# Exploring the QCD Phase Diagram with Functional RG

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Non-Perturbative Functional Methods in Quantum Field Theory

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Heviz, Hungary

# Outline

- Conjectured QCD Phase Diagram
- Quark-Meson Model Phase Diagram
  - ▷ Mean field approximation
  - ▷ Renormalization Group study
- Polyakov–Quark-Meson Model
- Three Flavor Quark-Meson Model

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QCD: two phase transitions:

1. restoration of chiral symmetry

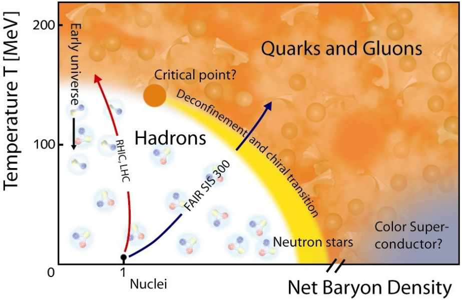
   $SU_{L+R}(N_f) \to SU_L(N_f) \times SU_R(N_f)$

   order parameter:

   $$\langle \bar{q}q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken, } T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase, } T > T_c \end{cases}$$

   associate limit: $m_q \to 0$

FAIR, Darmstadt




chiral transition: spontaneous restoration of global $SU_L(N_f) \times SU_R(N_f)$ at high $T$

# The conjectured QCD Phase Diagram

QCD: two phase transitions:

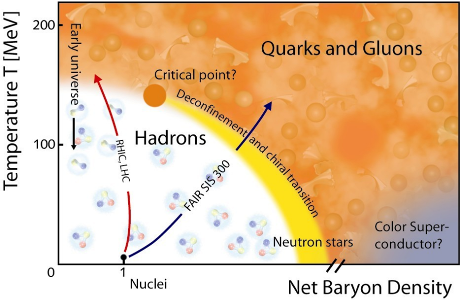
1. restoration of chiral symmetry
2. de/confinement

order parameter:

$$\frac{\langle \mathrm{tr}_c \mathcal{P}(\vec{x})\rangle}{N_c} \begin{cases} = 0 \Leftrightarrow \text{confined phase}, & T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase}, & T > T_c \end{cases}$$

associate limit: $m_q \to \infty$

FAIR, Darmstadt




$$\mathcal{P}(\vec{x}) = \mathcal{P} e^{i\int_0^\beta d\tau A_0(\tau,\vec{x})}$$

free energy of static quark antiquark pair:

$$\exp\left(-\frac{F_{\bar{q}q}(r,T)}{T}\right) = \langle \mathrm{tr}_c \mathcal{P}(x) \mathrm{tr}_c \mathcal{P}^\dagger(y)\rangle / N_c^2$$

QCD: two phase transitions:

1. restoration of chiral symmetry
2. de/confinement

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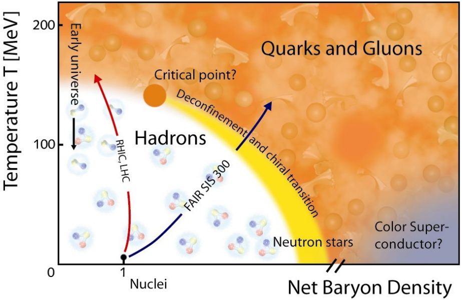


**effective models:**

1. Quark-meson model — or other models e.g. NJL
2. Polyakov–quark-meson model — or PNJL models

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- Lagrangian:

$$\mathcal{L}_{\mathrm{qm}} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

- Mean field analysis

- Lagrangian:

$$\mathcal{L}_{\text{qm}} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

- Mean field analysis

➜ partition function:

$$\mathcal{Z}(T,\mu) = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}\sigma\mathcal{D}\vec{\pi} \exp\left\{ i\int_0^{1/T} dt d^3x \left(\mathcal{L}_{\text{qm}} + \mu\bar{q}\gamma_0 q\right)\right\}.$$

$$\sigma \longrightarrow \langle\sigma\rangle \equiv \phi,$$

$$\pi \longrightarrow \langle\pi\rangle = 0,$$

integrate quark/antiquarks

- Lagrangian:

$$\mathcal{L}_{\text{qm}} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

- Mean field analysis

**Grand canonical potential**

$$\Omega(T,\mu) = -\frac{T\ln\mathcal{Z}}{V} = \frac{\lambda}{4}(\langle\sigma\rangle^2 - v^2)^2 - c\langle\sigma\rangle + \Omega_{\bar{q}q}(T,\mu)$$

