
Baryon-photon interaction in a covariant Faddeev approach

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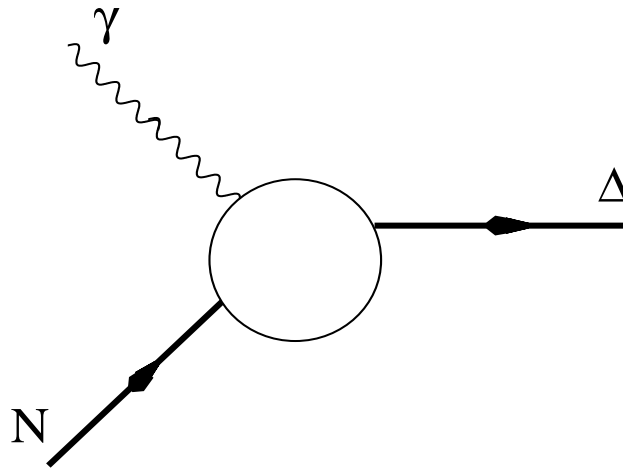
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Héviz, Hungary
Workshop

Outline

- Motivation:
baryonic structure → experimental advances.
- Back to fundamentals: [Poincare invariant Faddeev equations](#).
Ingredients: Quark propagator, gluon propagator, quark-gluon vertex ... DSEs.
Diquarks... BSEs.
Nucleon quark-core: solve a quark-diquark system.
- Calculated by now: M_N , nucleon FF, elmag. radii...
- Summary and Outlook (M_Δ , $N \rightarrow \Delta\gamma$, $\Sigma \rightarrow \Lambda\gamma$, π corrections).

Experimental advances

- Baryons are composite objects.
- Electromagnetic interaction:
present precise tests of the structure.
- Nucleon **electromagnetic form factors**, polarizabilities, strangeness content ($e^- p$ scattering).
- Electro- and photoproduction:
the proton and its lightest **resonance $\Delta(1232)$** .



- JLab-CEBAF, MIT-BATES, Mainz-MAMI, Brookhaven-LEGS.

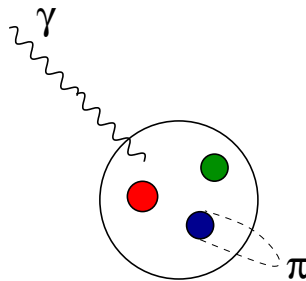
Theoretical efforts

$$EMR(\%) = \frac{E2}{M1} = -2.5 \pm 0.5 \quad (\text{MAMI})$$

$$CMR(\%) = \frac{C2}{M1} = -4.81 \pm 0.27 \quad (\text{LEGS})$$

Models

- Constituent quark models, pion cloud models....



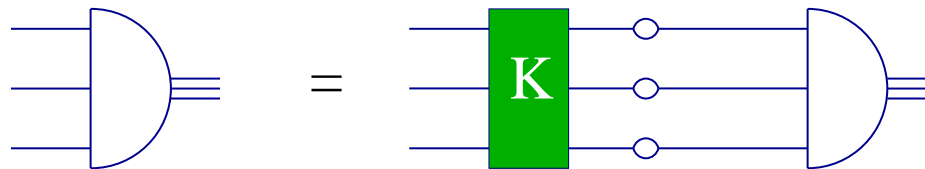
- Covariance, link to effective theories.

Do we understand the nucleon?

- Models offer a good description of data in low-energy regime.
- Baryon-quark vertex not trivial.
- Desired QCD-based solution.
- Confinement, DCSB, relativistic bound states ...

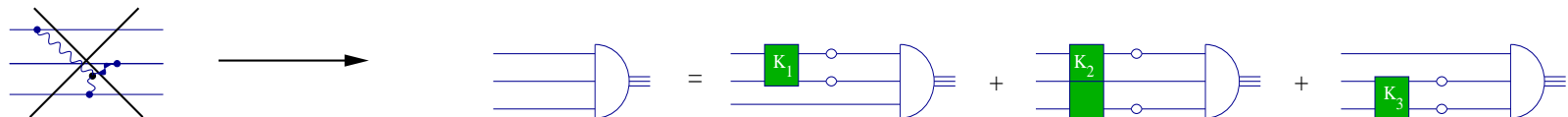
Faddeev Equations

- Baryons as composite objects of confined quarks and nonpointlike **diquarks**.
- Bound state poles in the 6-point Green-functions
→ homogenous integral equations for the baryon amplitudes.
- Solve iteratively if ingredients are known:
quark propagator, three-quark interaction kernel.



