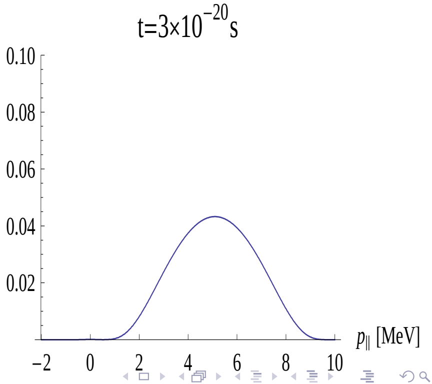


Single Particle Distribution Function $f(\mathbf{k}, t)$ for $k_\perp = 0$

Weak field: $E_0 = 0.1E_{\rm cr}$

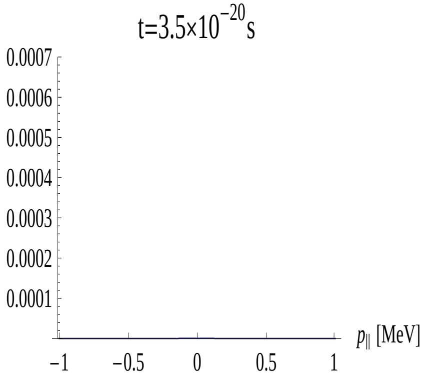


Strong field: $E_0 = E_{\rm cr}$:

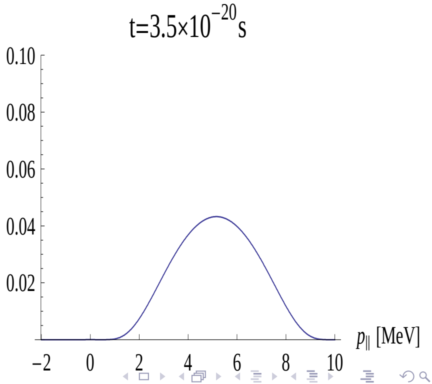


Single Particle Distribution Function $f(\mathbf{k}, t)$ for $k_\perp = 0$

Weak field: $E_0 = 0.1 E_{\mathrm{cr}}$

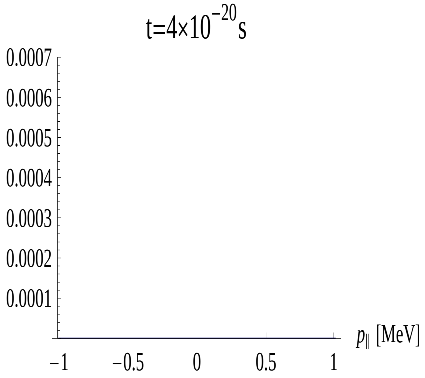


Strong field: $E_0 = E_{\mathrm{cr}}$:

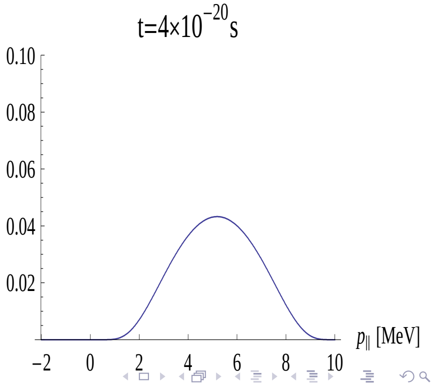


Single Particle Distribution Function $f(\mathbf{k}, t)$ for $k_\perp = 0$

Weak field: $E_0 = 0.1E_{\mathrm{cr}}$

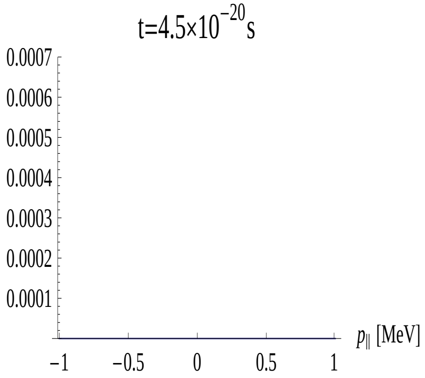


Strong field: $E_0 = E_{\mathrm{cr}}$:

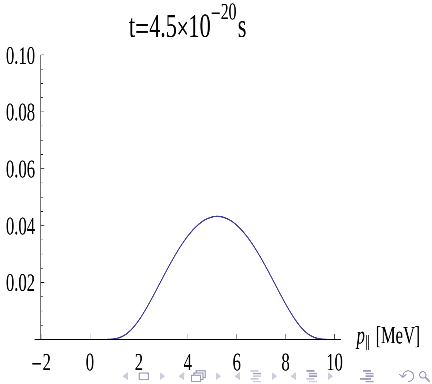


Single Particle Distribution Function $f(\mathbf{k}, t)$ for $k_\perp = 0$

Weak field: $E_0 = 0.1E_{cr}$

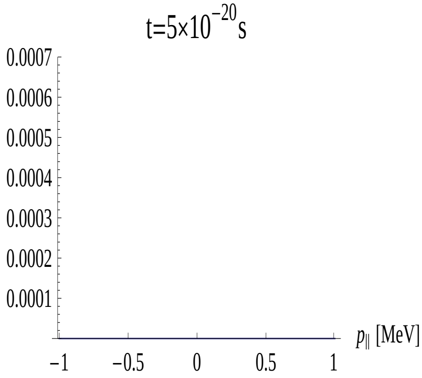


Strong field: $E_0 = E_{cr}$:

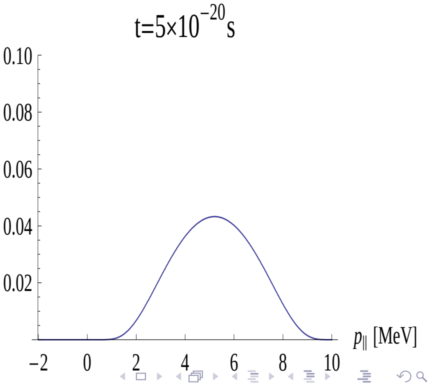


Single Particle Distribution Function $f(\mathbf{k}, t)$ for $k_\perp = 0$

Weak field: $E_0 = 0.1E_{\text{cr}}$

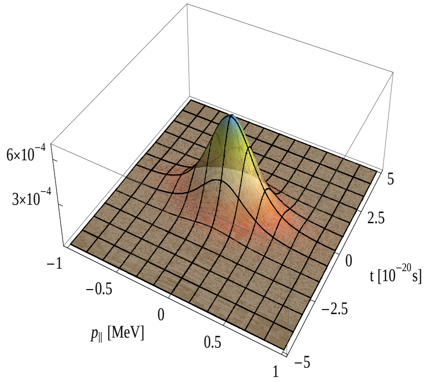


Strong field: $E_0 = E_{\text{cr}}$:



Single Particle Distribution Function $f(\mathbf{k}, t)$ for $k_\perp = 0$

Weak field: $E_0 = 0.1 E_{\mathrm{cr}}$

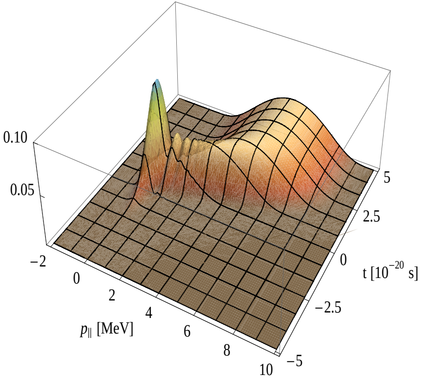


- Particle creation not only at rest: $-1\,\mathrm{MeV} \lesssim p_\parallel \lesssim 1\,\mathrm{MeV}$
- Approximately, still symmetry around $t = 0$

Single Particle Distribution Function $f(\mathbf{k}, t)$ for $k_\perp = 0$

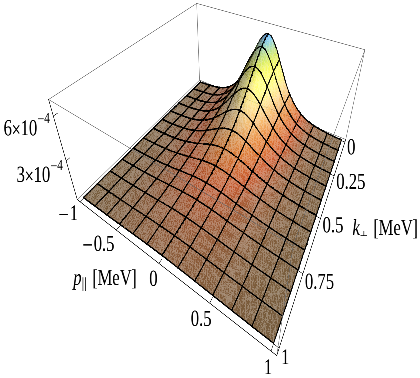
- Particle creation not only at rest: $-1\,\mathrm{MeV} \lesssim p_\| \lesssim 1\,\mathrm{MeV}$
- Electron-positron pairs are accelerated and drift away from each other
- Asymptotic distribution peaked around $p_\| \approx 5\,\mathrm{MeV}$
- No symmetry around $t = 0$