with

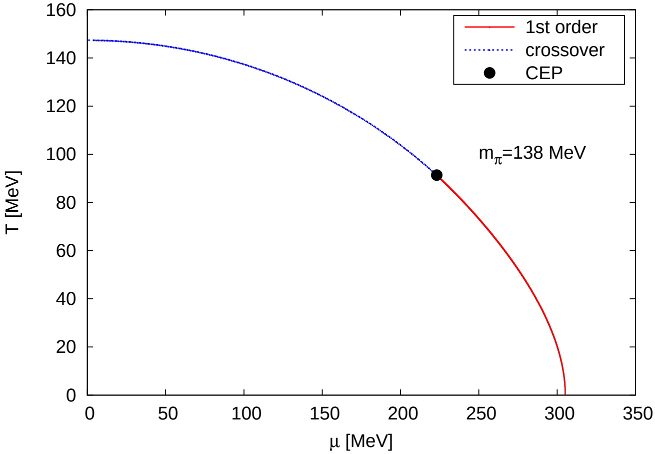
$$\Omega_{\bar{q}q}(T,\mu) = -2N_cN_fT\int\frac{d^3k}{(2\pi)^3}\left\{\ln(1+e^{-(E_q-\mu)/T}) + \ln(1+e^{-(E_q+\mu)/T})\right\}$$

[Scavenius et al. '01]

- Lagrangian:

$$\mathcal{L}_{\text{qm}} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

- Mean field analysis



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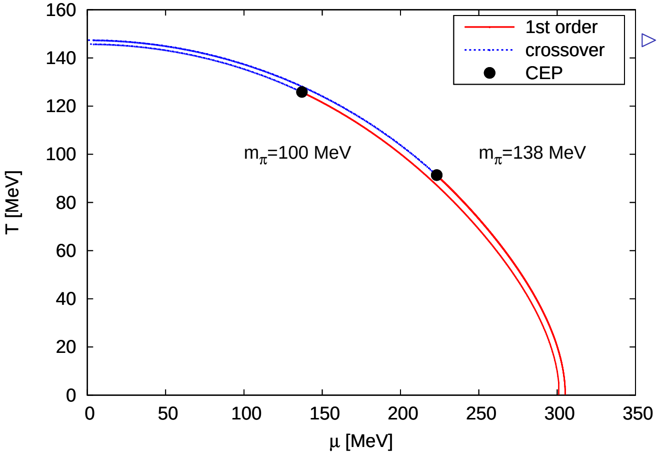
- Mean field analysis



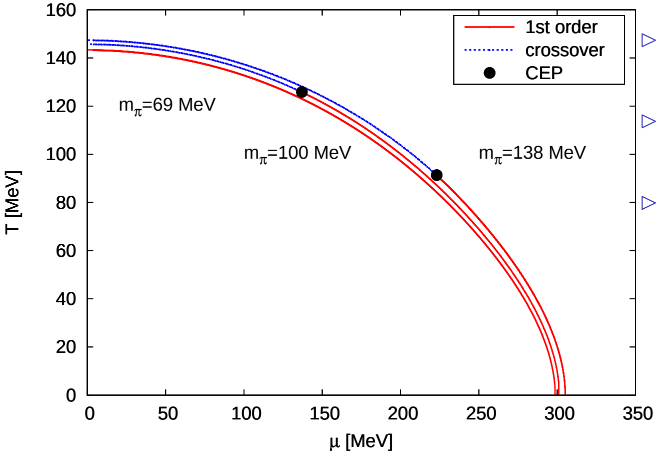

▷ as $m_q$ decreased

CEP $\rightarrow$ T-axis

- Lagrangian:

$$\mathcal{L}_{\rm qm} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2-v^2)^2 - c\sigma$$

- Mean field analysis




▷ as $m_q$ decreased
CEP $\rightarrow$ T-axis

▷ chiral limit
no CEP

▷ at $\mu = 0$
$O(4)$ scaling expected
i.e. 2nd-order PT

$\rightarrow$ truncation effect

$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$ ; $R_k$ regulators

**FRG (average effective action)** [Wetterich]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left( \frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

# RG analysis

- Ansatz for $\Gamma_k$:

$$\Gamma_k = \int d^4x \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + V_k(\phi^2)$$

$$V_{k=\Lambda}(\phi^2) = \tfrac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

# RG analysis

- Ansatz for $\Gamma_k$:

$$\Gamma_k = \int d^4x \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + V_k(\phi^2)$$