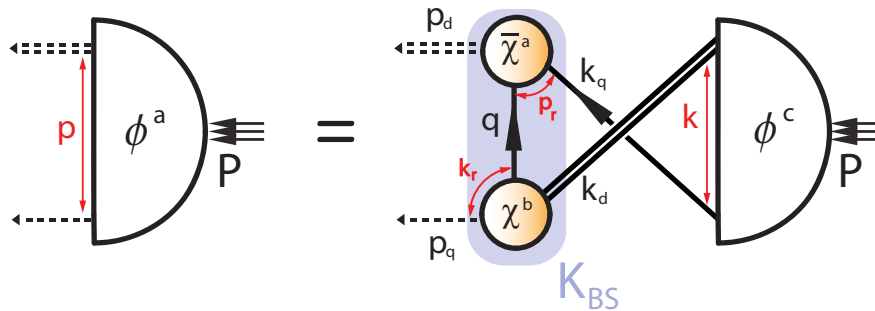
- Faddeev approximation:
retain only 2-particle interaction kernels (dominant structure in nucleon).

$$K = \sum_{i=1}^3 K_i \otimes S_i^{-1}$$



Faddeev equations

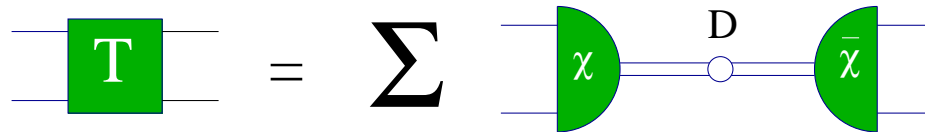
- Exploit the same attractive interaction for $1_c^{q\bar{q}}$ and $\bar{3}_c^{qq}$.
- Construct nucleon: $\bar{3}_c^{qq} \times 3_c^q \rightarrow \text{color-singlet}$.
- Binding in the nucleon:
quark exchange between the dormant quark and diquark.
- Solve numerically a quark-diquark BSE.



- Ingredients: quark propagator, 2-particle interaction kernel.

Diquarks

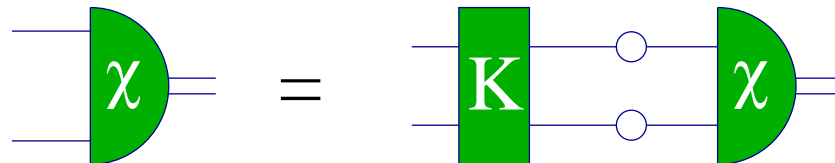
- q-q correlations dominant structure in the nucleon (supported by lattice).
- Approximation: two-quark separable correlations.



- Scalar and axial-vector correlations.

$$T_{qq}(p_1, p_2, P) \sim \chi(p_1)D(P)\bar{\chi}(p_2) + \chi^\mu(p_1)D^{\mu\nu}(P)\bar{\chi}^\nu(p_2)$$

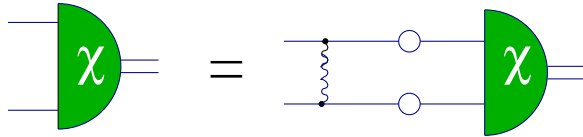
- Diquark homogenous Bethe-Salpeter equation.



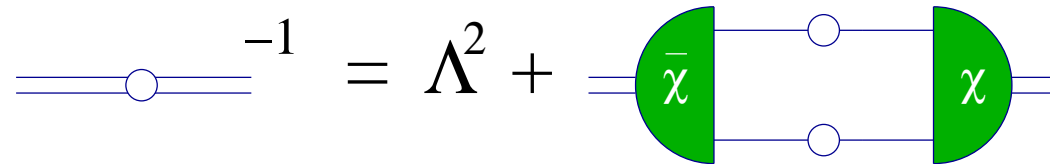
- Determines diquark amplitude on the mass-shell: $\chi = K G_0 \chi$

Diquark ingredients

- Solve using rainbow-ladder truncation:
one gluon exchange + vector-like quark-gluon vertex.



- Diquark propagator in ladder approximation.



$$D^{-1} = \bar{\chi} (K^{-1} - G_0) \chi$$

- Diquark propagator: calculated on-shell behavior,
parametrized off-shell behavior.

Quark ingredients

- Infinite coupled system of DSEs.

Quark propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}\text{---}^{-1} + \text{---}\bigcirc\text{---}$$

Gluon propagator:

$$\text{~~~~}\bigcirc\text{~~~~}^{-1} = \text{~~~~}\text{~~~~}^{-1} + \text{~~~~}\bigcirc\text{~~~~} + \text{~~~~}\bigcirc\text{~~~~} + \text{~~~~}\bigcirc\text{~~~~} + \text{~~~~}\bigcirc\text{~~~~} + \text{~~~~}\bigcirc\text{~~~~} + \text{~~~~}\bigcirc\text{~~~~}$$

Ghost propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}\text{---}^{-1} + \text{---}\bigcirc\text{---}$$

Ghost-gluon vertex:

$$\text{~~~~}\bigcirc\text{---} = \text{~~~~}\bigcirc\text{---} + \text{~~~~}\bigcirc\text{---}$$

Quark propagator

- Dressed quark propagator: solution of the DSE.

$$\text{---} \bigcirc \text{---}^{-1} = \text{---} \bigcirc \text{---}^{-1} + \text{---} \bigcirc \text{---} \text{---} \bigcirc \text{---}$$

