Phenomenological applications of the 2PI Hartree approximation

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January 25, 2008 Hévíz

Outline

- Renormalisation of the gap equations and the equation of state in general models
- The method of constructing the counter-terms for invariant tensor structures
- Example 1: SU(N)$\times$ SU(N) meson model, focusing on N$=3$ and N $\to \infty$
- Example 2: U(3)$\times$ U(3) meson model
- Solving the renormalised equations using phenomenological input
- Mass spectra at zero temperature

Renormalisation

- Consider a general Lagrangian:
  $L = \frac{1}{2}[\partial_\mu\sigma_a\partial^\mu\sigma_a + \partial_\mu\pi_a\partial^\mu\pi_a - \mu_S^2\sigma_a\sigma_a - \mu_P^2\pi_a\pi_a] - \frac{1}{3}F^S_{abcd}\sigma_a\sigma_b\sigma_c\sigma_d - \frac{1}{3}F^P_{abcd}\pi_a\pi_b\pi_c\pi_d - 2H_{ab,cd}\pi_a\pi_b\sigma_c\sigma_d$
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  M$= T_a(\sigma_a + i\pi_a)$

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- It fits the O(N), and also the SU(N)×SU(N) model with M= $T_a(\sigma_a + i\pi_a)$
- It is assumed, that only the sigma field has non-zero expectation value
- The 2PI effective potential is the following:
$V = \frac{1}{2}\mu^2_{ab,S}\bar\sigma_a\bar\sigma_b + \frac{1}{3}F^S_{abcd}\bar\sigma_a\bar\sigma_b\bar\sigma_c\bar\sigma_d - \frac{i}{2}\int_k[D^{-1S}_{ab}G^S_{ba} + D^{-1P}_{ab}G^P_{ba}] - -\frac{i}{2}\int_k[lnG^{-1S}_{aa} + lnG^{-1P}_{aa}] + V_2 + V^{ct}$

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- In Hartree approximation V_2 contains the double bubble diagrams in the theory
- V^{ct} contains the counter-terms

Renormalisation

- Counter-term structure: $V^{\text{ct}} = V_4^{\text{ct}} + V_2^{\text{ct}} + V_0^{\text{ct}}$
- $V_4^{\text{ct}} = \frac{1}{2}\delta\tilde{\mu}^2_{ab,S}\bar{\sigma}_a\bar{\sigma}_b + \frac{1}{3}\delta\tilde{F}_{abcd}\bar{\sigma}_a\bar{\sigma}_b\bar{\sigma}_c\bar{\sigma}_d$
- $V_2^{\text{ct}} = \frac{1}{2}\delta\hat{\mu}^2_{ab,S}\int_k G^S_{ba}(k) + \frac{1}{2}\delta\hat{\mu}^2_{ab,P}\int_k G^P_{ba}(k) +$
  $4\delta\hat{F}_{abcd}\int_k G^S_{ab}(k)\bar{\sigma}_c\bar{\sigma}_d + 4\delta\hat{H}_{abcd}\int_k G^P_{ab}(k)\bar{\sigma}_c\bar{\sigma}_d$
- $V_0^{\text{ct}} = \delta F^S_{abcd}\int_k G^S_{ab}(k)\int_p G^S_{cd}(p) +$
  $\delta F^P_{abcd}\int_k G^P_{ab}(k)\int_p G^P_{cd}(p) + 2\delta H_{abcd}\int_k G^S_{ab}(k)\int_p G^P_{cd}(p)$
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- 9 different counter tensors
- The stationary conditions: $\frac{\delta V}{\delta G^{P,S}_{ab}} = 0$, $\frac{\delta V}{\delta\bar\sigma_a} = 0$ give the gap equations and the equation of state
- One has to separate the finite parts from the divergences, in the tadpole integrals M_0 renormalisation scale has to be introduced