Strong field: $E_0 = E_{\mathrm{cr}}$



Single Particle Distribution Function $f(\mathbf{k}, 0)$

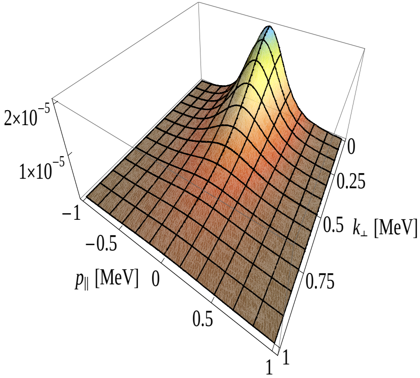
Weak field: $E_0 = 0.1 E_{\rm cr}$



- Particle creation for perpendicular momenta: $k_\perp \lesssim 1\,\mathrm{MeV}$
- Distribution function approximately exponentially damped as function of $\mathbf{k}_\perp^2$

Single Particle Distribution Function $f(\mathbf{k}, 2 \cdot 10^{-20} s)$

Weak field: $E_0 = 0.1 E_{\mathrm{cr}}$

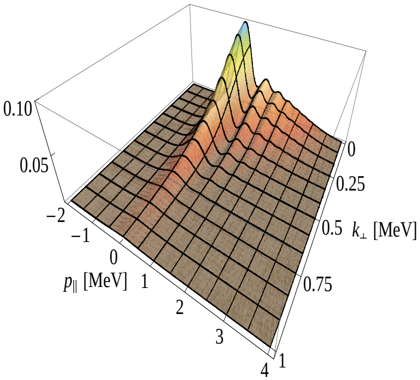


- Particle-antiparticle pairs are annihilated for weakening electric field again

Single Particle Distribution Function $f(\mathbf{k}, 0)$

- Particle creation for perpendicular momenta: $k_\perp \lesssim 1\,\mathrm{MeV}$
- Distribution function approximately exponentially damped as function of $\mathbf{k}_\perp^2$
- Particle-antiparticle pairs are accelerated for $k_\perp \lesssim 0.5\,\mathrm{MeV}$

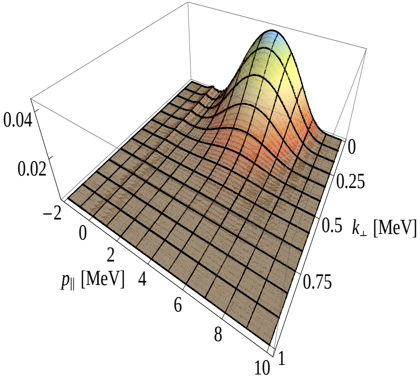
Strong field: $E_0 = E_{\mathrm{cr}}$



Single Particle Distribution Function $f(\mathbf{k}, 2 \cdot 10^{-20}\,s)$

- Asymptotic distribution peaked around $p_{||} \approx 5\,\mathrm{MeV}$ with perpendicular momenta $k_{\perp} \lesssim 0.5\,\mathrm{MeV}$

Strong field: $E_0 = E_{\mathrm{cr}}$

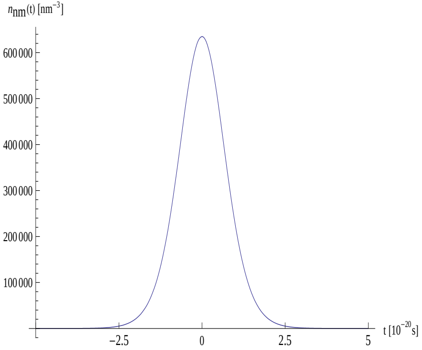


Field Strength Dependence of $n_{\rm nm}(t)$

- Non-vanishing number density $n_{\rm nm}(\infty)$ even for $E_0 = 0.1E_{\rm cr}$
- Ratio $n_{\rm nm}(\infty)/n_{\rm nm}(t_{\max})$ increases as a function of E_0
- The peak of the number density $n_{\rm nm}(t_{\max})$ is shifted to later times for $E_0 \gtrsim 0.9E_{\rm cr}$

$n_{\rm nm}(\infty)[{\rm nm}^{-3}]$	6
$n_{\rm nm}(\infty)/n_{\rm nm}(t_{\max})$	$9.4 \cdot 10^{-6}$
$t_{\max}[10^{-20}{\rm s}]$	0.005

$E_0 = 0.1E_{\rm cr}$

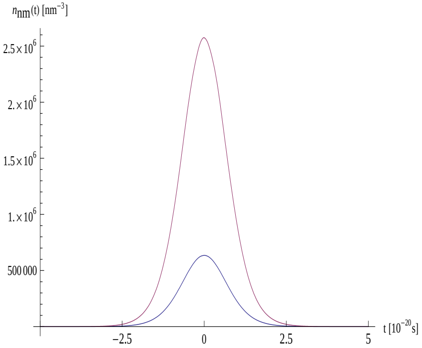


Field Strength Dependence of $n_{\rm nm}(t)$

- Non-vanishing number density $n_{\rm nm}(\infty)$ even for $E_0 = 0.1E_{\rm cr}$
- Ratio $n_{\rm nm}(\infty)/n_{\rm nm}(t_{\rm max})$ increases as a function of E_0
- The peak of the number density $n_{\rm nm}(t_{\rm max})$ is shifted to later times for $E_0 \gtrsim 0.9E_{\rm cr}$

$n_{\rm nm}(\infty)[\rm nm^{-3}]$	36
$n_{\rm nm}(\infty)/n_{\rm nm}(t_{\rm max})$	$1.4 \cdot 10^{-5}$
$t_{\rm max}[10^{-20}\rm s]$	-0.010

$E_0 = 0.2E_{\rm cr}$

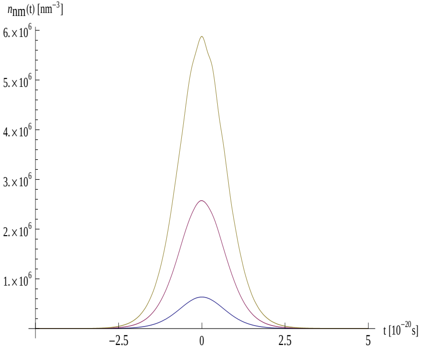


Field Strength Dependence of $n_{\text{nm}}(t)$

- Non-vanishing number density $n_{\text{nm}}(\infty)$ even for $E_0 = 0.1E_{\text{cr}}$
- Ratio $n_{\text{nm}}(\infty)/n_{\text{nm}}(t_{\max})$ increases as a function of E_0
- The peak of the number density $n_{\text{nm}}(t_{\max})$ is shifted to later times for $E_0 \gtrsim 0.9E_{\text{cr}}$

$n_{\text{nm}}(\infty)[\text{nm}^{-3}]$	$2.7 \cdot 10^3$
$n_{\text{nm}}(\infty)/n_{\text{nm}}(t_{\max})$	$4.6 \cdot 10^{-4}$
$t_{\max}[10^{-20}\text{s}]$	-0.005

$E_0 = 0.3E_{\text{cr}}$

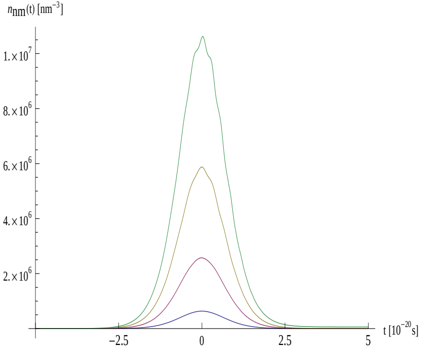


Field Strength Dependence of $n_{\rm nm}(t)$

- Sizeable number density $n_{\rm nm}(\infty)$ for $E_0 \gtrsim 0.4E_{\rm cr}$
- Ratio $n_{\rm nm}(\infty)/n_{\rm nm}(t_{\max})$ increases as a function of E_0
- The peak of the number density $n_{\rm nm}(t_{\max})$ is shifted to later times for $E_0 \gtrsim 0.9E_{\rm cr}$