- General form: $S(p) = i \not{p} \sigma_v(p^2) - \sigma_s(p^2) = -\frac{1}{iA(p^2) + B(p^2)}$

- Gluon dressing reflected in the quark mass function $M(p^2) = \frac{B(p^2)}{A(p^2)}$

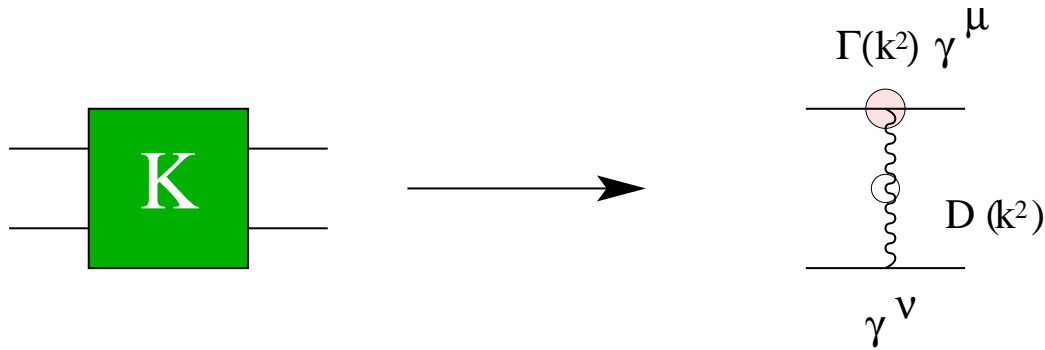
- Asymptotic freedom: $M(p^2) \rightarrow m_q$ perturbative quark propagator.

Rainbow-ladder truncation

- Solve quark DSE using rainbow truncation:

Quark gluon vertex ansatz $i\Gamma_\mu = i\gamma_\mu \times \Gamma(k^2)$

Note: consistent BSE q-q kernel.



- Advantage: simple truncation, quark DSE and q-q BSE kernel fulfil AV-WTI.

Gluon propagator, running α

- Dressed gluon propagator.

$$D_{\mu\nu}(k) = D(k^2) D_{\mu\nu}^{free}(k^2)$$

- Dressings $D(k^2)$ and $\Gamma(k^2)$ absorbed in the running coupling:

$$\alpha_{eff}(k^2) \sim D(k^2) \Gamma(k^2)$$

constraint by the correct UV behaviour $\rightarrow \frac{\pi \gamma_m}{\ln \frac{k^2}{\Lambda_{QCD}^2}}$

and strong enough in IR to generate D χ SB.