## flow for grand canonical potential [BJS, J.Wambach]

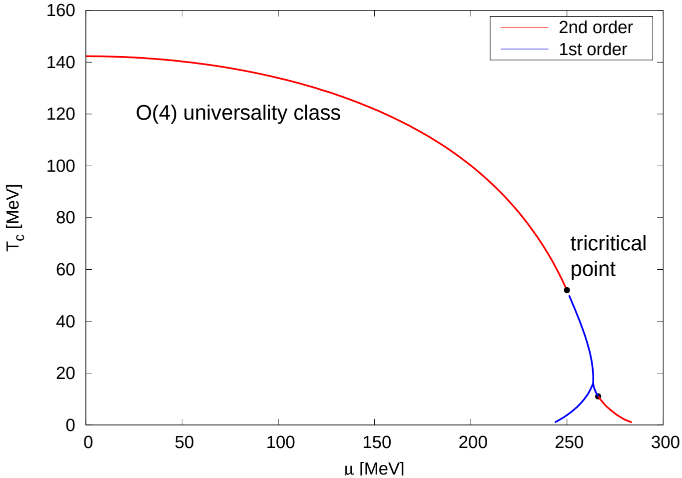
$$\begin{aligned}\partial_t\Omega_k(T,\mu;\phi) \quad = \quad & \frac{k^4}{12\pi^2}\left[\frac{3}{E_\pi}\coth\left(\frac{E_\pi}{2T}\right) + \frac{1}{E_\sigma}\coth\left(\frac{E_\sigma}{2T}\right)\right. \\ & \left. - \frac{2N_cN_f}{E_q}\left\{\tanh\left(\frac{E_q-\mu}{2T}\right) + \tanh\left(\frac{E_q+\mu}{2T}\right)\right\}\right]\end{aligned}$$

$$E_\pi^2 = 1 + 2\Omega_k'/k^2\ , \qquad E_\sigma^2 = 1 + 2\Omega_k'/k^2 + 4\phi^2\Omega_k''/k^2\ , \qquad E_q^2 = 1 + g^2\phi^2/k^2$$

$$\phi \sim \langle\bar{q}q\rangle\ , \qquad \Omega_k' = \partial\Omega_k/\partial\phi \qquad \text{etc}$$

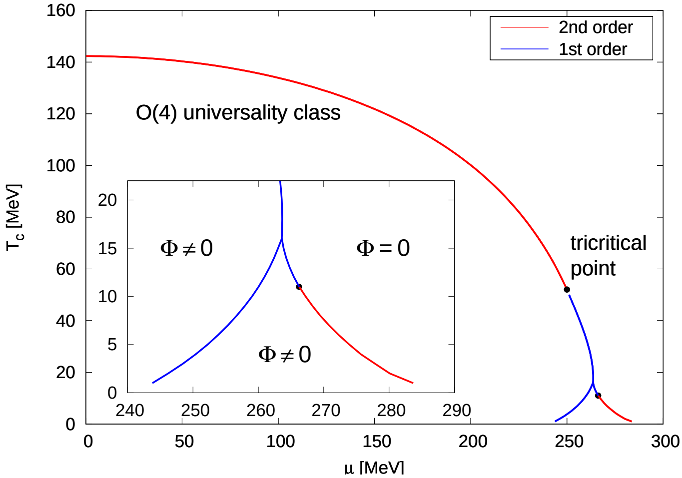
- quark fluctuations: chiral symmetry breaking
- meson fluctuations: chiral symmetry restoration

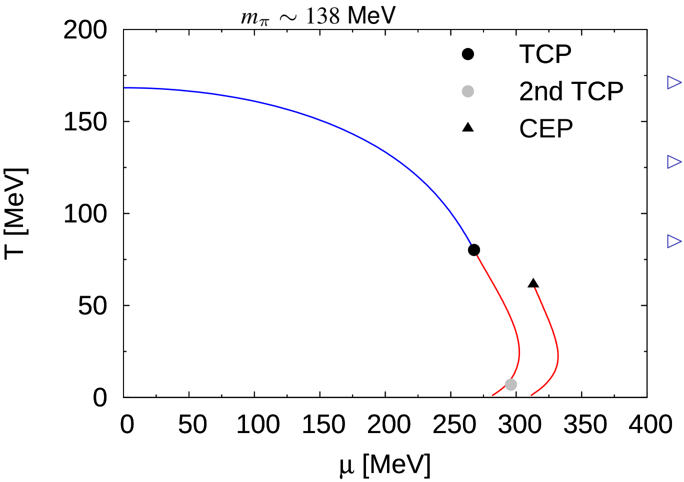
$O(4) \sim SU(2) \times SU(2)$ chiral limit



# Chiral Phase Diagram $N_f = 2$ & $m_q \sim 280$ MeV

$O(4) \sim SU(2) \times SU(2)$ chiral limit no spinodal lines!