$n_{\rm nm}(\infty)[{\rm nm}^{-3}]$	$5.9 \cdot 10^4$
$n_{\rm nm}(\infty)/n_{\rm nm}(t_{\max})$	$5.6 \cdot 10^{-3}$
$t_{\max}[10^{-20}{\rm s}]$	0.025

$$E_0 = 0.4E_{\rm cr}$$

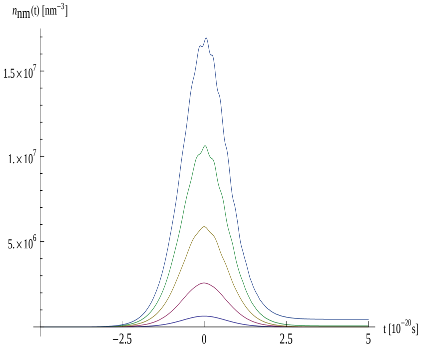


Field Strength Dependence of $n_{\mathrm{nm}}(t)$

- Sizeable number density $n_{\mathrm{nm}}(\infty)$ for $E_0 \gtrsim 0.4E_{\mathrm{cr}}$
- Ratio $n_{\mathrm{nm}}(\infty)/n_{\mathrm{nm}}(t_{\mathrm{max}})$ increases as a function of E_0
- The peak of the number density $n_{\mathrm{nm}}(t_{\mathrm{max}})$ is shifted to later times for $E_0 \gtrsim 0.9E_{\mathrm{cr}}$

$n_{\mathrm{nm}}(\infty)[\mathrm{nm}^{-3}]$	$4.5 \cdot 10^{5}$
$n_{\mathrm{nm}}(\infty)/n_{\mathrm{nm}}(t_{\mathrm{max}})$	$2.6 \cdot 10^{-2}$
$t_{\mathrm{max}}[10^{-20}\mathrm{s}]$	0.055

$E_0 = 0.5E_{\mathrm{cr}}$

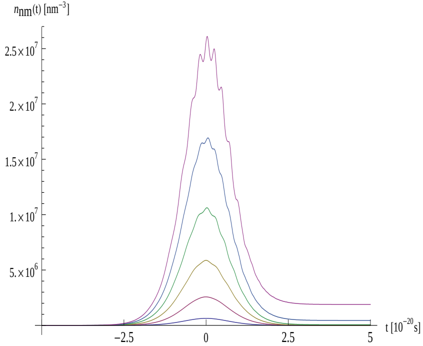


Field Strength Dependence of $n_{\rm nm}(t)$

- Sizeable number density $n_{\rm nm}(\infty)$ for $E_0 \gtrsim 0.4E_{\rm cr}$
- Ratio $n_{\rm nm}(\infty)/n_{\rm nm}(t_{\max})$ increases as a function of E_0
- The peak of the number density $n_{\rm nm}(t_{\max})$ is shifted to later times for $E_0 \gtrsim 0.9E_{\rm cr}$

$n_{\rm nm}(\infty)[{\rm nm}^{-3}]$	$1.9 \cdot 10^{6}$
$n_{\rm nm}(\infty)/n_{\rm nm}(t_{\max})$	$7.2 \cdot 10^{-2}$
$t_{\max}[10^{-20}{\rm s}]$	0.035

$E_0 = 0.6E_{\rm cr}$

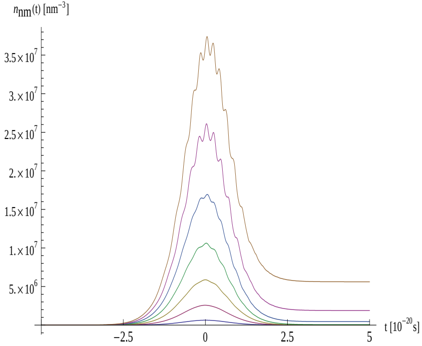


Field Strength Dependence of $n_{\mathrm{nm}}(t)$

- Sizeable number density $n_{\mathrm{nm}}(\infty)$ for $E_0 \gtrsim 0.4 E_{\mathrm{cr}}$
- Ratio $n_{\mathrm{nm}}(\infty)/n_{\mathrm{nm}}(t_{\max})$ increases as a function of E_0
- The peak of the number density $n_{\mathrm{nm}}(t_{\max})$ is shifted to later times for $E_0 \gtrsim 0.9 E_{\mathrm{cr}}$

$n_{\mathrm{nm}}(\infty)[\mathrm{nm}^{-3}]$	$5.6 \cdot 10^6$
$n_{\mathrm{nm}}(\infty)/n_{\mathrm{nm}}(t_{\max})$	0.15
$t_{\max}[10^{-20}\mathrm{s}]$	0.050

$E_0 = 0.7 E_{\mathrm{cr}}$

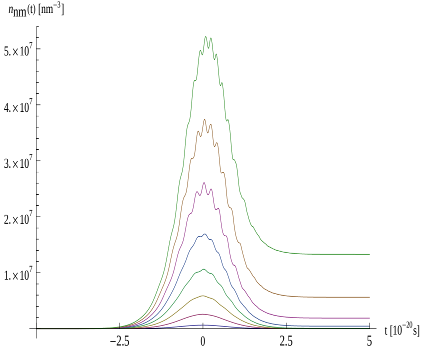


Field Strength Dependence of $n_{\mathrm{nm}}(t)$

- Sizeable number density $n_{\mathrm{nm}}(\infty)$ for $E_0 \gtrsim 0.4E_{\mathrm{cr}}$
- Ratio $n_{\mathrm{nm}}(\infty)/n_{\mathrm{nm}}(t_{\mathrm{max}})$ increases as a function of E_0
- The peak of the number density $n_{\mathrm{nm}}(t_{\mathrm{max}})$ is shifted to later times for $E_0 \gtrsim 0.9E_{\mathrm{cr}}$

$n_{\mathrm{nm}}(\infty)[\mathrm{nm}^{-3}]$	$1.3 \cdot 10^7$
$n_{\mathrm{nm}}(\infty)/n_{\mathrm{nm}}(t_{\mathrm{max}})$	0.25
$t_{\mathrm{max}}[10^{-20}\mathrm{s}]$	0.080

$E_0 = 0.8E_{\mathrm{cr}}$

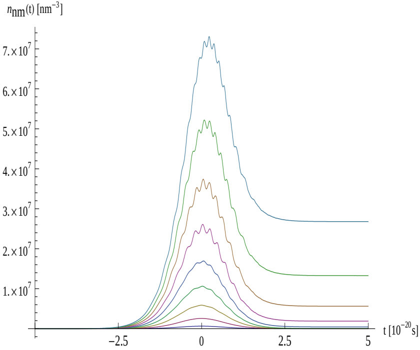


Field Strength Dependence of $n_{\mathrm{nm}}(t)$

- Sizeable number density $n_{\mathrm{nm}}(\infty)$ for $E_0 \gtrsim 0.4 E_{\mathrm{cr}}$
- Ratio $n_{\mathrm{nm}}(\infty)/n_{\mathrm{nm}}(t_{\max})$ increases as a function of E_0
- The peak of the number density $n_{\mathrm{nm}}(t_{\max})$ is shifted to later times for $E_0 \gtrsim 0.9 E_{\mathrm{cr}}$

$n_{\mathrm{nm}}(\infty)[\mathrm{nm}^{-3}]$	$2.7 \cdot 10^7$
$n_{\mathrm{nm}}(\infty)/n_{\mathrm{nm}}(t_{\max})$	0.37
$t_{\max}[10^{-20}\mathrm{s}]$	0.225

$E_0 = 0.9 E_{\mathrm{cr}}$

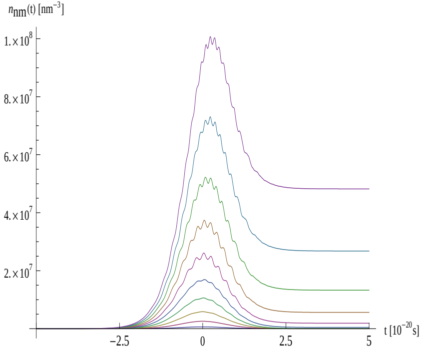


Field Strength Dependence of $n_{\mathrm{nm}}(t)$

- Sizeable number density $n_{\mathrm{nm}}(\infty)$ for $E_0 \gtrsim 0.4 E_{\mathrm{cr}}$
- Ratio $n_{\mathrm{nm}}(\infty)/n_{\mathrm{nm}}(t_{\mathrm{max}})$ increases as a function of E_0
- The peak of the number density $n_{\mathrm{nm}}(t_{\mathrm{max}})$ is shifted to later times for $E_0 \gtrsim 0.9 E_{\mathrm{cr}}$

$n_{\mathrm{nm}}(\infty)[\mathrm{nm}^{-3}]$	$4.8 \cdot 10^7$
$n_{\mathrm{nm}}(\infty)/n_{\mathrm{nm}}(t_{\mathrm{max}})$	0.48
$t_{\mathrm{max}}[10^{-20}\mathrm{s}]$	0.230