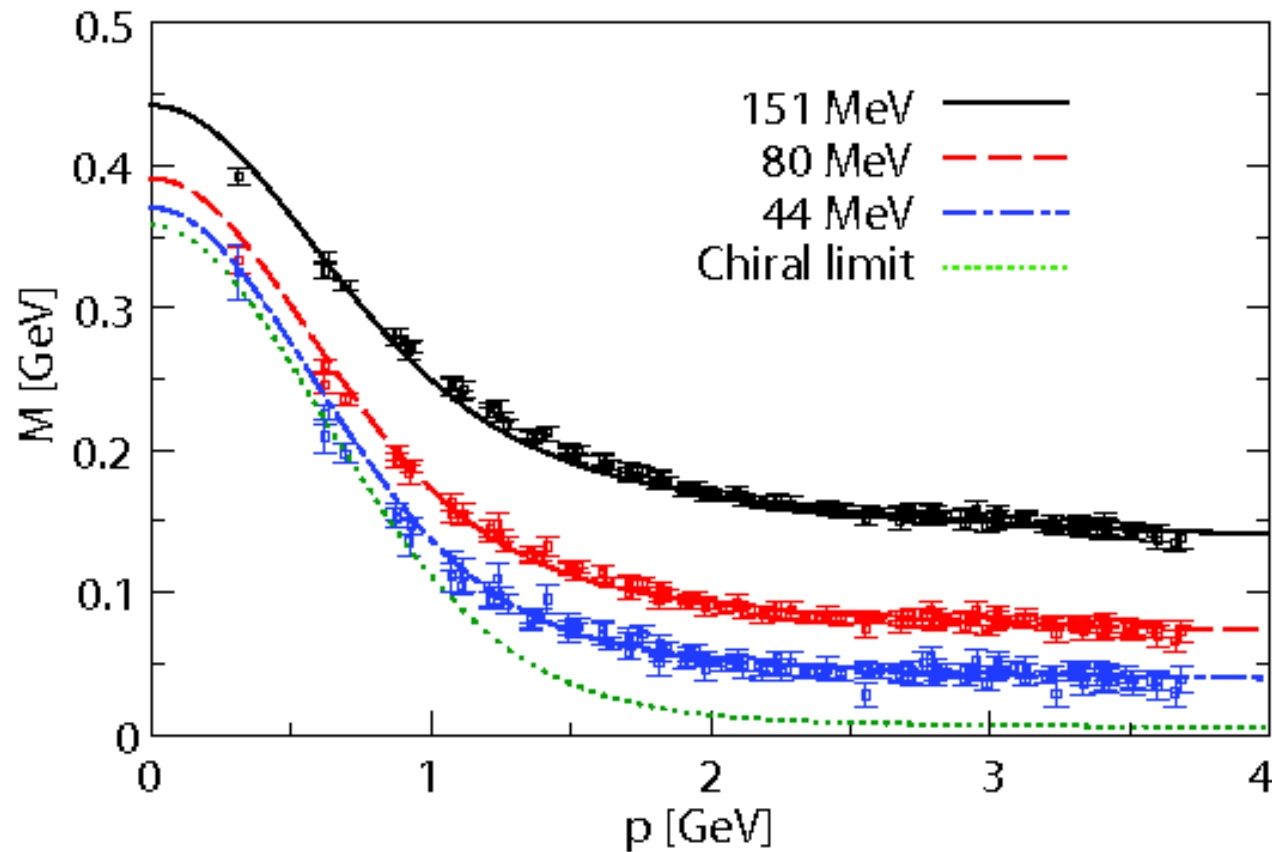
- Dressed quark propagator

$$S^{-1}(p, \mu^2) \sim Z_2(\mu^2, \Lambda^2) S_0^{-1}(p, \Lambda^2) - Z_2(\mu^2, \Lambda^2) \int_q^\Lambda d^4q \alpha_{eff}(k^2) D_{\mu\nu}^{free} \gamma_\mu S(q, \mu^2) i\gamma_\nu$$

P. Maris, P.C. Tandy Phys. Rev. C 60, 055214 (1999).

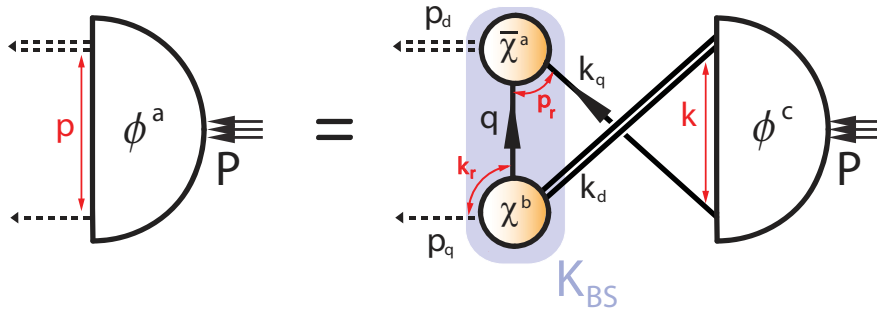
$M(p)$ in rainbow-ladder

- Quark mass function fixed by lattice calculations
P.O. Bowman *et al.* Phys. Rev. D 66, 014505 (2002).



Nucleon BSE

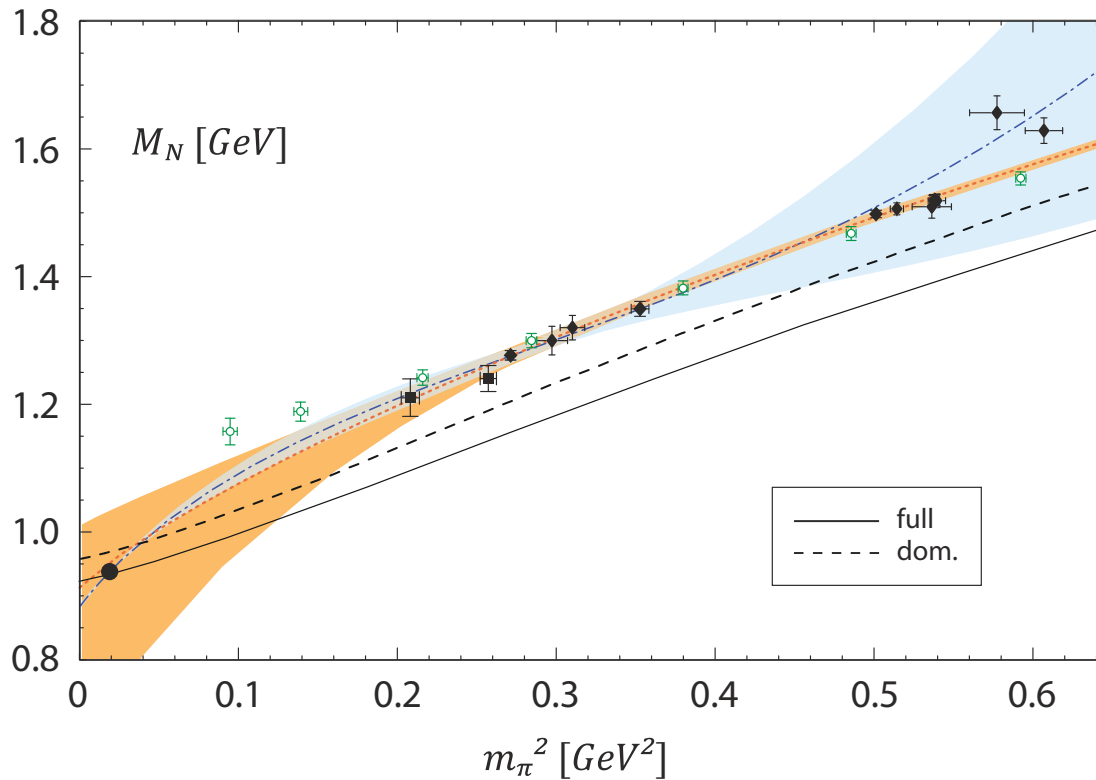
$$\Phi(p, P)^a = \sum_{b,c} \int \frac{d^4k}{(2\pi)^2} \chi^b(k_r, k_d) S^T(q) \bar{\chi}^{aT}(p_r, p_d) S(k_q) D^{bc}(k_d) \Phi(k, P)$$