Renormalisation

- With the notation
  $M^{S,P}_{cd} = O^{S,P}_{ce} O^{S,P}_{de} \tilde{M}^2_e, \quad \int_k G^{S,P}_{cd} = O^{S,P}_{ce} O^{S,P}_{de} T(\tilde{M}^2_{S;P,e}),$
  the finite parts are:
- P gap: $M^2_{ab,P} =$
  $\mu^2_{ab,P} + 4H_{abcd} O^S_{ce} O^S_{de} T_F(\tilde{M}^2_{S,e}) + 4F^P_{abcd} O^P_{ce} O^P_{de} T_F(\tilde{M}^2_{P,e})$
- S gap: $M^2_{ab,S} = \mu^2_{ab,S} + 4F^S_{abcd}\bar{\sigma}_c\bar{\sigma}_d +$
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- Eq. of state: $M^2_{ab,S}\bar{\sigma}_b = \frac{8}{3}F^S_{abcd}\bar{\sigma}_b\bar{\sigma}_c\bar{\sigma}_d$

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- P gap: $M_{ab,P}^2 =$
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- Eq. of state: $M_{ab,S}^2 \bar{\sigma}_b = \frac{8}{3} F_{abcd}^S \bar{\sigma}_b \bar{\sigma}_c \bar{\sigma}_d$
- Types of divergences: $\bar{\sigma}$ independent, $\bar{\sigma}$ dependent overall divergences, T_F dependent subdivergences → it must be ensured that all types of these expressions vanish

Renormalisation

- $\delta F^S_{abcd}, \delta F^P_{abcd}, \delta H_{abcd}$ determines $\delta \hat{F}_{abcd}, \delta \hat{H}_{abcd}$ and $\delta \tilde{F}_{abcd}$
- The 3 mass counter tensors can be expressed with the above 6

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- The non-trivial problem is to solve the following equations:
- $\Big(\delta F^{S/P}_{abmn} + 4T_d[(F^{S/P}_{abcd} + \delta F^{S/P}_{abcd})F^{S/P}_{cdmn} + (H_{ab,cd} + \delta H_{ab,cd})H_{cd,mn}]\Big) O^{S/P}_{me} O^{S}_{ne} T_F(\tilde{M}^2_{S/P,e}) = 0$
- $\Big(\delta H_{ab,mn} + 4T_d[(F^{S/P}_{abcd} + \delta F^{S/P}_{abcd})H_{cd,mn} + (H_{ab,cd} + \delta H_{ab,cd})F^{P/S}_{cd,mn}]\Big) O^{P/S}_{me} O^{P/S}_{ne} T_F(\tilde{M}^2_{P/S,e}) = 0$

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- Assuming that the the spectrum contains enough different masses, the projecting is irrelevant

Renormalisation

- The coupling and the counter tensors are linear combinations of independent rank-4 invariant tensors (t^α) of the symmetry group
- $F^{S/P}_{abcd} = \sum_\alpha f^{S/P}_\alpha t^\alpha_{abcd}, \quad H_{abcd} = \sum_\alpha h_\alpha t^\alpha_{abcd}$
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- $\delta F^{S/P}_{abcd} = \sum_\alpha \delta f^{S/P}_\alpha t^\alpha_{abcd}, \quad \delta H_{abcd} = \sum_\alpha \delta h_\alpha t^\alpha_{abcd}$
- It is useful to work out a multiplication table for these invariants: $t^\alpha_{abcd} t^\beta_{cdef} = \sum_\gamma g_{\alpha\beta\gamma} t^\gamma_{abef}$
- After determing the $g_{\alpha\beta\gamma}$ coefficients, $\delta f^{S/P}_\alpha, \delta h_\alpha$ counterterms can be easily expressed, since the equations are linear

SU(N)×SU(N) meson model, N=3 case

- In this model the four-point coupling tensors $F^{S/P}_{abcd}$ and H_{abcd} can be written in the following form:
$F^S_{abcd} = F^P_{abcd} = \frac{g_1}{4}\delta_{ab}\delta_{cd} + \frac{g_1}{4}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) + \frac{g_2}{8}d_{abm}d_{cdm} + \frac{g_2}{8} + (d_{acm}d_{bdm} + d_{adm}d_{bcm})$
- $H_{ab,cd} = \frac{1}{4}(g_1 + \frac{2g_2}{N})\delta_{ab}\delta_{cd} - \frac{g_2}{4N}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) + \frac{3g_2}{8}d_{abm}d_{cdm} - \frac{g_2}{8}(d_{acm}d_{bdm} + d_{adm}d_{bcm})$
- Definition of d tensors: $\{\lambda_i, \lambda_j\} = \frac{4}{N}\delta_{ij} + 2d_{ijk}\lambda_k$