$E_0 = E_{\mathrm{cr}}$

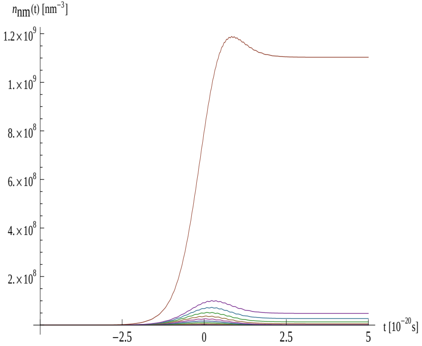


Field Strength Dependence of $n_{\rm nm}(t)$

- Sizeable number density $n_{\rm nm}(\infty)$ for $E_0 \gtrsim 0.4 E_{\rm cr}$
- Ratio $n_{\rm nm}(\infty)/n_{\rm nm}(t_{\max})$ increases as a function of E_0
- The peak of the number density $n_{\rm nm}(t_{\max})$ is shifted to later times for $E_0 \gtrsim 0.9 E_{\rm cr}$

$n_{\rm nm}(\infty)[{\rm nm}^{-3}]$	$1.1 \cdot 10^9$
$n_{\rm nm}(\infty)/n_{\rm nm}(t_{\max})$	0.92
$t_{\max}[10^{-20}{\rm s}]$	0.830

$E_0 = 2E_{\rm cr}$



Low-Density / Markovian approximation of $n_{\mathrm{nm}}(t)$

Solve Markovian equation for the production rate:

$$\dot{f}_{\mathrm{m}}(\mathbf{k},t) = [1 - 2f_{\mathrm{m}}(\mathbf{k},t)] \frac{eE(t)\epsilon_\perp}{2\omega_{\mathbf{p}}^2(t)} \int_{t_0}^{t} \mathrm{d}t' \, \frac{eE(t')\epsilon_\perp}{\omega_{\mathbf{p}}^2(t')} \times \cos\left(2\int_{t'}^{t} \mathrm{d}\tau \, \omega_{\mathbf{p}}(\tau)\right)$$

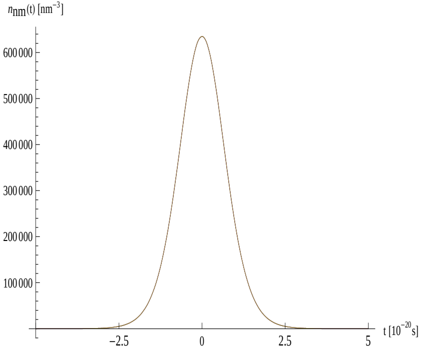
Solve low-density equation for the production rate:

$$\dot{f}_{\mathrm{ld}}(\mathbf{k},t) = \frac{eE(t)\epsilon_\perp}{2\omega_{\mathbf{p}}^2(t)} \int_{t_0}^{t} \mathrm{d}t' \, \frac{eE(t')\epsilon_\perp}{\omega_{\mathbf{p}}^2(t')} \times \cos\left(2\int_{t'}^{t} \mathrm{d}\tau \, \omega_{\mathbf{p}}(\tau)\right)$$

Low-Density / Markovian approximation of $n_{\mathrm{nm}}(t)$

- Approximations show the correct characteristics
- Approximations describe $n_{\mathrm{nm}}(t_{\mathrm{max}})$ for $E_0 \lesssim 0.9E_{\mathrm{cr}}$ well
- Approximations overestimate $n_{\mathrm{nm}}(\infty)$ for $E_0 \gtrsim 0.3E_{\mathrm{cr}}$

$$E_0 = 0.1E_{\mathrm{cr}}$$

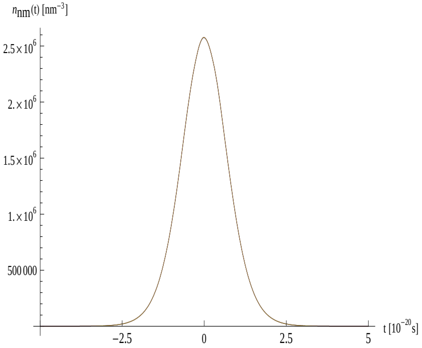


	l-d	Markov
$n(t_{\mathrm{max}})/n_{\mathrm{nm}}(t_{\mathrm{max}})$	1.000	1.000
$n(\infty)/n_{\mathrm{nm}}(\infty)$	1.000	1.000

Low-Density / Markovian approximation of $n_{\rm nm}(t)$

- Approximations show the correct characteristics
- Approximations describe $n_{\rm nm}(t_{\rm max})$ for $E_0 \lesssim 0.9E_{\rm cr}$ well
- Approximations overestimate $n_{\rm nm}(\infty)$ for $E_0 \gtrsim 0.3E_{\rm cr}$

$E_0 = 0.2E_{\rm cr}$

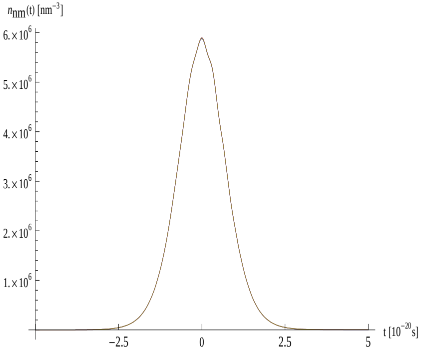


	l-d	Markov
$n(t_{\rm max})/n_{\rm nm}(t_{\rm max})$	1.001	1.001
$n(\infty)/n_{\rm nm}(\infty)$	1.053	1.053

Low-Density / Markovian approximation of $n_{\mathrm{nm}}(t)$

- Approximations show the correct characteristics
- Approximations describe $n_{\mathrm{nm}}(t_{\max})$ for $E_0 \lesssim 0.9E_{\mathrm{cr}}$ well
- Approximations overestimate $n_{\mathrm{nm}}(\infty)$ for $E_0 \gtrsim 0.3E_{\mathrm{cr}}$

$$E_0 = 0.3E_{\mathrm{cr}}$$



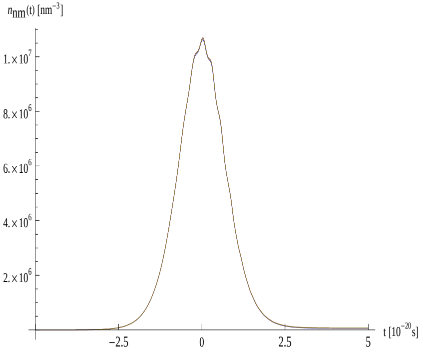
	l-d	Markov
$n(t_{\max})/n_{\mathrm{nm}}(t_{\max})$	1.003	1.002
$n(\infty)/n_{\mathrm{nm}}(\infty)$	1.201	1.201

Low-Density / Markovian approximation of $n_{\mathrm{nm}}(t)$

- Approximations show the correct characteristics
- Approximations describe $n_{\mathrm{nm}}(t_{\max})$ for $E_0 \lesssim 0.9E_{\mathrm{cr}}$ well
- Approximations overestimate $n_{\mathrm{nm}}(\infty)$ for $E_0 \gtrsim 0.3E_{\mathrm{cr}}$

	l-d	Markov
$n(t_{\max})/n_{\mathrm{nm}}(t_{\max})$	1.007	1.005
$n(\infty)/n_{\mathrm{nm}}(\infty)$	1.228	1.228

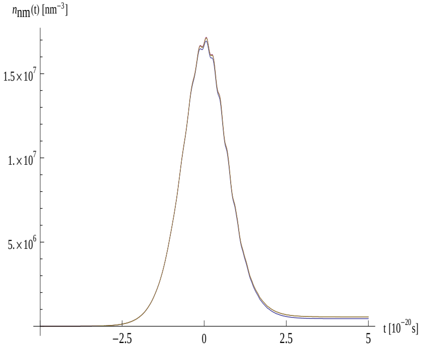
$$E_0 = 0.4E_{\mathrm{cr}}$$



Low-Density / Markovian approximation of $n_{\mathrm{nm}}(t)$

- Approximations show the correct characteristics
- Approximations describe $n_{\mathrm{nm}}(t_{\mathrm{max}})$ for $E_0 \lesssim 0.9E_{\mathrm{cr}}$ well
- Approximations overestimate $n_{\mathrm{nm}}(\infty)$ for $E_0 \gtrsim 0.3E_{\mathrm{cr}}$

$$E_0 = 0.5E_{\mathrm{cr}}$$

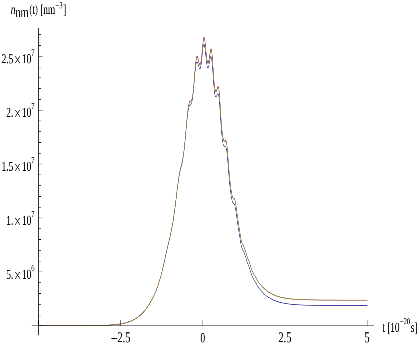


	l-d	Markov
$n(t_{\mathrm{max}})/n_{\mathrm{nm}}(t_{\mathrm{max}})$	1.013	1.010
$n(\infty)/n_{\mathrm{nm}}(\infty)$	1.247	1.246