- Decomposition of Faddeev amplitudes in Dirac space
- in explicit calculations use full diquark amplitudes.

Nucleon mass

- $M_N = 0.93 \text{ GeV}$ at physical point $m_\pi = 138 \text{ MeV}$
 $M_N^{exp} = 0.94 \text{ GeV}$.

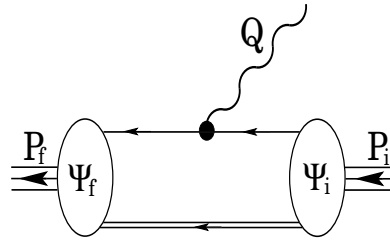


G. Eichman, A. Krassnigg, M. Schwinzerl, R. Alkofer, arxiv:0712.2666

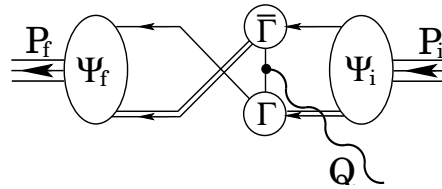
Lattice data; chiral extrapolation methods.

Electromagnetic interaction

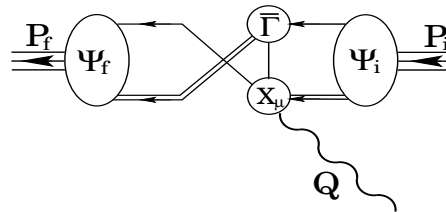
- Ward-Takahashi identity: more than impulse approx. is needed.
- Electromagnetic transition from Ax. to S diquark satisfies gauge invariance alone.



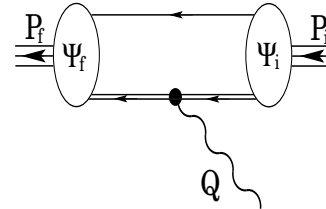
(a)



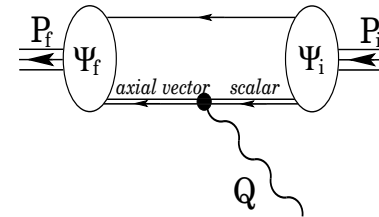
(b)



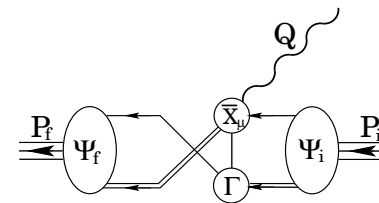
(c)



(d)



(e)