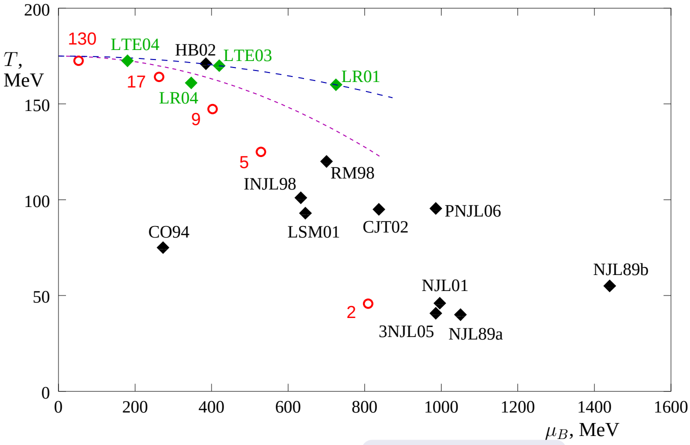

- bending usual for RG
- 2nd tricritical point
- features
  parameter independent
  but locations TCP/CEP
  parameter dependent

[BJS,Wambach '05 & '06]

# Charts of QCD Critical End Points

model studies vs. lattice simulations

Black points: models    Lines & green points: lattice    Red points: Freezeout points for HIC



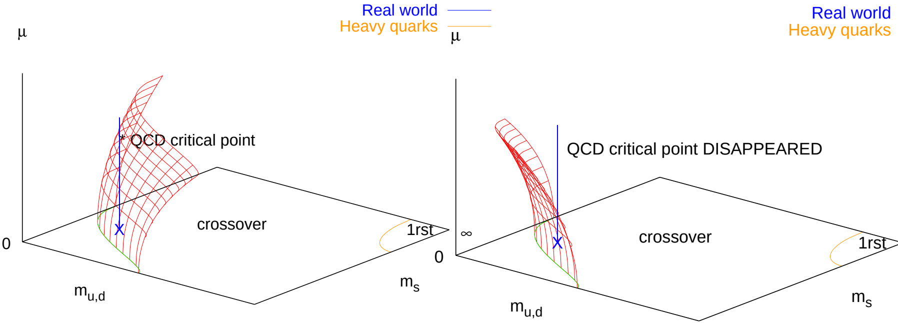

Stephanov '05 & '07

lattice methods:

- reweighting
- imaginary $\mu_B$
- Taylor expansion around $\mu_B = 0$

large $m_q$ sensitivity?

if $m_s \to 0$
$\Rightarrow$ 1st-order




standard scenario: $m_c(\mu)$ increasing

non-standard scenario: $m_c(\mu)$ decreasing

[de Forcrand, Philipsen '05]

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- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$
- Polyakov loop potential:

Polyakov 1978

Pisarski 2000

$$\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$$

$$\frac{\mathcal{U}(\phi, \bar{\phi})}{T^4} = -\frac{b_2(T, T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

Ratti, Weise et al. 2004

Dumitru, Pisarski 2004

Friman, Redlich, Sasaki 2006

$\Rightarrow$ first-order transition at $T_0 = 270$ MeV

in presence of dynamical quarks: $T_0 = T_0(N_f)$

BJS, Pawlowski, Wambach, 2007

| $N_f$ | 0 | 1 | 2 | 2+1 | 3 |
|---|---|---|---|---|---|
| $T_0$ [MeV] | 270 | 240 | 208 | 187 | 178 |

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$
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$\Rightarrow$ first-order transition at $T_0 = 270$ MeV

$\mu \neq 0: \quad T_0 = T_0(N_f, \mu)$

BJS, Pawlowski, Wambach, 2007

$$\bar{\phi} \neq \phi^*$$

- grand canonical potential:

$$\Omega(T,\mu) = \mathcal{U}(\phi,\bar{\phi}) + V_{\text{renorm}}(\langle\sigma\rangle,\vec{0}) + \Omega_{\bar{q}q}(T,\mu)$$

with fermi contribution:

$$\Omega_{\bar{q}q} = -2N_f T \int \frac{d^3p}{(2\pi)^3} \left\{ \ln\left[1+3(\phi+\bar{\phi}e^{-(E_p-\mu)/T})e^{-(E_p-\mu)/T} + e^{-3(E_p-\mu)/T}\right] + \ln\left[1+3(\bar{\phi}+\phi e^{-(E_p+\mu)/T})e^{-(E_p+\mu)/T} + e^{-3(E_p+\mu)/T}\right] \right\}$$

