Retarded propagator presentation (and consequences of it) of out of equilibrium QFT

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R/A formalism in early 1990ies

P. Aurenche and T.B. Becherawy, P. Aurenche, E. Petitgirard and T. del Rio Gaztelurrutia, M.A. van Eijck and Ch. G. van Weert, T. Ewans,F. Guerin

- mainly for equilibrium
- propagators R/A, vertices include density distributions
- not directly connected to present work

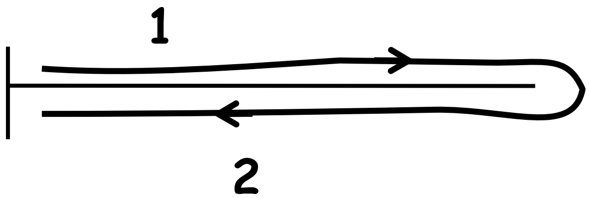
Infinite (Keldysh) time path - pinching

T. Altherr and D. Seibert, P. F. Bedaque, M. leBellac and H. Mabilat, A. Niegawa, ID, A. Jakovac,…

Finite time path - no adiabatic switching of interaction

- Boyanovsky with collaborators

Green functions



$D_C(p)$

$$D_{11}(p_0, \vec{p}) = \frac{i}{p_0 - \vec{p}^{\,2} - m^2 + 2i\varepsilon |p_0|} + 2\pi n_B \, \delta(p^2 - m^2)$$

$$D_{12} = 2\pi \, \delta(p^2 - m^2)\left[(1 + n_B)\Theta(-p_0) + n_B \, \Theta(p_0)\right]$$

$$D_{21} = 2\pi \, \delta(p^2 - m^2)\left[(1 + n_B)\Theta(p_0) + n_B \, \Theta(-p_0)\right]$$

$$D_{22} = D_{11}^{*}$$

$$D_{11} - D_{12} - D_{21} + D_{22} = 0$$

$$n_B \equiv n_B(\omega_p) \quad or \quad n_B(|p_0|), \quad \omega_p = \sqrt{(\vec{p}^{\,2} + m^2)}$$

R, A, K Basis

$$D_R(p) = -D_{11} + D_{12} = \frac{-i}{p^2 - m^2 + 2i\varepsilon\, p_0}$$

$$D_A(p) = -D_{11} + D_{21} = \frac{-i}{p^2 - m^2 - 2i\varepsilon\, p_0} \quad \equiv D_R(-p)$$

$$D_K = D_{11} + D_{22} = 2\pi\delta(p^2 - m^2)(1 + 2n_B)$$

Rename index “2” into “-1”, then in (1,-1) basis

$$D_{\mu\upsilon}(p) = \frac{1}{2}\left[D_K - \upsilon D_R - \mu D_A\right]$$

Projection to R/A basis

Two point function $A(x, y)$

Finite time path $A(x, y) = \Theta(x_0)\,\Theta(y_0)\,\overline{A}(x, y)$

$$1 = \Theta(x_0 - y_0) + \Theta(y_0 - x_0)$$

$$A(x, y) = A_R(x, y) + A_A(x, y)$$

$$A_R(x, y) = 0(x_0 - y_0)\,A(x, y) = \Theta(x_0 - y_0)\,\Theta(y_0)\,\overline{A}(x, y)$$

$$A_A \ldots$$

Projected functions $\overline{A}(x, y) = \overline{A}(x - y)$

Fourier and Wigner transforms

$$\overline{A}(x-y)=(2\pi)^{-4}\int d^4p\, e^{-ip(x-y)}\,\overline{A}(p)$$

$$\Theta(x_0)\,\Theta(y_0)=\int dp_0\, P_{X_0}(p_0,p_0')\,e^{-ip_0s_0}$$

$$P_{X_0}(p_0,p_0')=\frac{\Theta(X_0)}{2\pi}\int_{-2X_0}^{2X_0} ds_0\, e^{is_0(p_0-p_0')}$$

$$X_0=\frac{x_0+y_0}{2}\qquad s=x_0-y_0$$

$$\Theta(x_0-y_0)\,\Theta(y_0)=\int dp_0\, P_{X_0,R}(p_0,p_0')\,e^{-is_0(p_0-p_0')}$$

$$P_{X_0,R}(p_0,p_0')=\frac{\Theta(X_0)}{2\pi}\int_{0}^{2X_0} ds_0\, e^{is_0(p_0-p_0')}$$