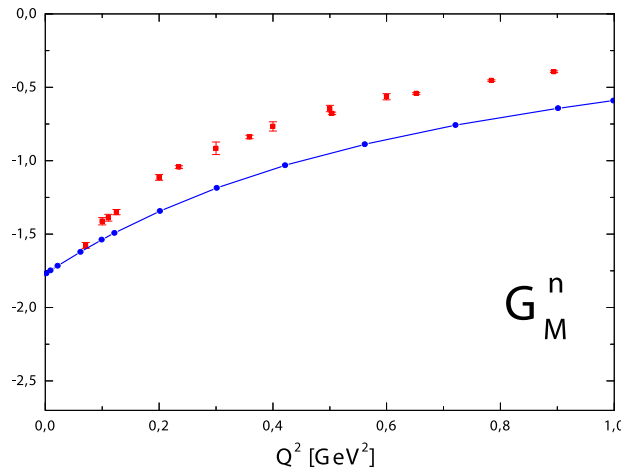
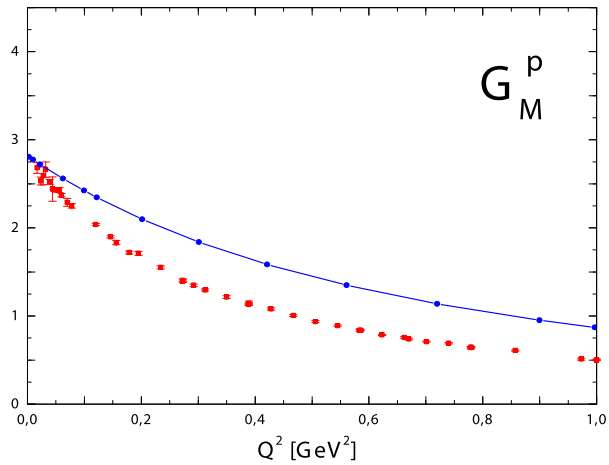
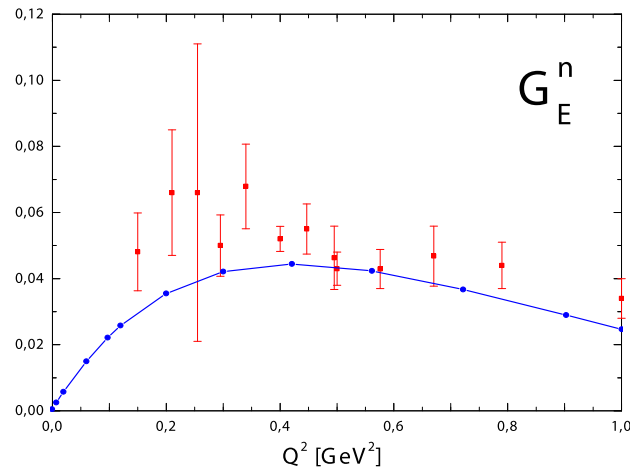
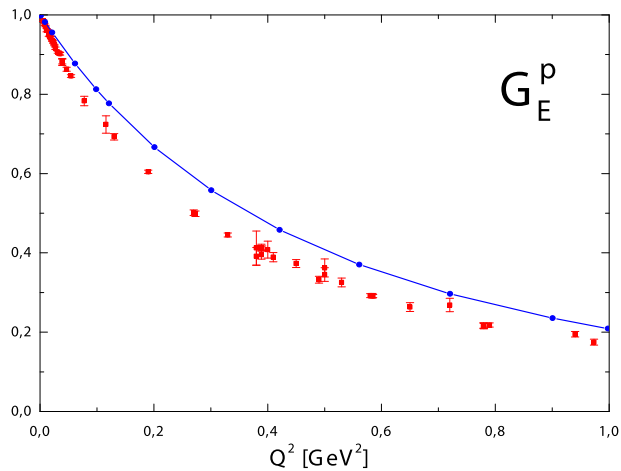


(f)

- General current:

$$\langle J^\mu \rangle = \int \bar{\Phi} \{ D S \Gamma_q^\mu S + D \Gamma_{diq.}^\mu D S + D S K^\mu S D \} \Phi$$

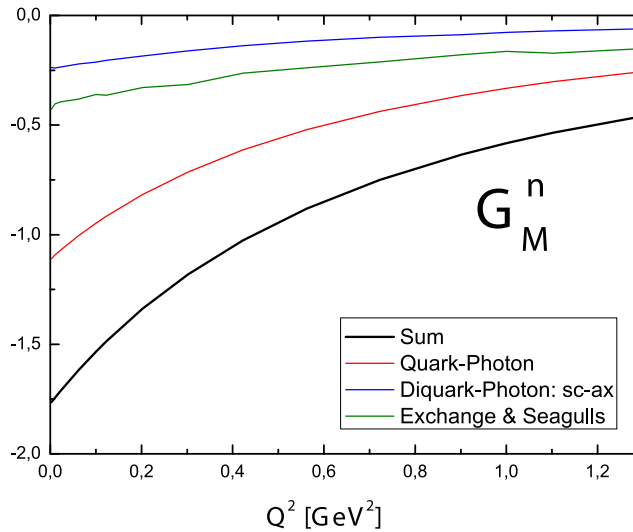
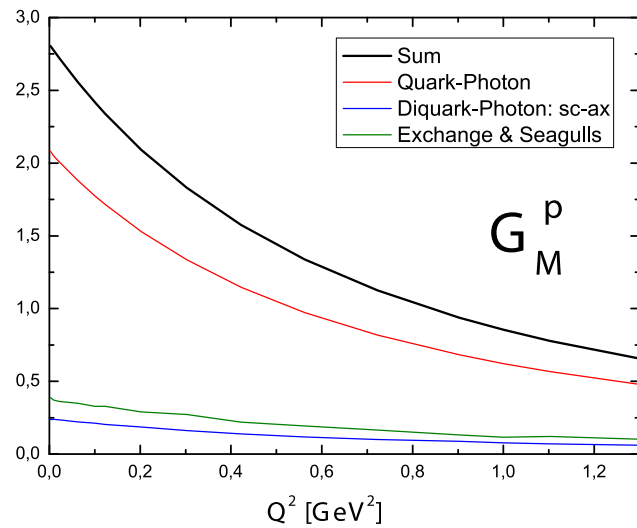
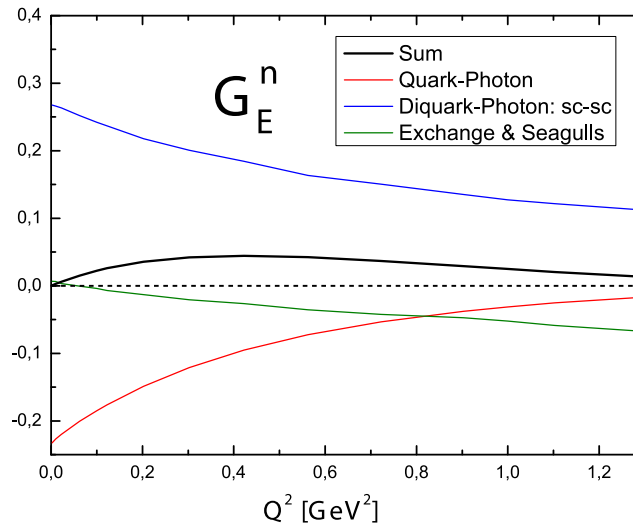
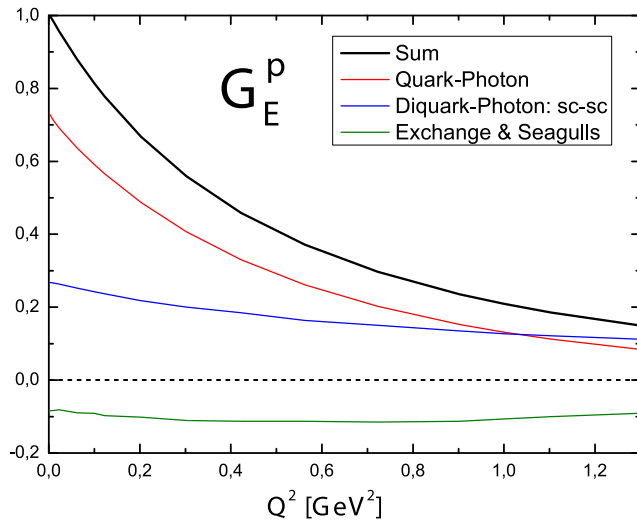
Results