$$E_p = \sqrt{p^2+m_q^2}$$

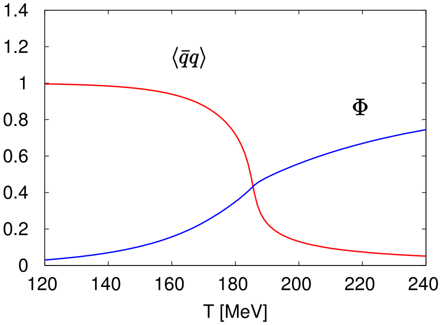
- three EoM:

$$\frac{\partial\Omega}{\partial\sigma} = 0\,, \qquad \frac{\partial\Omega}{\partial\phi} = 0\,, \qquad \frac{\partial\Omega}{\partial\bar{\phi}} = 0\,.$$

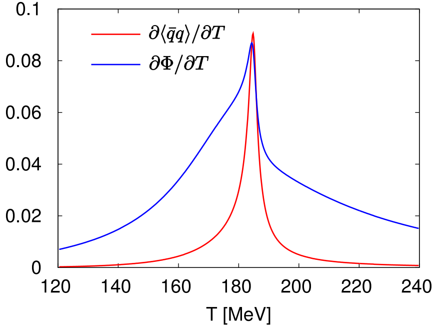
[BJS, Pawlowski, Wambach '07]

Numerical results:

order parameters




$T$-derivatives of order parameters

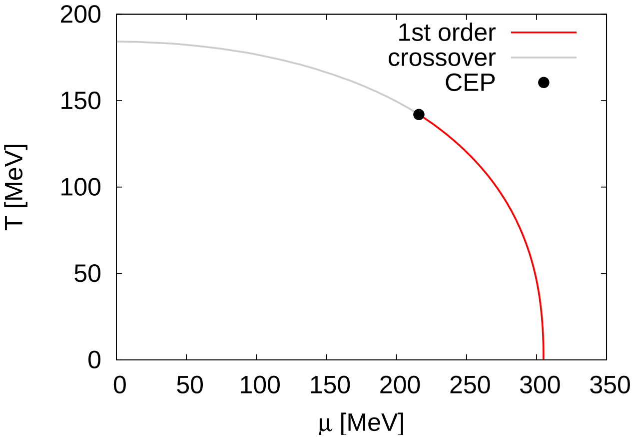


[BJS, Pawlowski, Wambach '07]

- for PQM model

in mean field approximation

chiral transition and 'deconfinement' coincide



[BJS, Pawlowski, Wambach '07]

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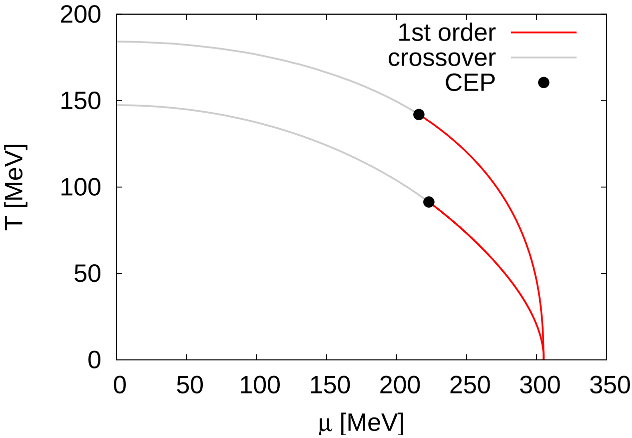
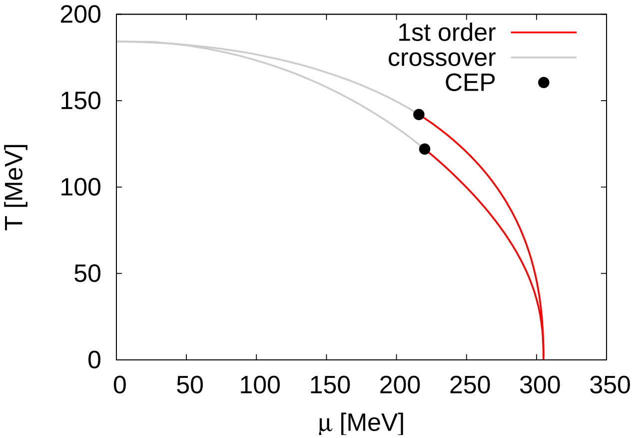