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- Special case N $=3$: there are only 3 invariants, because of the relation:
  $d_{abm}d_{cdm} + d_{acm}d_{bdm} + d_{adm}d_{bcm} = \frac{1}{3}(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})$
- One realises that $\delta F^P = \delta F^S$, therefore there are 6 counter terms: $\delta f_1, \delta f_2, \delta f_3, \delta h_1, \delta h_2, \delta h_3$ and 6 equations

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- $\delta f_1 = -8T_d[5f(f+\delta f_1) + f(f+\delta f_2) + 4h_1(h_1+\delta h_1) + h_1(h_2+\delta h_2) + h_2(h_1+\delta h_1)]$
- $\delta f_2 = -8T_d[f(f+\delta f_2) + h_2(h_2+\delta h_2)]$
- $\delta f_3 = -8T_d[f\delta f_3 + h_3(h_2+\delta h_2) + (h_2+5h_3/6)(h_3+\delta h_3)]$
- $\delta h_1 = -8T_d[(4h_1+h_2)(f+\delta f_1) + h_1(f+\delta f_2) + 5f(h_1+\delta h_1) + f(h_2+\delta h_2)]$
- $\delta h_2 = -8T_d[h_2(f+\delta f_2) + f(h_2+\delta h_2)]$
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- They are linear, easy to solve them

SU(N)×SU(N) meson model, N $\to \infty$ case

- The general SU(N) tensors' multiplication table contains only one product which scales with N^2, this is the only one which counts while solving the equations for counter terms
- $\delta_{ab}\delta_{cd} * \delta_{cd}\delta_{ef} = (N^2 - 1)\delta_{ab}\delta_{ef}$

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- In this case there will be 2 counter terms: δf_1, δh_1
- $\delta f_1 + 4 T_d N^2((f_1 + \delta f_1) f_1 + (h_1 + \delta h_1) h_1) = 0$
- $\delta h_1 + 4 T_d N^2((f_1 + \delta f_1) h_1 + (h_1 + \delta h_1) f_1) = 0$

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- Since at large N $f_1 = h_1 = g_1/4$, one can consistently choose $\delta h_1 = \delta f_1 =: \delta g_1 \to$ one single counterterm is enough to renormalise the theory
- We get $\delta g_1 = -\frac{2N^2 T_d g_1^2}{1+2N^2 T_d g_1}$, which coincides with the O(2N^2) model's coupling counterterm at large N

U(3)×U(3) model

- The group algebra modifies as: $\{\lambda_a, \lambda_b\} = d_{abc}\lambda_c$ with $d_{ab0} = \sqrt{\frac{2}{N}}\delta_{ab}$
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- The F and H tensors can not be expressed with 3 invariant tensors closing under multiplication
- The minimum set of invariants has to contain 9 invariant tensors
- It means that 18 different counter terms have to be determined

U(3)×U(3) model

- The used invariant tensors:
- $$t^1_{abcd} = \delta_{ab}\delta_{cd}(1-\delta_{a0})(1-\delta_{b0})(1-\delta_{c0})(1-\delta_{d0}),$$
  $$t^2_{abcd} = (\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{cb})(1-\delta_{a0})(1-\delta_{b0})(1-\delta_{c0})(1-\delta_{d0}),$$
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  $t^3_{abcd} = d_{abm}d_{cdm}(1-\delta_{a0})(1-\delta_{b0})(1-\delta_{c0})(1-\delta_{d0})$,
- $t^4_{abcd} = \delta_{ab}(1-\delta_{a0})(1-\delta_{b0})\delta_{c0}\delta_{d0}$,
  $t^5_{abcd} = \delta_{cd}(1-\delta_{c0})(1-\delta_{c0})\delta_{a0}\delta_{b0}$,
  $t^6_{abcd} =$
  $\delta_{ad}\delta_{b0}\delta_{c0}(1-\delta_{a0})(1-\delta_{d0})+\delta_{ac}\delta_{b0}\delta_{d0}(1-\delta_{a0})(1-\delta_{c0})+$
  $\delta_{bd}\delta_{a0}\delta_{c0}(1-\delta_{b0})(1-\delta_{d0})+\delta_{bc}\delta_{a0}\delta_{d0}(1-\delta_{b0})(1-\delta_{c0})$,
  $t^7_{abcd} = d_{acd}\delta_{b0}(1-\delta_{a0})(1-\delta_{c0})(1-\delta_{d0})+$
  $d_{abd}\delta_{c0}(1-\delta_{a0})(1-\delta_{b0})(1-\delta_{d0})$,
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- $t^9_{abcd} = \delta_{a0}\delta_{b0}\delta_{c0}\delta_{d0}.$