Low-Density / Markovian approximation of $n_{\mathrm{nm}}(t)$

- Approximations show the correct characteristics
- Approximations describe $n_{\mathrm{nm}}(t_{\max})$ for $E_0 \lesssim 0.9 E_{\mathrm{cr}}$ well
- Approximations overestimate $n_{\mathrm{nm}}(\infty)$ for $E_0 \gtrsim 0.3 E_{\mathrm{cr}}$

$E_0 = 0.6 E_{\mathrm{cr}}$

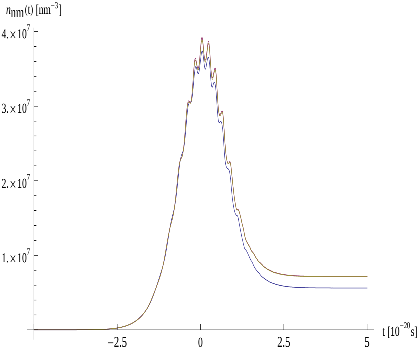


	l-d	Markov
$n(t_{\max})/n_{\mathrm{nm}}(t_{\max})$	1.025	1.019
$n(\infty)/n_{\mathrm{nm}}(\infty)$	1.263	1.260

Low-Density / Markovian approximation of $n_{\rm nm}(t)$

- Approximations show the correct characteristics
- Approximations describe $n_{\rm nm}(t_{\rm max})$ for $E_0 \lesssim 0.9E_{\rm cr}$ well
- Approximations overestimate $n_{\rm nm}(\infty)$ for $E_0 \gtrsim 0.3E_{\rm cr}$

$$E_0 = 0.7E_{\rm cr}$$

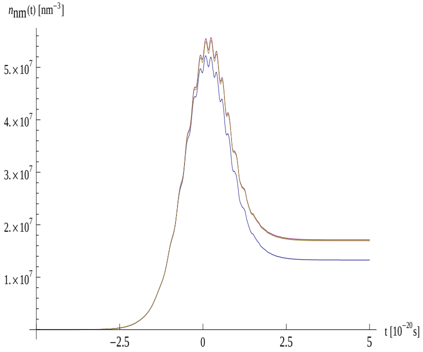


	l-d	Markov
$n(t_{\rm max})/n_{\rm nm}(t_{\rm max})$	1.049	1.040
$n(\infty)/n_{\rm nm}(\infty)$	1.277	1.269

Low-Density / Markovian approximation of $n_{\rm nm}(t)$

- Approximations show the correct characteristics
- Approximations describe $n_{\rm nm}(t_{\rm max})$ for $E_0 \lesssim 0.9E_{\rm cr}$ well
- Approximations overestimate $n_{\rm nm}(\infty)$ for $E_0 \gtrsim 0.3E_{\rm cr}$

$$E_0 = 0.8E_{\rm cr}$$

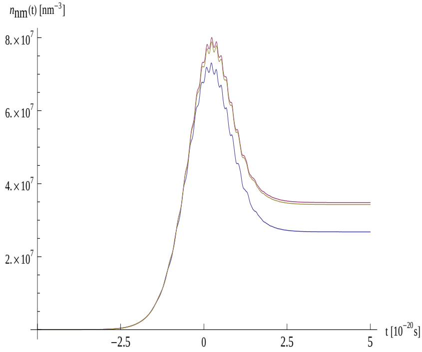


	l-d	Markov
$n(t_{\rm max})/n_{\rm nm}(t_{\rm max})$	1.061	1.050
$n(\infty)/n_{\rm nm}(\infty)$	1.289	1.276

Low-Density / Markovian approximation of $n_{\rm nm}(t)$

- Approximations show the correct characteristics
- Approximations describe $n_{\rm nm}(t_{\rm max})$ for $E_0 \lesssim 0.9E_{\rm cr}$ well
- Approximations overestimate $n_{\rm nm}(\infty)$ for $E_0 \gtrsim 0.3E_{\rm cr}$

$$E_0 = 0.9E_{\rm cr}$$



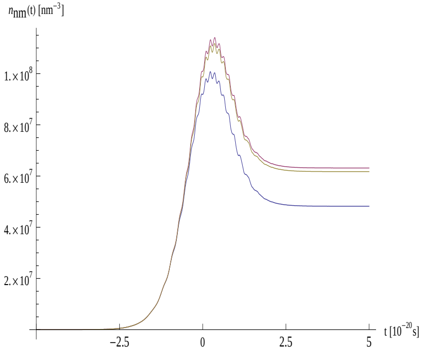
	l-d	Markov
$n(t_{\rm max})/n_{\rm nm}(t_{\rm max})$	1.095	1.079
$n(\infty)/n_{\rm nm}(\infty)$	1.300	1.279

Low-Density / Markovian approximation of $n_{\mathrm{nm}}(t)$

- Approximations show the correct characteristics
- Approximations describe $n_{\mathrm{nm}}(t_{\mathrm{max}})$ for $E_0 \lesssim 0.9E_{\mathrm{cr}}$ well
- Approximations overestimate $n_{\mathrm{nm}}(\infty)$ for $E_0 \gtrsim 0.3E_{\mathrm{cr}}$

	l-d	Markov
$n(t_{\mathrm{max}})/n_{\mathrm{nm}}(t_{\mathrm{max}})$	1.132	1.100
$n(\infty)/n_{\mathrm{nm}}(\infty)$	1.310	1.280

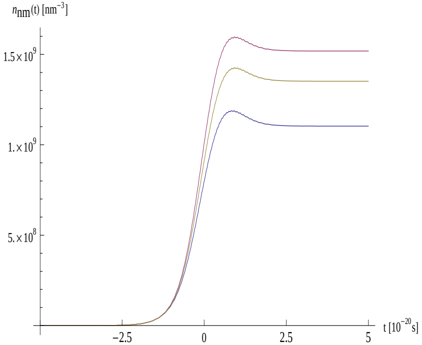
$E_0 = E_{\mathrm{cr}}$



Low-Density / Markovian approximation of $n_{\rm nm}(t)$

- Approximations show the correct characteristics
- Approximations describe $n_{\rm nm}(t_{\max})$ for $E_0 \lesssim 0.9E_{\rm cr}$ well
- Approximations overestimate $n_{\rm nm}(\infty)$ for $E_0 \gtrsim 0.3E_{\rm cr}$

$$E_0 = 2E_{\rm cr}$$



	l-d	Markov
$n(t_{\max})/n_{\rm nm}(t_{\max})$	1.343	1.199
$n(\infty)/n_{\rm nm}(\infty)$	1.377	1.225

Backreaction Mechanism

Solve full non-Markovian equation for the production rate:

$$\dot{f}_{\text{back}}(\mathbf{k},t) = \frac{e\,E(t)\epsilon_\perp}{2\omega_\mathbf{p}^2(t)} \int_{t_0}^{t} \mathrm{d}t' \, \frac{e\,E(t')\epsilon_\perp}{\omega_\mathbf{p}^2(t')} \left[1 - 2f_{\text{back}}(\mathbf{k},t')\right] \times \cos\left(2\int_{t'}^{t} \mathrm{d}\tau\,\omega_\mathbf{p}(\tau)\right)$$

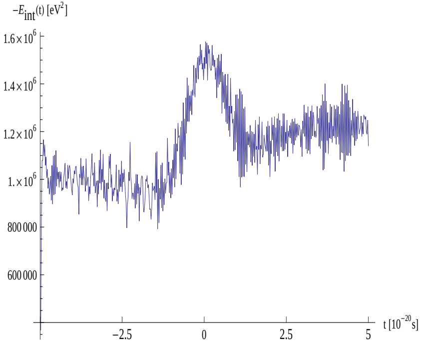
Include the backreaction mechanism in the calculation:

$$\dot{E}_{\text{int}}(t) = -4e \int \frac{\mathrm{d}^3k}{(2\pi)^3} \left[\frac{p_\parallel(t)}{\omega_\mathbf{p}(t)} f_{\text{back}}(\mathbf{k},t) + \frac{\omega_\mathbf{p}(t)}{eE(t)} \dot{f}_{\text{back}}(\mathbf{k},t) - \frac{e\dot{E}(t)\epsilon_\perp^2}{8\,\omega_\mathbf{p}^5(t)}\right]$$