Exp. data
by P. Grabmayr,
2005.

G. Eichman, A. Krassnigg, M. Schwinzerl, R. Alkofer, arxiv:0712.2666

Results



$q - \gamma$ coupling
is dominant

G. Eichman, A. Krassnigg, M. Schwinzerl, R. Alkofer, arxiv:0712.2666

Results

Electromagnetic radii of nucleon

	r_E^p	r_E^n	r_M^p	r_M^n	[fm]
exp	0.87	0.34	0.86	0.88	
calc.	0.67	0.13	0.58	0.57	

Magnetic moments of nucleon.

	μ_p	μ_n	[n.m]
exp	2.79	-1.91	
calc.	2.52	-1.55	

Summary

- Main message: develop QCD - based understanding of the nucleon structure.
- Poincare covariant DSE-BSE approach to baryons.
- Relativistic bound states, dynamical chiral symmetry breaking...
- Quark-diquark picture describes well nucleon quark-core.
- Rainbow-ladder truncation: consistent resolution of DSE and BSE.
Calculation of nucleon static properties and FF.
- No baryon observables as input.

Outlook

- Study Δ (1232): spin- $\frac{3}{2}$ particle, fully flavour symmetric \rightarrow axial-vector diquarks only!
- By now, using specific ansätze for the quark propagator and the diquark correlations:
 $M_{\Delta} = 1.004 \div 1.007 \text{ GeV}$
M. Oettel, R. Alkofer, L. von Smekal, Eur. Phys. J. A8: 553-566 (2000)
- Extension to $N - \Delta\gamma$:
 - highly non-trivial!
 - long disputed electromagnetic ratios $\frac{E2}{M1}, \frac{C2}{M1}$ positive and few %
 - compare to the diagonal $\Sigma \rightarrow \Lambda\gamma$: importance of axial-vector correlations.
- Consistently include π -clouds
 - expected to dominate low-energy observables.

-
- $m_{sc} \simeq 0.67 \text{ GeV}$
 - $m_{av} \simeq 0.88 \text{ GeV}$
 - Chiral limit: $m_{av} - m_{sc} \simeq 0.21 \text{ GeV}$