Solving the finite gap equations and the equation of state

- The earlier model can be extended:
$$\Delta L = c\Big[det(\lambda_a(\sigma_a + i\pi_a)) + det(\lambda_a(\sigma_a + i\pi_a))^\dagger\Big] + Tr\Big[\lambda_a h_a\Big(\lambda_b(\sigma_b + i\pi_b) + (\lambda_b(\sigma_b + i\pi_b))^\dagger\Big)\Big]$$
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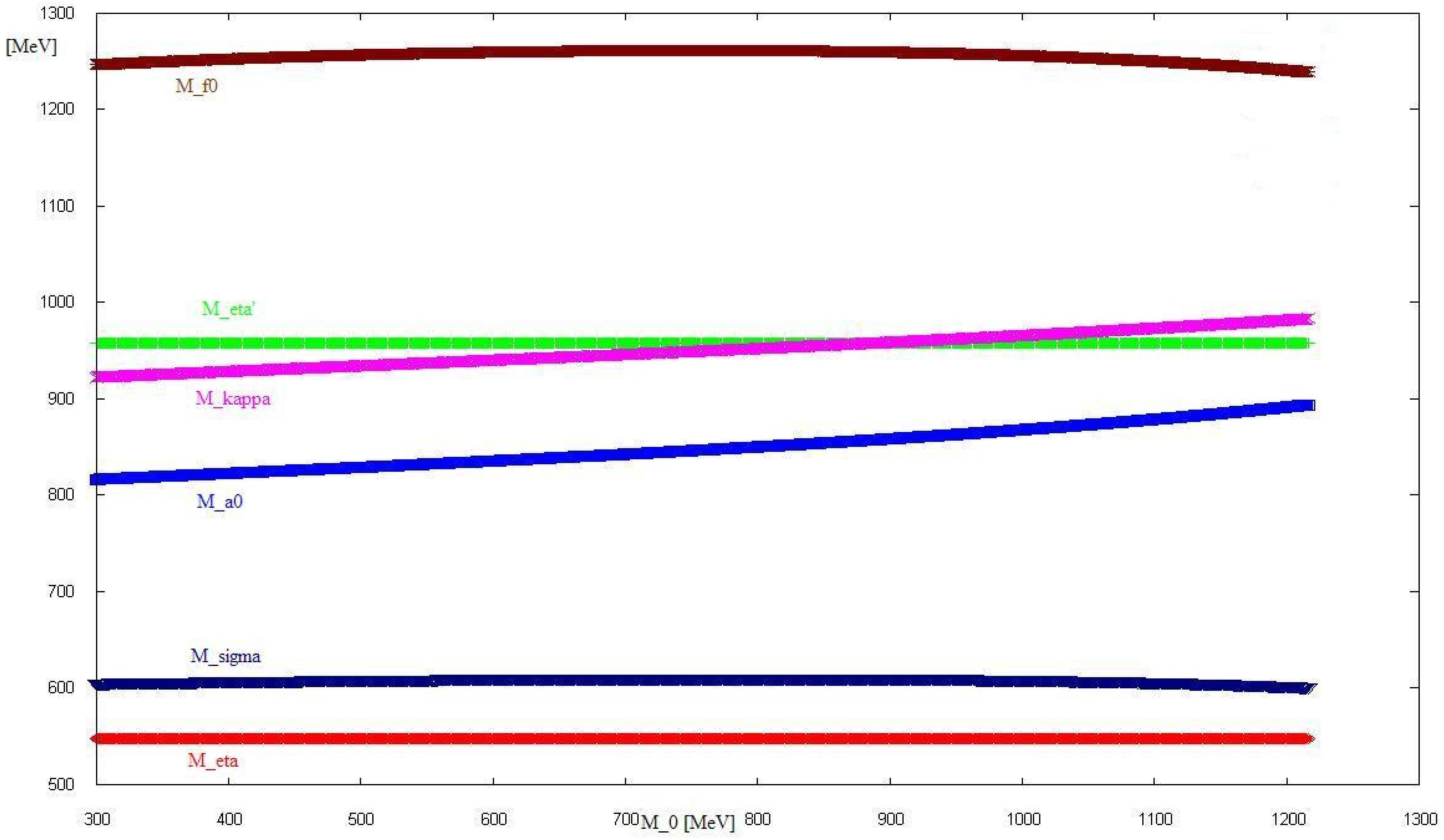
- There are 8 gap equations + 2 equations for the angles of the diagonalizing matrices + 2 equations of state → 12 equations have to be solved simultaneously
- The main goal is to get the condensates and the masses as a function of the temperature
- The first step: parametrisation → change the variables, fix 4 masses at zero temperature and a given renormalisation scale