- for PQM model
- for QM model
  (lower lines)

[BJS, Pawlowski, Wambach '07]

in mean field approximation

chiral transition and 'deconfinement' coincide

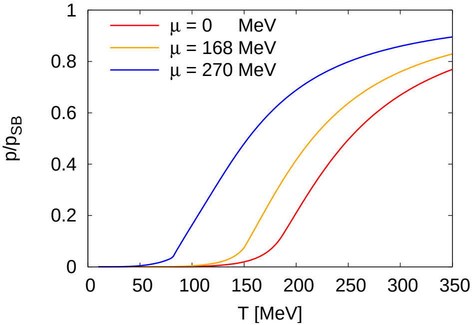


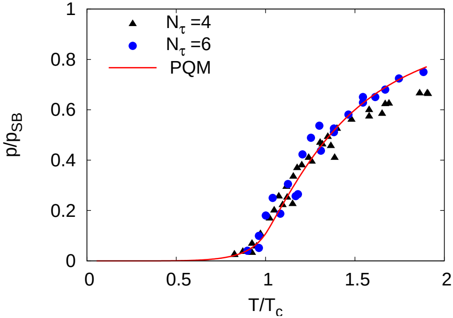

- for PQM model
- for PQM model with $\mu$-modification in Polyakov loop potential (lower lines)

- perturbative pressure of QCD with $N_f$ massless quarks

$$\frac{p}{T^4} = (N_c^2 - 1)\frac{\pi^2}{45} + N_f\left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu}{T}\right)^4\right] .$$

- $N_f = 2$: lattice: $N_\tau = 4, 6$ ; $\mu = 0$




[Ali Khan et al. '01]

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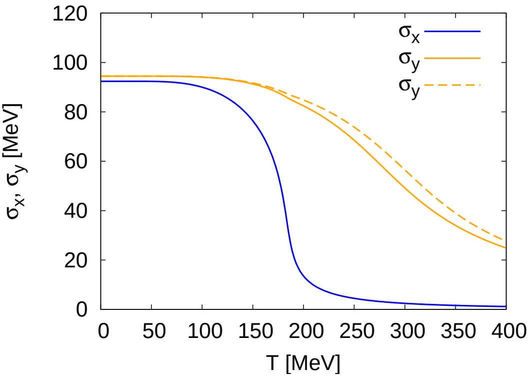
[BJS, M. Wagner, work in progress]

➜ two condesates: nonstrange $\sigma_x(T, \mu_f)$ and strange $\sigma_y(T, \mu_f)$

with (solid) and without (dashed) $U(1)_A$ anomaly




▷ almost no effect due $U(1)_A$ anomaly

▷ $T_c$ depends on $m_\sigma$

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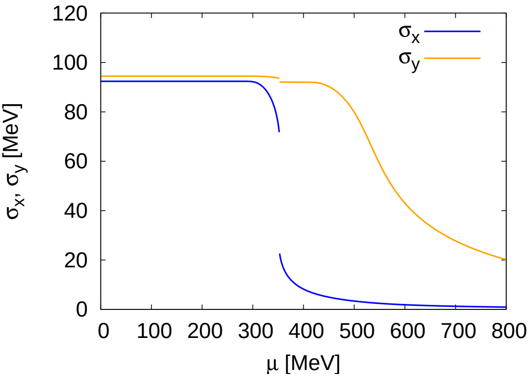
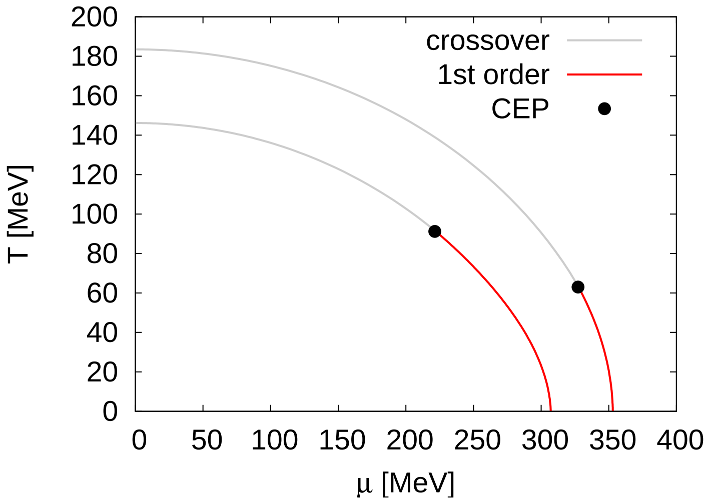