Backreaction Mechanism - Internal Electric Field

- For $E_0 \lesssim 0.3 E_{\mathrm{cr}}$: $t \approx 0$
- Virtual particle creation for $E \gtrsim 2 \cdot 10^{10}\,\mathrm{eV}^2$
- Backreaction mechanism becomes imporant just for for field strenghts $E_0 \gtrsim E_{\mathrm{cr}}$

$E_0 = 0.1 E_{\mathrm{cr}}$

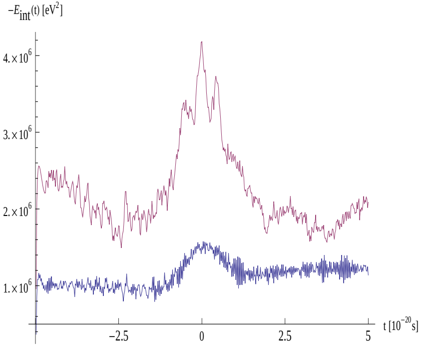


$E_{\mathrm{int}}(5 \cdot 10^{-20}\,\mathrm{s})$ [eV^2]	$-1.2 \cdot 10^6$
$\lvert E_{\mathrm{int}}(5 \cdot 10^{-20}\,\mathrm{s}) \rvert / E_{\mathrm{cr}}$	$1.3 \cdot 10^{-6}$

Backreaction Mechanism - Internal Electric Field

- For $E_0 \lesssim 0.3 E_{\text{cr}}$: $t \approx 0$
- Virtual particle creation for $E \gtrsim 2 \cdot 10^{10}\,\text{eV}^2$
- Backreaction mechanism becomes imporant just for for field strenghts $E_0 \gtrsim E_{\text{cr}}$

$E_0 = 0.2 E_{\text{cr}}$



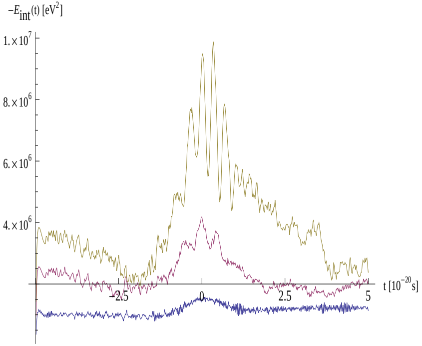
$E_{\text{int}}(5 \cdot 10^{-20}\,\text{s})$ [eV^2]	$-2.3 \cdot 10^6$
$\lvert E_{\text{int}}(5 \cdot 10^{-20}\,\text{s}) \rvert / E_{\text{cr}}$	$2.4 \cdot 10^{-6}$

Backreaction Mechanism - Internal Electric Field

- For $E_0 \lesssim 0.3E_{\rm cr}$: $t \approx 0$
- Virtual particle creation for $E \gtrsim 2 \cdot 10^{10}\,\mathrm{eV}^2$
- Backreaction mechanism becomes imporant just for for field strenghts $E_0 \gtrsim E_{\rm cr}$

$E_{\rm int}(5 \cdot 10^{-20}\,\mathrm{s})\ [\mathrm{eV}^2]$	$-5.2 \cdot 10^6$
$\lvert E_{\rm int}(5 \cdot 10^{-20}\,\mathrm{s})\rvert / E_{\rm cr}$	$2.8 \cdot 10^{-6}$

$E_0 = 0.3E_{\rm cr}$

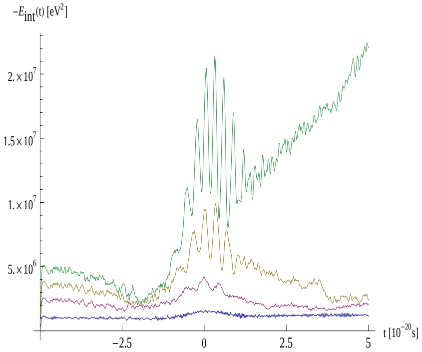


Backreaction Mechanism - Internal Electric Field

- For $E_0 \gtrsim 0.3E_{\text{cr}}$: $t = 5 \cdot 10^{-20}\,\text{s}$
- Virtual particle creation for $E \gtrsim 2 \cdot 10^{10}\,\text{eV}^2$
- Backreaction mechanism becomes imporant just for for field strenghts $E_0 \gtrsim E_{\text{cr}}$

$E_{\text{int}}(5 \cdot 10^{-20}\,\text{s})\ [\text{eV}^2]$	$-2.5 \cdot 10^7$
$\lvert E_{\text{int}}(5 \cdot 10^{-20}\,\text{s}) \rvert / E_{\text{cr}}$	$2.6 \cdot 10^{-5}$

$E_0 = 0.4E_{\text{cr}}$

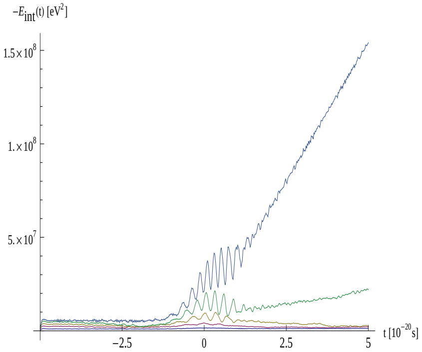


Backreaction Mechanism - Internal Electric Field

- For $E_0 \gtrsim 0.3E_{\rm cr}$: $t = 5 \cdot 10^{-20}\,{\rm s}$
- Virtual particle creation for $E \gtrsim 2 \cdot 10^{10}\,{\rm eV}^2$
- Backreaction mechanism becomes imporant just for for field strenghts $E_0 \gtrsim E_{\rm cr}$

$E_{\rm int}(5 \cdot 10^{-20}\,{\rm s})\ [{\rm eV}^2]$	$-1.6 \cdot 10^8$
$\lvert E_{\rm int}(5 \cdot 10^{-20}\,{\rm s}) \rvert / E_{\rm cr}$	$1.8 \cdot 10^{-4}$

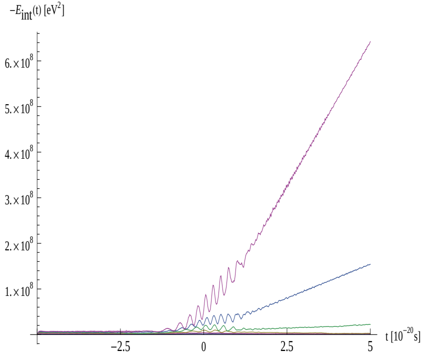
$E_0 = 0.5E_{\rm cr}$



Backreaction Mechanism - Internal Electric Field

- For $E_0 \gtrsim 0.3 E_{\rm cr}$: $t = 5 \cdot 10^{-20}$ s
- Virtual particle creation for $E \gtrsim 2 \cdot 10^{10}\,\mathrm{eV}^2$
- Backreaction mechanism becomes imporant just for for field strenghts $E_0 \gtrsim E_{\rm cr}$

$E_0 = 0.6 E_{\rm cr}$



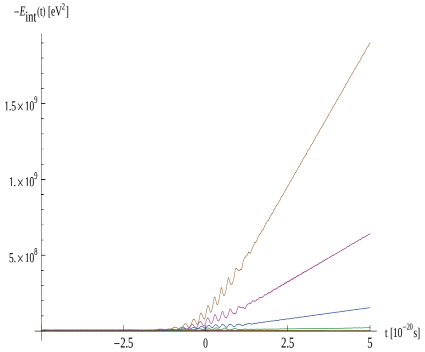
$E_{\rm int}(5 \cdot 10^{-20}\,\mathrm{s})$ [eV^2]	$-6.4 \cdot 10^8$
$\lvert E_{\rm int}(5 \cdot 10^{-20}\,\mathrm{s}) \rvert / E_{\rm cr}$	$7.5 \cdot 10^{-4}$

Backreaction Mechanism - Internal Electric Field

- For $E_0 \gtrsim 0.3 E_{\mathrm{cr}}$: $t = 5 \cdot 10^{-20}\,\mathrm{s}$
- Virtual particle creation for $E \gtrsim 2 \cdot 10^{10}\,\mathrm{eV}^2$
- Backreaction mechanism becomes imporant just for for field strenghts $E_0 \gtrsim E_{\mathrm{cr}}$

$E_{\mathrm{int}}(5 \cdot 10^{-20}\,\mathrm{s})\ [\mathrm{eV}^2]$	$-1.9 \cdot 10^9$
$\lvert E_{\mathrm{int}}(5 \cdot 10^{-20}\,\mathrm{s}) \rvert / E_{\mathrm{cr}}$	$2.2 \cdot 10^{-3}$