Summary

- Renormalization procedure for various types of multicomponent scalar models
- The explicit construction of the counter terms
- 2 examples: SU(N)$\times$SU(N) and U(3)$\times$U(3) models
- Solving the finite equations, the mass spectrum scale dependence at zero temperature
- Near future: thermodynamics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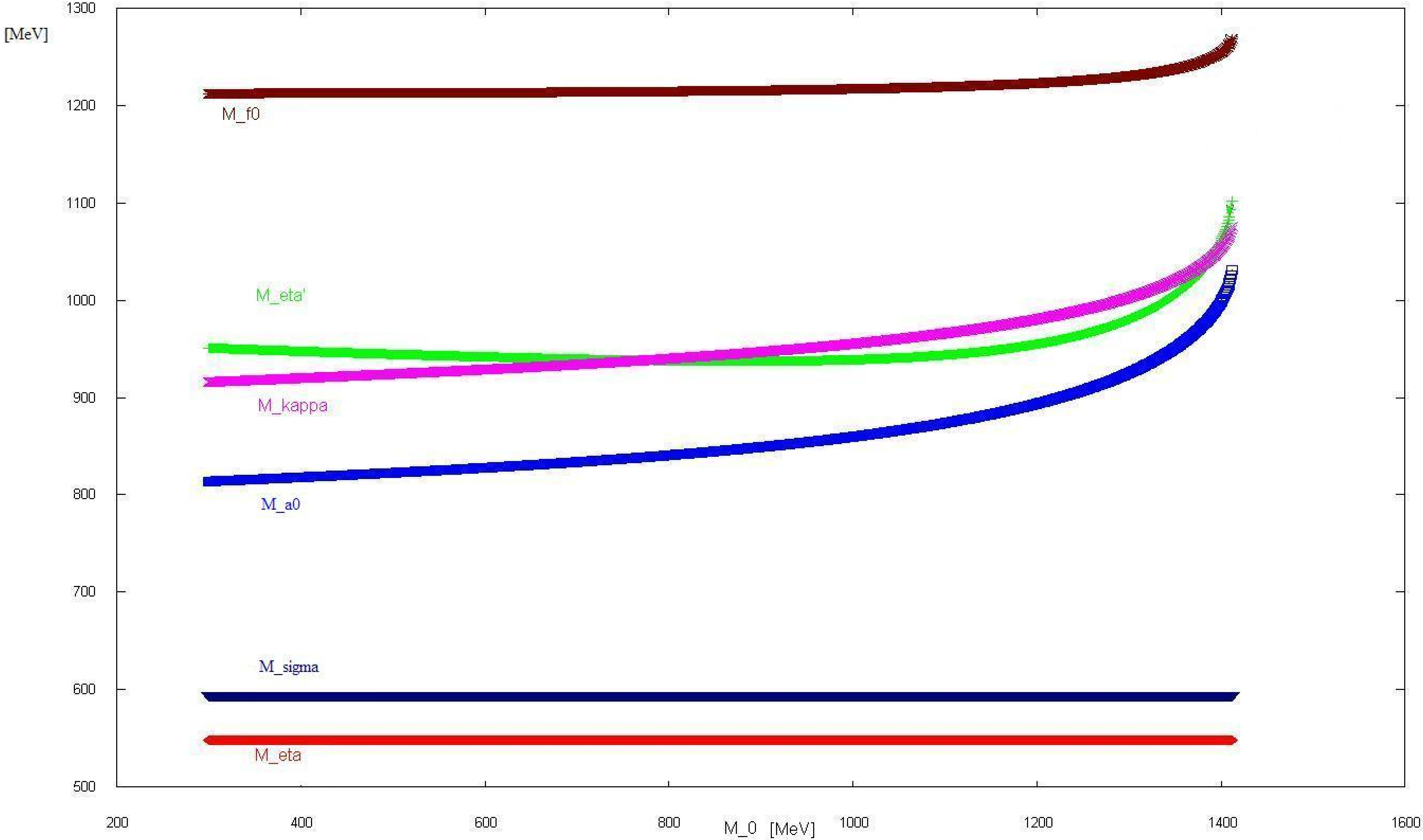
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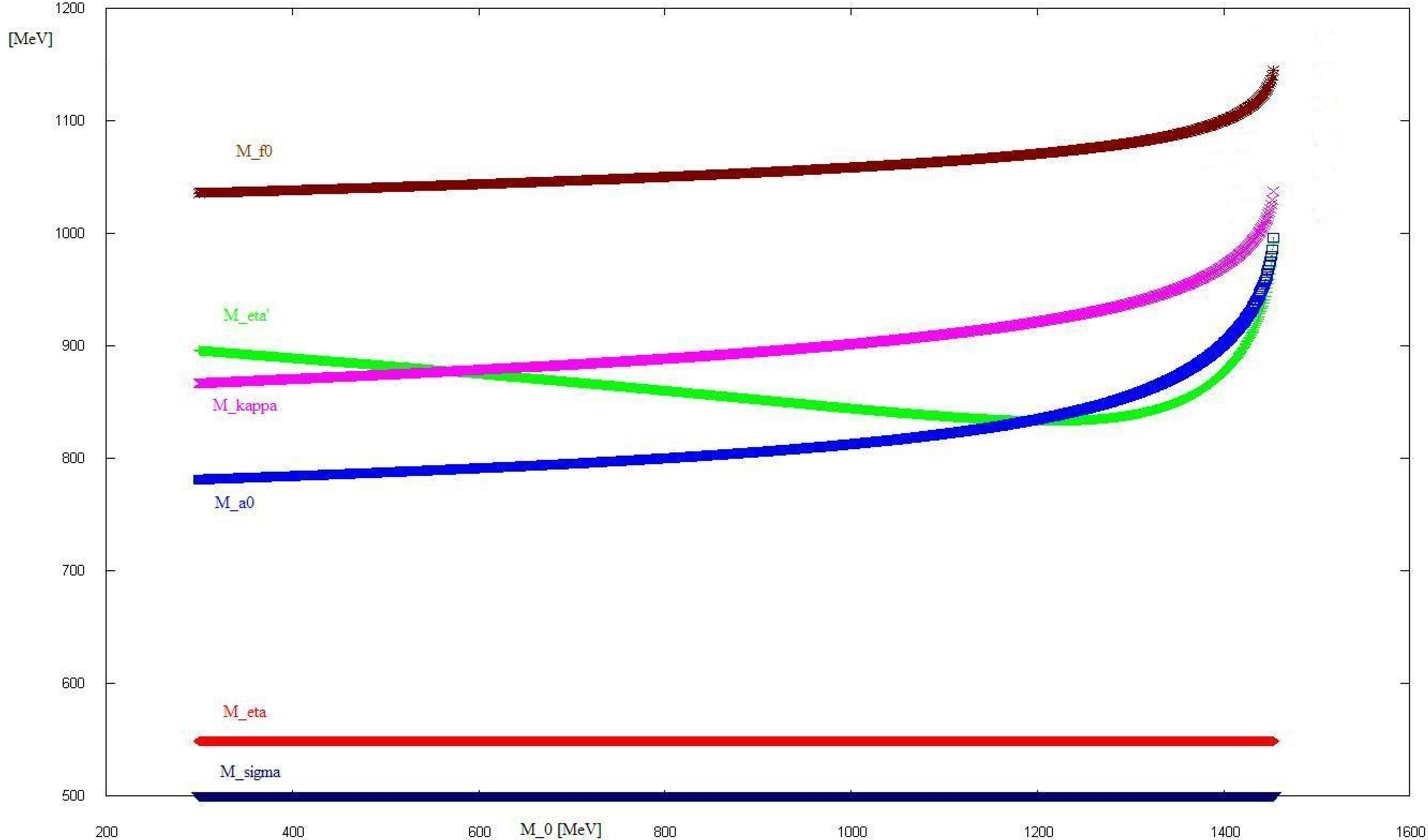