- ▷ almost no effect due $U(1)_A$ anomaly
- ▷ $T_c$ depends on $m_\sigma$
- ▷ $\mu \equiv \mu_q = \mu_s$
- ▷ $\mu_c$ depends on $m_\sigma$

# Phase diagram $N_f = 3$

for $\mu \equiv \mu_q = \mu_s$

$m_\sigma = 600$ MeV (lower lines) and $m_\sigma = 800$ MeV



# In-medium meson masses

- genuine problem of linear sigma model (w/ and w/o quarks) finite T

  $\rightarrow$ negative meson masses

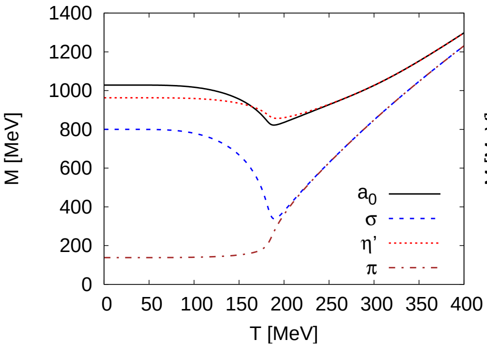
- generalize tree-level Ward identities to finite $T, \mu_f$

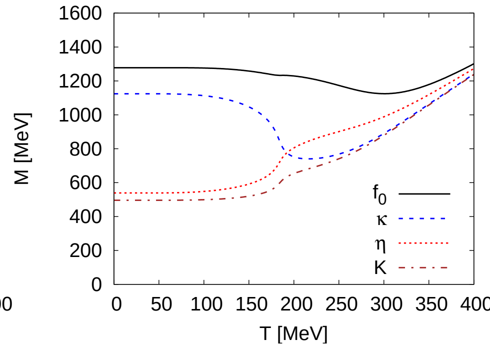
$$h_x = f_\pi m_\pi^2 \qquad \rightarrow \qquad h_x = f_\pi(T,\mu_f) m_\pi^2(T,\mu_f)$$

similar for strange sector

$$h_y = \sqrt{2} f_K m_K^2 - \frac{1}{\sqrt{2}} f_\pi m_\pi^2$$

masses with $U(1)_A$ anomaly



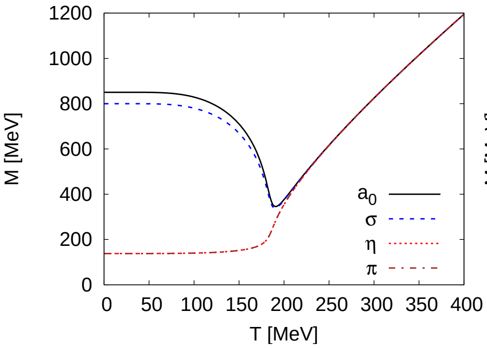
▷ generalize tree-level Ward identities to finite $T, \mu_f$

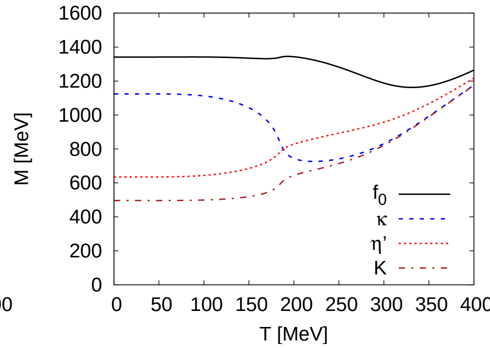
$$h_x = f_\pi m_\pi^2 \qquad \longrightarrow \qquad h_x = f_\pi(T,\mu_f) m_\pi^2(T,\mu_f)$$

similar for strange sector

$$h_y = \sqrt{2} f_K m_K^2 - \frac{1}{\sqrt{2}} f_\pi m_\pi^2$$

masses without $U(1)_A$ anomaly



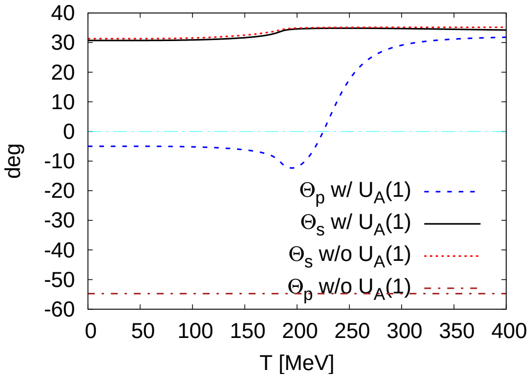
In vacuum: physical ($\eta$,$\eta'$) close to ($\eta_8$,$\eta_0$)