$$A(X,Y) = (2\pi)^{-4} \int d^4 p \, e^{-ip(x-y)}$$

$$\int dp_0' \, P_{X_0}(p_0, p_0') \overline{A}(p_0', \vec{p})$$

$$A_R(X,Y) = \left(2\pi\right)^{-4} \int d^4 p \, e^{-ip(x-y)}$$

$$\int dp_0' P_{X_0,R}(p_0, p_0') \overline{A}(p_0', \vec{p})$$

$$X_0 \to \infty$$

$$D_K(p) = -D_{K,R}(p) + D_{K,A}(p)$$

scalar

$$D_{K,R}(p) = -\frac{p_0}{\omega_p}(1+2n(\omega_p))D_R(p)$$

spinor

$$S_{K,R}(p) = -(1-2n(\omega_p))D_R(p)(\omega_p\gamma_0 - \frac{\vec{p}p_0}{\omega_p}\vec{\gamma} + \frac{mp_0}{\omega_p})$$

$I.D.,\ Phys.\mathrm{Re}v.D\,63\,(2001), 024011$

heuristic rule

$$sign(p_0) \to \left(\frac{p_0}{\omega_p}\right)^{2n+1}, \quad n = 0, \pm 1, \pm 2,..$$

Particle number

$$N_p = a^+_{\vec{p}}\, a_{\vec{p}}$$

Number of particles at the time X_0

$$\left\langle 2N_{\vec{p}} + 1 \right\rangle_{X_0} = Tr\left[\left(2N_{\vec{p}} + 1 \right) \rho \right]_{X_0}$$

$$= \frac{\omega_p}{\pi} \int dp_0 \int dp'_0\, \frac{\Theta(X_0)}{8\pi i}\, \frac{e^{2iX_0(p_0 - p'_0)} - 1}{p_0 - p'_0}$$

$$\left[G_{K,X_0}(p_0, \vec{p}) + G_{R,X_0}(p_0, \vec{p}) \right]$$

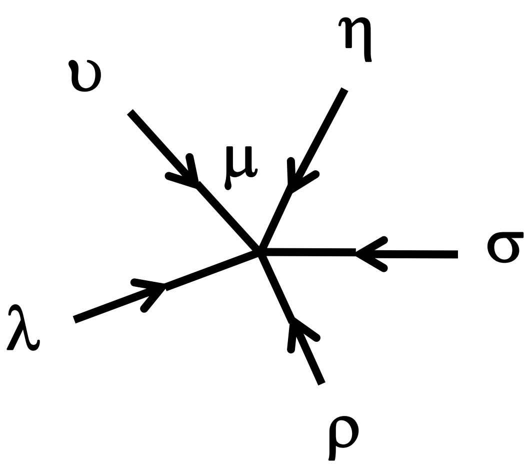
$$\left\langle (2N_{\vec{p}_1} + 1)(2N_{\vec{p}_2} + 1)\ldots \right\rangle_{X_0} = \ldots$$

- **inclusive**
- **correspond to $|M|^2$**
- **at t $\to \infty$ corresponds to "Fermi's golden rule"**

Absence of “source” vertices

$$\sum_{\mu=-1}^{1} \mu\, D^{(1)}_{\upsilon\mu}\ D^{(2)}_{\lambda\mu}\ D^{(3)}_{\rho\mu}\ D^{(4)}_{\sigma\mu}\ D^{(5)}_{\eta\mu}$$

$$= \sum_{\mu=-1}^{1} \mu\left(D^{(1)}_{K} - \mu D^{(1)}_{R} - \upsilon D^{(1)}_{A}\right)\left(D^{(2)}_{K} - \mu D^{(2)}_{R} - \lambda D^{(2)}_{A}\right)\ldots$$

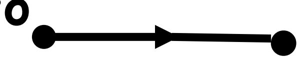


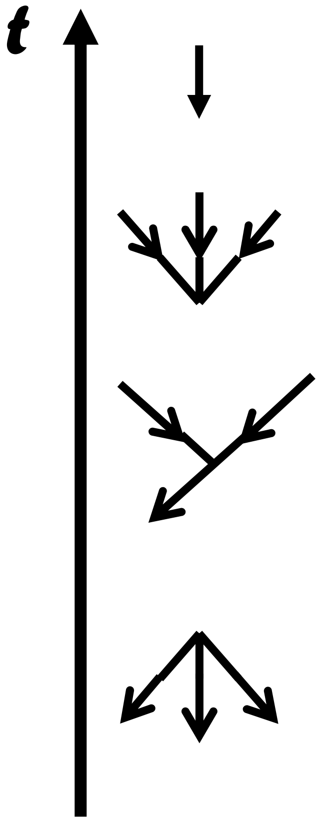
Only *even power* of μ contributes

- ⇒ **at least one D_R !!!!**
- ⇒ **no contribution from “source” vertices**
- ⇒ **internal vertex cannot have local maximal time**
- ⇒ **no closed diagrams**