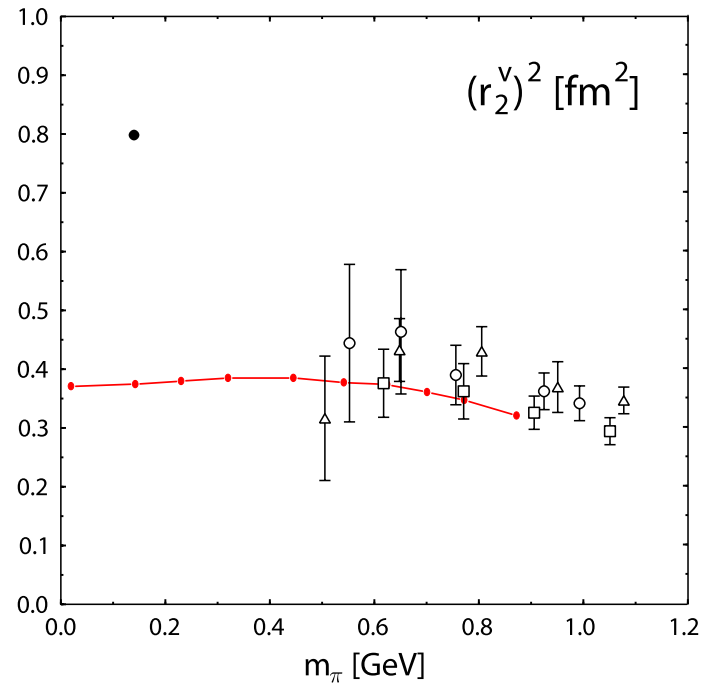
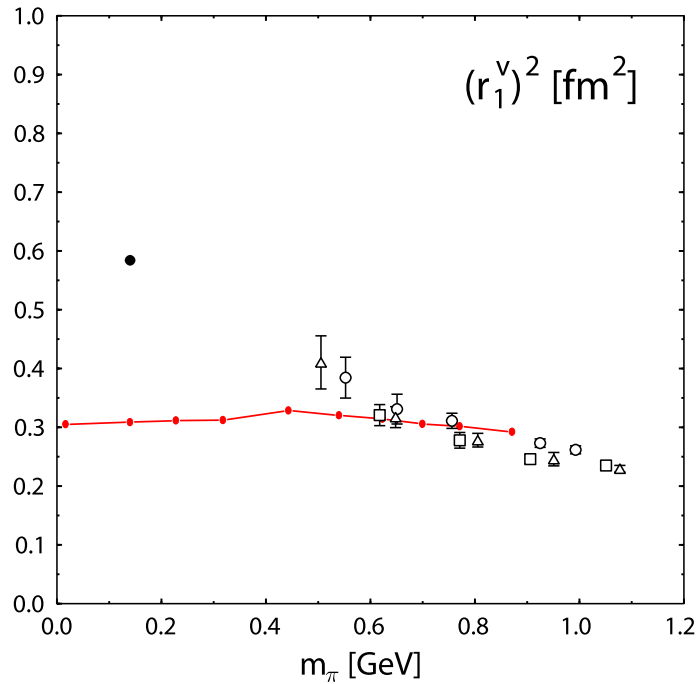
 - Compared to the lattice values
 - 0.14 GeV - C. Alexandrou *et al.* Phys. Rev. Lett. 97, 222002 (2006)
 - 0.29 GeV - R. Babich *et al.* hep-lat/0701023

Few... technicalities

- Decompose nucleon Faddeev amplitudes in Dirac space.
- Reduce the 4-dimensional eqs. to a coupled system of 1-dimensional eqs. via Chebyshev expansions of:
 - Dirac coefficients
 - propagator matrix: $S_q D_{diq}$
 - quark-exchange kernel.
- Solve quark-diquark BSE:
obtain nucleon mass and quark-diquark amplitudes on the nucleon mass-shell.

Results

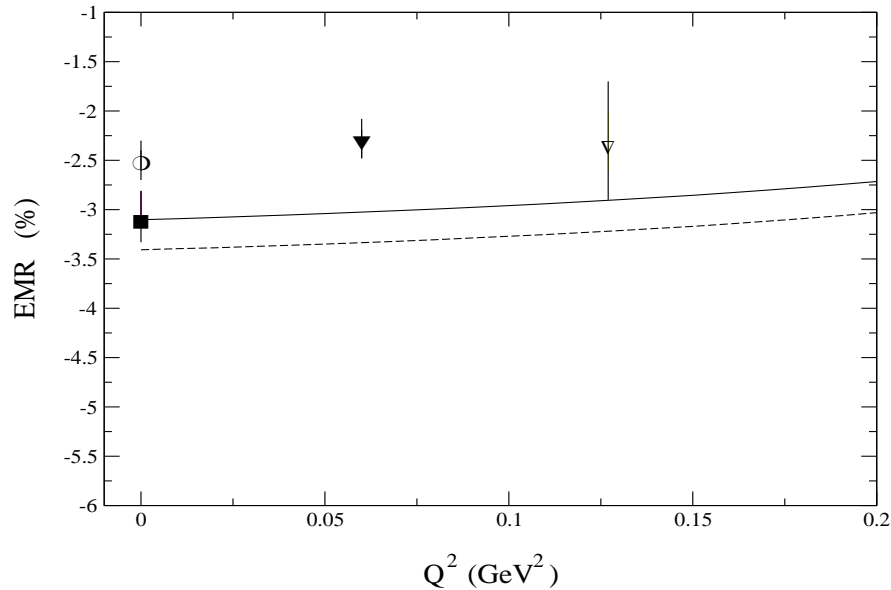
Isovector charge radii



G. Eichman, A. Krassnigg, M. Schwinzerl, R. Alkofer, arxiv:0712.2666

Lattice points Phys. Rev. D 71, 034508 (2005).

EMR and CMR



*filled triangle - A1 Collab.
filled box - LEGS Collab.
opened circle - MAMI
opened triangle - OOPS
Collab.*

*filled triangle - A1 Collab.
opened triangle - OOPS Collab.
filled box - MAMI
filled circle - MAMI*