$E_0 = 0.7 E_{\mathrm{cr}}$

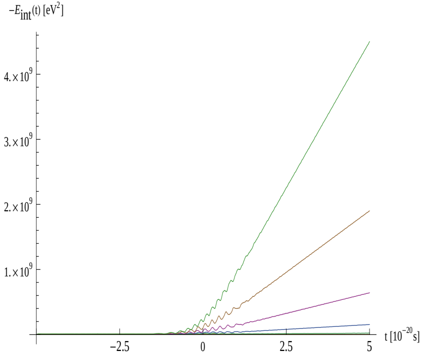


Backreaction Mechanism - Internal Electric Field

- For $E_0 \gtrsim 0.3 E_{\mathrm{cr}}$: $t = 5 \cdot 10^{-20}\,\mathrm{s}$
- Virtual particle creation for $E \gtrsim 2 \cdot 10^{10}\,\mathrm{eV}^2$
- Backreaction mechanism becomes imporant just for for field strenghts $E_0 \gtrsim E_{\mathrm{cr}}$

$E_{\mathrm{int}}(5 \cdot 10^{-20}\,\mathrm{s})\ [\mathrm{eV}^2]$	$-4.5 \cdot 10^9$
$\lvert E_{\mathrm{int}}(5 \cdot 10^{-20}\,\mathrm{s}) \rvert / E_{\mathrm{cr}}$	$5.2 \cdot 10^{-3}$

$E_0 = 0.8 E_{\mathrm{cr}}$

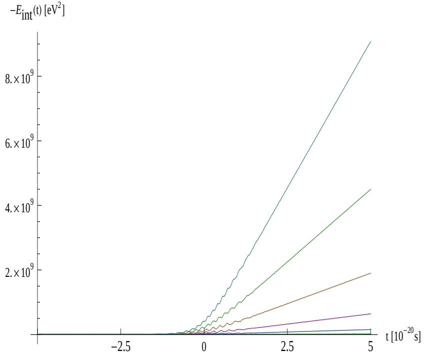


Backreaction Mechanism - Internal Electric Field

- For $E_0 \gtrsim 0.3 E_{\rm cr}$: $t = 5 \cdot 10^{-20}\,\mathrm{s}$
- Virtual particle creation for $E \gtrsim 2 \cdot 10^{10}\,\mathrm{eV}^2$
- Backreaction mechanism becomes imporant just for for field strenghts $E_0 \gtrsim E_{\rm cr}$

$E_{\rm int}(5 \cdot 10^{-20}\,\mathrm{s})\ [\mathrm{eV}^2]$	$-9.1 \cdot 10^9$
$\lvert E_{\rm int}(5 \cdot 10^{-20}\,\mathrm{s}) \rvert / E_{\rm cr}$	0.011

$E_0 = 0.9 E_{\rm cr}$

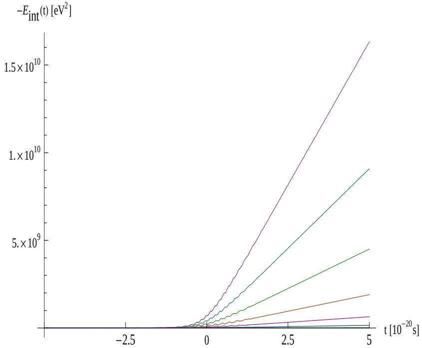


Backreaction Mechanism - Internal Electric Field

- For $E_0 \gtrsim 0.3 E_{\rm cr}$: $t = 5 \cdot 10^{-20}\,{\rm s}$
- Virtual particle creation for $E \gtrsim 2 \cdot 10^{10}\,{\rm eV}^2$
- Backreaction mechanism becomes imporant just for for field strenghts $E_0 \gtrsim E_{\rm cr}$

$E_{\rm int}(5 \cdot 10^{-20}\,{\rm s})\ [{\rm eV}^2]$	$-1.6 \cdot 10^{10}$
$\left\lvert E_{\rm int}(5 \cdot 10^{-20}\,{\rm s}) \right\rvert / E_{\rm cr}$	0.019

$E_0 = E_{\rm cr}$

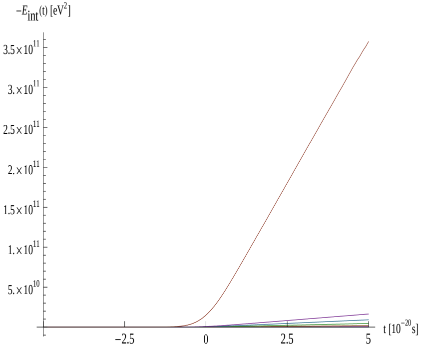


Backreaction Mechanism - Internal Electric Field

- For $E_0 \gtrsim 0.3E_{\mathrm{cr}}$: $t = 5 \cdot 10^{-20}\,\mathrm{s}$
- Virtual particle creation for $E \gtrsim 2 \cdot 10^{10}\,\mathrm{eV}^2$
- Backreaction mechanism becomes imporant just for for field strenghts $E_0 \gtrsim E_{\mathrm{cr}}$

$E_{\mathrm{int}}(5 \cdot 10^{-20}\,\mathrm{s})\ [\mathrm{eV}^2]$	$-3.6 \cdot 10^{11}$
$\lvert E_{\mathrm{int}}(5 \cdot 10^{-20}\,\mathrm{s}) \rvert / E_{\mathrm{cr}}$	0.415

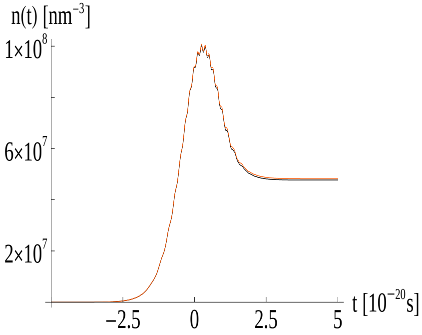
$E_0 = 2E_{\mathrm{cr}}$



Backreaction Mechanism - Number Density $n_{\text{back}}(t)$

- For $E_0 \lesssim E_{\text{cr}}$: Backreaction mechanism is marginal
- Particle creation → asymptotic particles?!
- Long-term evolution of the single particle distribution function?!

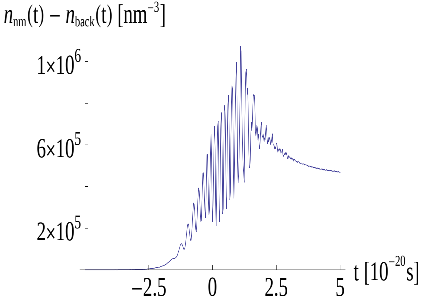
$$E_0 = E_{\text{cr}}$$



Backreaction Mechanism - Number Density $n_{\mathrm{back}}(t)$

$$E_0 = E_{\mathrm{cr}}$$

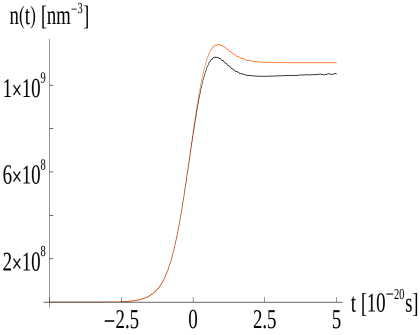
- For $E_0 \lesssim E_{\mathrm{cr}}$: Backreaction mechanism is marginal
- Particle creation $\longrightarrow$ asymptotic particles?!
- Long-term evolution of the single particle distribution function?!



Backreaction Mechanism - Number Density $n_{\mathrm{back}}(t)$

- For $E_0 \gtrsim E_{\mathrm{cr}}$: Backreaction mechanism is important
- Particle creation $\rightarrow$ asymptotic particles?!
- Long-term evolution of the single particle distribution function?!

$$E_0 = 2E_{\mathrm{cr}}$$



Backreaction Mechanism - Number Density $n_{\text{back}}(t)$

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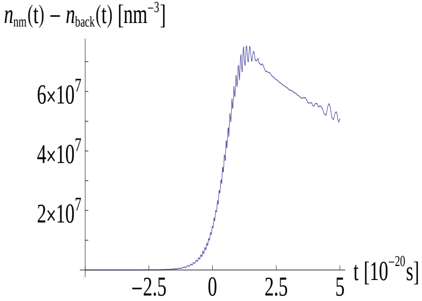
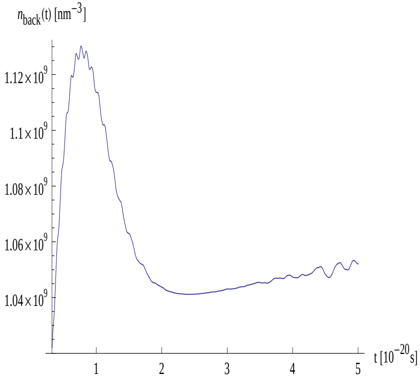
$$E_0 = 2E_{\text{cr}}$$



Backreaction Mechanism - Number Density $n_{\mathrm{back}}(t)$

$$E_0 = 2E_{\mathrm{cr}}$$

- For $E_0 \gtrsim E_{\mathrm{cr}}$: Backreaction mechanism is important
- Particle creation $\longrightarrow$ asymptotic particles?!
- Long-term evolution of the single particle distribution function?!