Graphical representation

Turn all propagators to retarded;
ignore all the other properties

All lines retarded x_0  y_0 $x_0 > y_0$



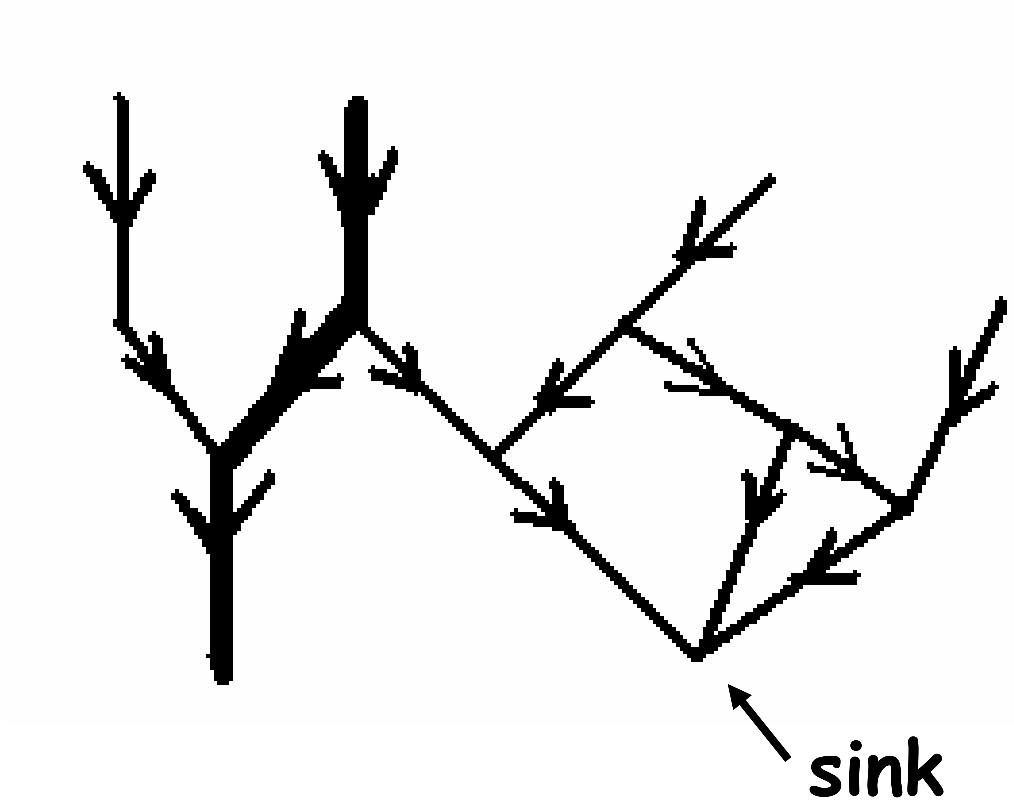
R

sink

normal

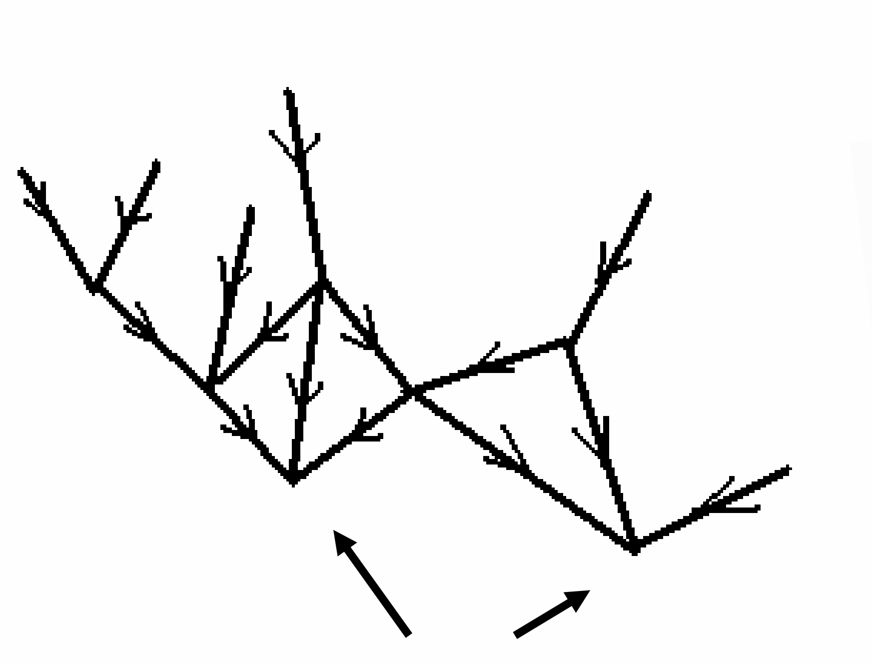
source
(absent)

arbitrary graph



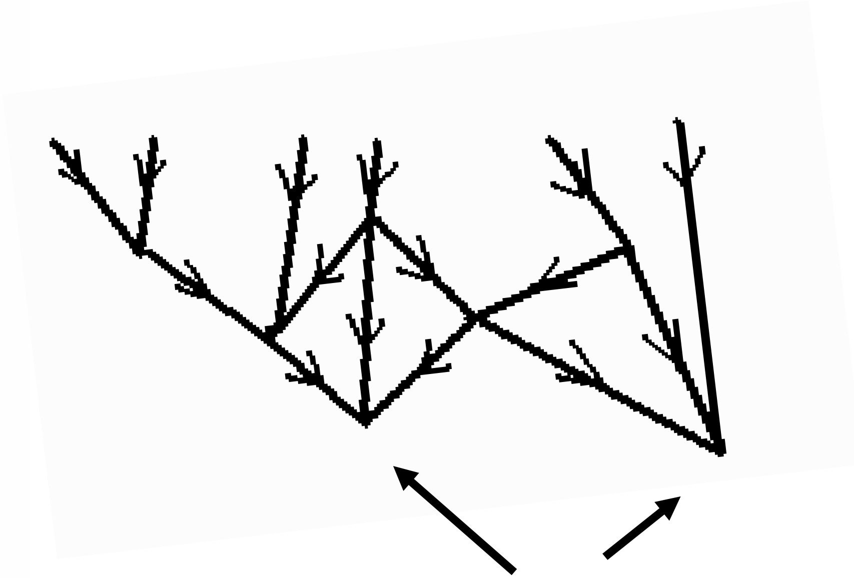
graphs with outgoing line vanish at equal time

contributing graph



sink

equal time

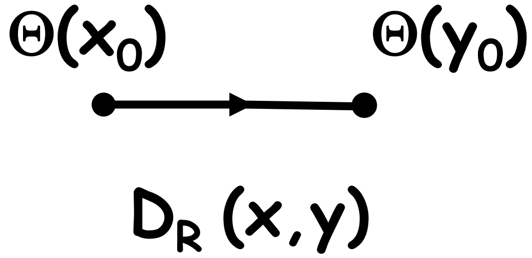