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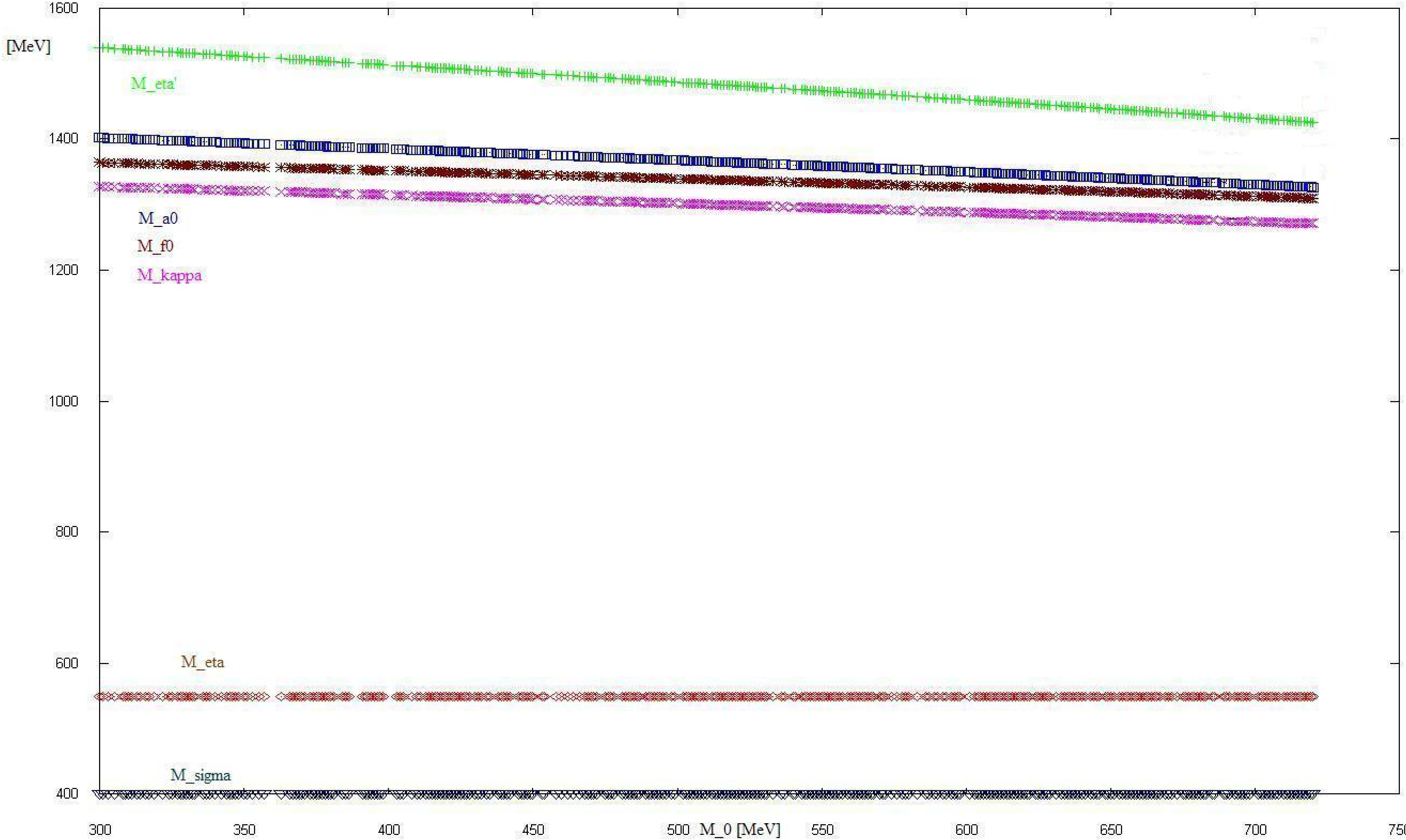
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