➜ mixing angle $\theta_p$

➜ pseudoscalar and scalar mixing angles

as a function of $T$ (for $\mu = 0$)

with and without $U(1)_A$ anomaly

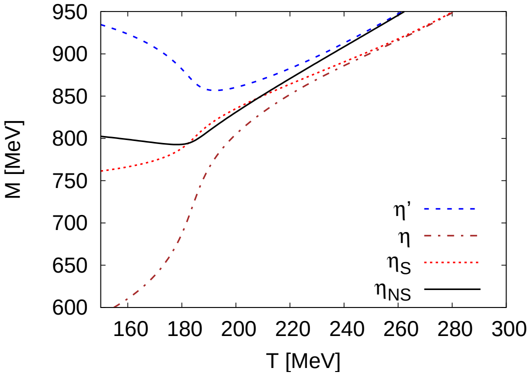



▷ without $U(1)_A$ anomaly $\rightarrow$

In vacuum: physical ($\eta$,$\eta'$) close to ($\eta_8$,$\eta_0$)

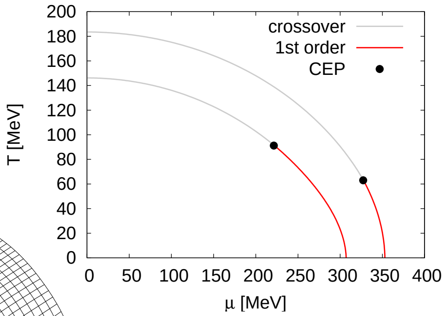
➜ mixing angle $\theta_p$

➜ identity switching above $T_c$

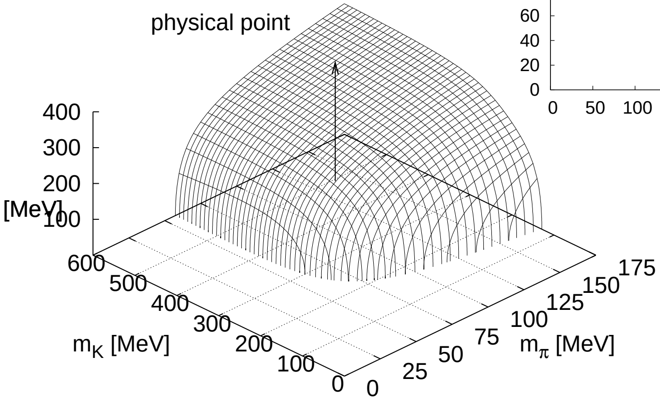



- ▷ without $U(1)_A$ anomaly $\rightarrow$
- ▷ $\eta' \rightarrow \eta_{\text{NS}}$
- ▷ $\eta \rightarrow \eta_{\text{S}}$ for $T > 250$ MeV
- ▷ no Witten-Veneziano relation has been used

$m_\sigma = 600$ MeV (lower lines) and $m_\sigma = 800$ MeV




chiral critical surface:



Summary & Outlook

# Summary

Quark-meson model study for $N_F = 2$

➜ Mean field versus RG

Influence of fluctuations on phase diagram

Findings:

- ▷ MF phase diagram: no TCP (in chiral limit) found
- ▷ RG phase diagram: two TCP's (in chiral limit) & CEP found
- ▷ Size of critical region via susceptibilities: "compressed" with fluctuations

Quark-meson model study for $N_F = 3$

➜ preliminary Mean field approximation

no need for Optimized Perturbation Theory

with and without axial anomaly

Polyakov–quark-meson model study for $N_F = 2$

➜ only mean-field approximation

Findings:

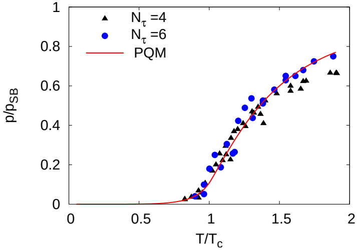
- ▷ Parameter in Polyakov loop potential: $T_0 \Rightarrow T_0(N_f, \mu)$

  pure gauge: $T_0 \sim 270$ MeV
  $N_f = 2$: $T_0 \sim 210$ MeV

- ▷ Chiral & deconfinement transition coincide

- ▷ Mean-field approximation encouraging

  Quark-meson model is renormalizable
  $\rightarrow$ no UV cutoff parameter (cf. PNJL model)




## Outlook

- ▷ include quark-meson dynamics in PQM model with RG

- ▷ include glue dynamics with RG $\rightarrow$ full QCD
  (step by step)