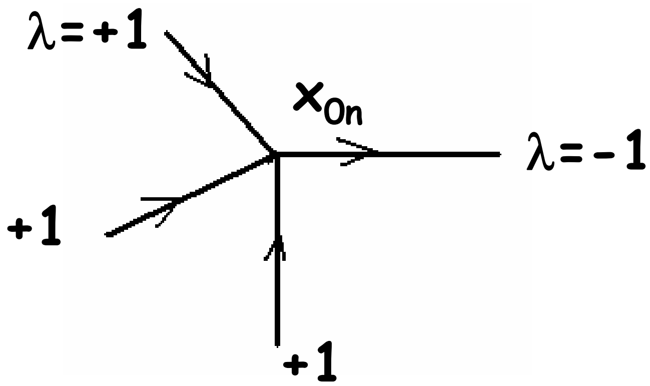
sink

6 point Greens function

external lines in pairs $G_R(p_0, \vec{p}),\ G_R(p_0', -\vec{p})$

Integrate time variables of internal vertices





$$D_R(x,y) = (2\pi)^{-4} \int d^4x \, e^{-ip(x-y)} \, G_R(p_0, \vec{p})$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} dx_{0n} \Theta(x_{0n}) e^{i\sum \lambda_i \, p_{0i} \, x_{0n}} = \frac{i}{\sum \lambda_i \, p_{0i} + i\varepsilon}$$