Pulse Length Dependence of $n_{\rm nm}(t)$

Solve full non-Markovian equation for the production rate:

$$\dot{f}_{\rm nm}(\mathbf{k}, t) = \frac{e\,E(t)\epsilon_\perp}{2\omega_{\mathbf{p}}^2(t)} \int_{t_0}^{t} \mathrm{d}t' \, \frac{e\,E(t')\epsilon_\perp}{\omega_{\mathbf{p}}^2(t')} \left[1 - 2f_{\rm nm}(\mathbf{k}, t')\right] \times \cos\left(2 \int_{t'}^{t} \mathrm{d}\tau \, \omega_{\mathbf{p}}(\tau)\right)$$

Ignore the backreaction mechanism!

Choose longer pulse lengths:

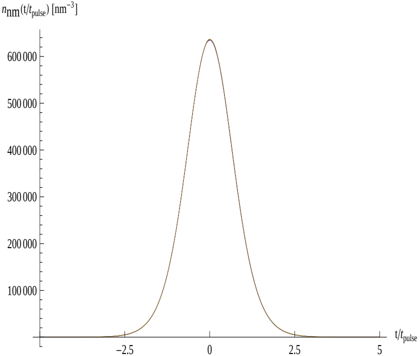
$$t_{\rm pulse,2} = 2 \cdot 10^{-19}\,\mathrm{s} = 2t_{\rm pulse,1}$$
$$t_{\rm pulse,4} = 4 \cdot 10^{-19}\,\mathrm{s} = 4t_{\rm pulse,1}$$

Pulse Length Dependence of $n_{\rm nm}(t)$

- $E_0 = 0.1E_{\rm cr}$: $n_{\rm nm}(\bar{t})$ nearly identical for all pulse lengths
- $E_0 = 0.4E_{\rm cr}$: More oscillations
- $E_0 \geq 0.7E_{\rm cr}$: $n_{\rm nm}(\bar{t})$ increases
- $E_0 = 2E_{\rm cr}$: Proportionality $n_{\rm nm,j}(\infty) \approx j \cdot n_{\rm nm}(\infty)$

	2	4
$n(t_{\max})/n_{\rm nm}(t_{\max})$	1.002	1.003
$n(\infty)/n_{\rm nm}(\infty)$	1.051	1.037

$E_0 = 0.1E_{\rm cr}$



Pulse Length Dependence of $n_{\text{nm}}(t)$

- $E_0 = 0.1E_{\text{cr}}$: $n_{\text{nm}}(\bar{t})$ nearly identical for all pulse lengths
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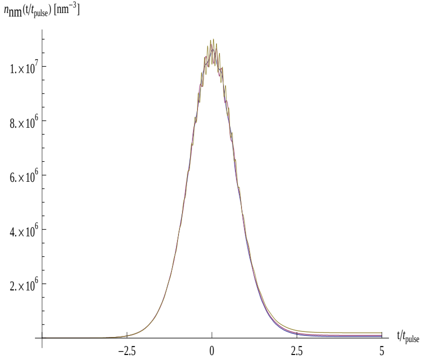
- $E_0 \geq 0.7E_{\text{cr}}$: $n_{\text{nm}}(\bar{t})$ increases
- $E_0 = 2E_{\text{cr}}$: Proportionality $n_{\text{nm,j}}(\infty) \approx j \cdot n_{\text{nm}}(\infty)$

	2	4
$n(t_{\text{max}})/n_{\text{nm}}(t_{\text{max}})$	1.018	1.035
$n(\infty)/n_{\text{nm}}(\infty)$	1.724	3.316

$E_0 = 0.4E_{\text{cr}}$

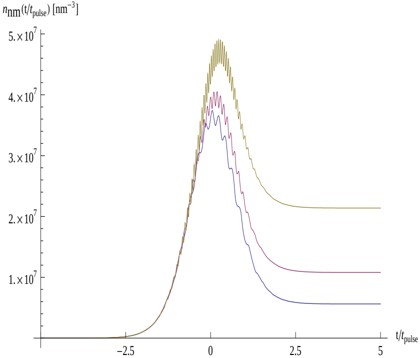


Pulse Length Dependence of $n_{\rm nm}(t)$

- $E_0 = 0.1E_{\rm cr}$: $n_{\rm nm}(\bar{t})$ nearly identical for all pulse lengths
- $E_0 = 0.4E_{\rm cr}$: More oscillations
- $E_0 \geq 0.7E_{\rm cr}$: $n_{\rm nm}(\bar{t})$ increases
- $E_0 = 2E_{\rm cr}$: Proportionality $n_{\rm nm,j}(\infty) \approx j \cdot n_{\rm nm}(\infty)$

	2	4
$n(t_{\rm max})/n_{\rm nm}(t_{\rm max})$	1.083	1.314
$n(\infty)/n_{\rm nm}(\infty)$	1.922	3.806

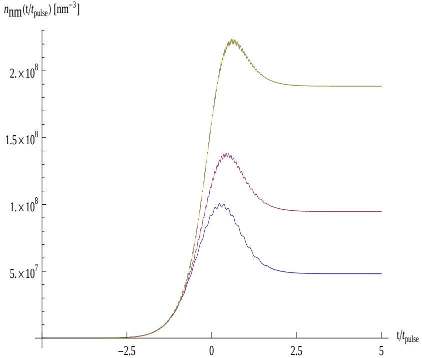
$E_0 = 0.7E_{\rm cr}$



Pulse Length Dependence of $n_{\mathrm{nm}}(t)$

- $E_0 = 0.1E_{\mathrm{cr}}$: $n_{\mathrm{nm}}(\bar{t})$ nearly identical for all pulse lengths
- $E_0 = 0.4E_{\mathrm{cr}}$: More oscillations
- $E_0 \geq 0.7E_{\mathrm{cr}}$: $n_{\mathrm{nm}}(\bar{t})$ increases
- $E_0 = 2E_{\mathrm{cr}}$: Proportionality $n_{\mathrm{nm,j}}(\infty) \approx j \cdot n_{\mathrm{nm}}(\infty)$

$$E_0 = E_{\mathrm{cr}}$$



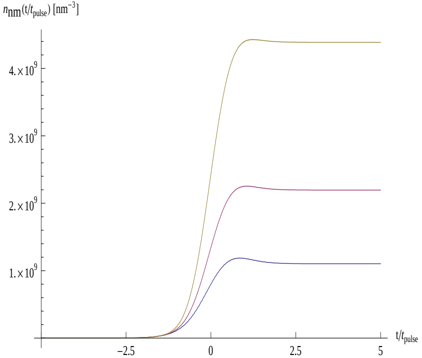
	2	4
$n(t_{\max})/n_{\mathrm{nm}}(t_{\max})$	1.374	2.220
$n(\infty)/n_{\mathrm{nm}}(\infty)$	1.964	3.910

Pulse Length Dependence of $n_{\mathrm{nm}}(t)$

- $E_0 = 0.1E_{\mathrm{cr}}$: $n_{\mathrm{nm}}(\bar{t})$ nearly identical for all pulse lengths
- $E_0 = 0.4E_{\mathrm{cr}}$: More oscillations
- $E_0 \geq 0.7E_{\mathrm{cr}}$: $n_{\mathrm{nm}}(\bar{t})$ increases
- $E_0 = 2E_{\mathrm{cr}}$: Proportionality $n_{\mathrm{nm,j}}(\infty) \approx j \cdot n_{\mathrm{nm}}(\infty)$

	2	4
$n(t_{\max})/n_{\mathrm{nm}}(t_{\max})$	1.898	3.728
$n(\infty)/n_{\mathrm{nm}}(\infty)$	1.991	3.976

$E_0 = 2E_{\mathrm{cr}}$



Outline

Summary

- Particle creation is peaked around $p_{\|} = 0$ and occurs for perpendicular momenta $k_\perp \lesssim 1\,\mathrm{MeV}$
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- Including collisional effects by means of a relaxation time approximation
- Extending the pulse length to the order $t_{\mathrm{pulse}} \approx 10^{-15}\,\mathrm{s}$
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