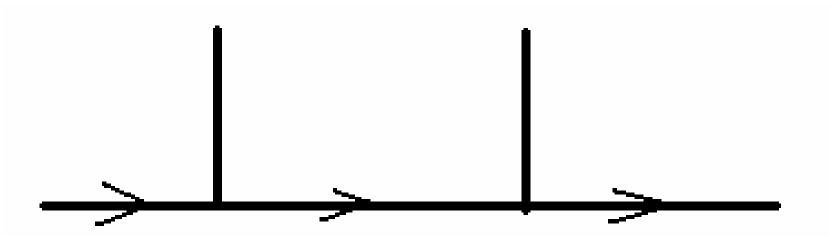
Remaining exponentials connected to external vertices, x_{0je}

$$\prod_j e^{-i\sum_i \lambda_{ij}\, p_{0ije}\, x_{0\,je}}$$

Equal time procedure $x_{0je} \rightarrow t$

$$\lim_{x_{0\,je}\rightarrow t} \quad \text{implies} \quad \int \prod_{ej} dp_{0ije} e^{-i\sum_j \lambda_{ij}\, p_{0ij}\, x_{0\,je}} D\left(p_{0ije}\right)$$

P_0 integrals



j-1 j j+1

$$\int \frac{dp_{0\,j}}{2\pi} F_{R,\varepsilon}\left(p_{0\,j}\right)\left[\left(-p_{0\,j} + p_{0\,j-1} + \lambda_1 q_{01} + i\varepsilon\right)\left(-p_{0\,j+1} + p_{0\,j} + \lambda_2 q_{02} + i\varepsilon\right)\right]^{-1}$$

close from above

$$= \frac{-iF_{R,2\varepsilon}\left(p_{oj-1} + \lambda_1 q_{01}\right)}{\left[-p_{0\,j+1} + p_{0\,j-1} + \lambda_1 q_{01} + \lambda_2 q_{02} + 2i\varepsilon\right]}$$

$\varepsilon > 0$ small finite number, $2\varepsilon \neq \varepsilon$!!!!

Energy nonconservation at sink

sink i $$\sum_j p_{0ijs} \neq 0$$

Conservation of nonconservation

$$\sum_i \sum_j p_{0ijs} = \sum_k \sum_l p_{0kle}$$

Equal time limit $x_{0\,je} \to t$

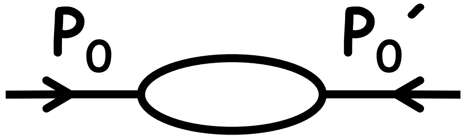
$$\frac{\prod_j e^{-it\sum_i p_{0ije}}}{\prod_k \left(\sum_l p_{0kls} + i\varepsilon \right)} = \prod_k \frac{e^{-it\sum_l p_{0kls}}}{\sum_l p_{0kls} + i\varepsilon}$$

Regularisation

$$\frac{e^{-it\sum p_{0kls}}}{\sum_l p_{0kls} + i\varepsilon} = \frac{e^{-it\sum_l p_{0kls}} - 1}{\sum_l p_{0kls} + i\varepsilon} + \frac{1}{\sum_l p_{0kls} + i\varepsilon}$$

vanishes !!!

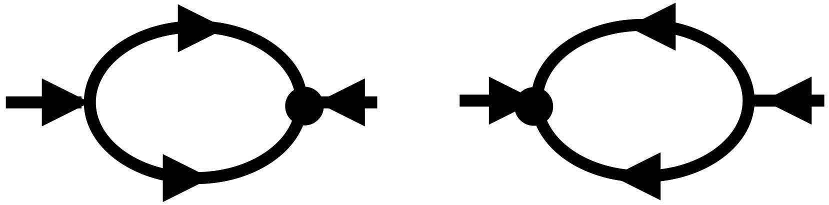
- remains only energy nonconserving term
- works only at equal time!
- no pinching



two denominators have different energy variables

Examples of diagrams

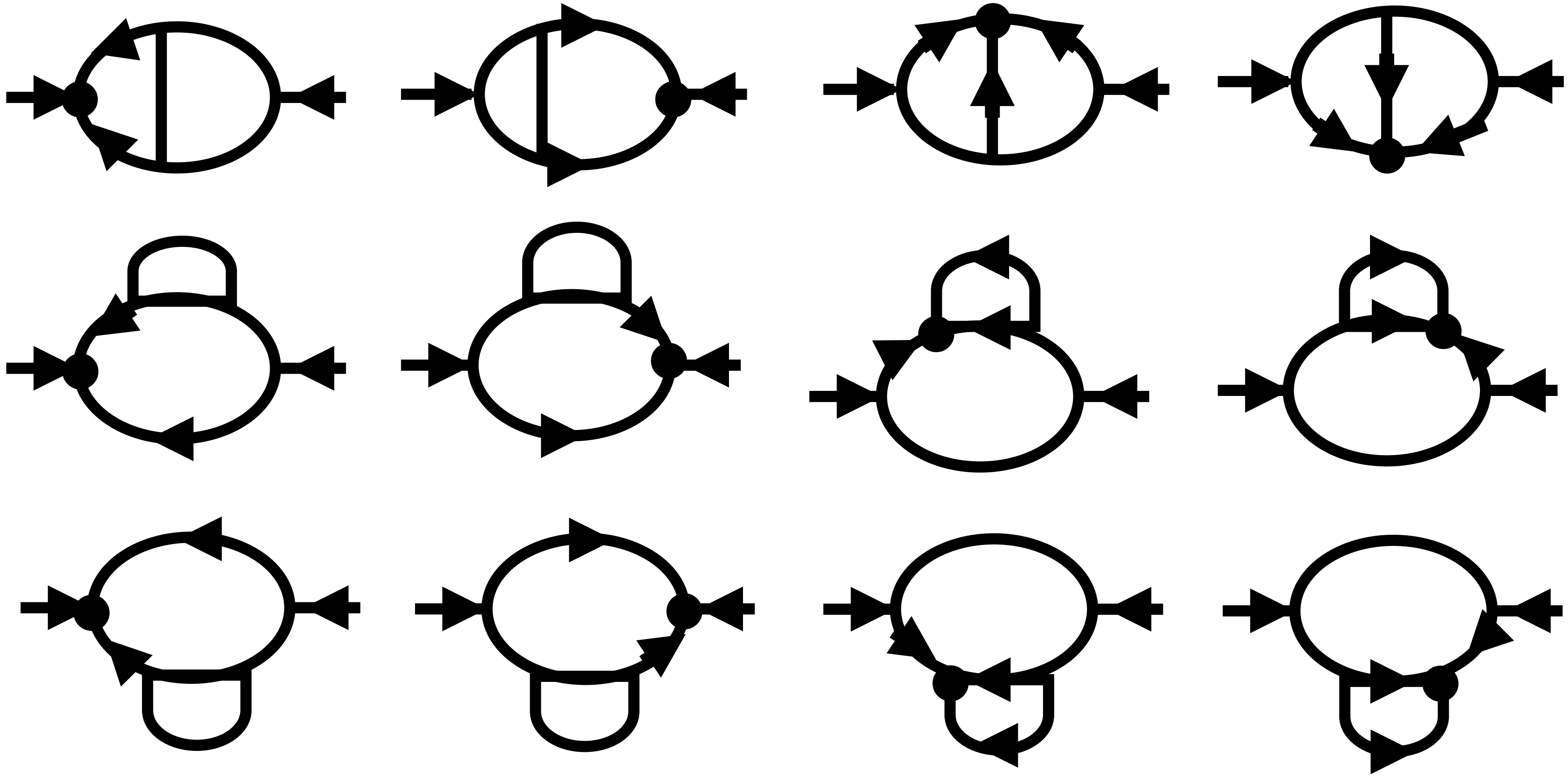
g^2 order



Photon production from QGP

pairing of nonsymmetric diagrams

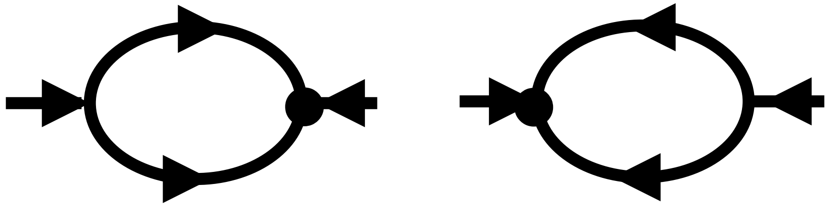
g^4 order



Examples of diagrams

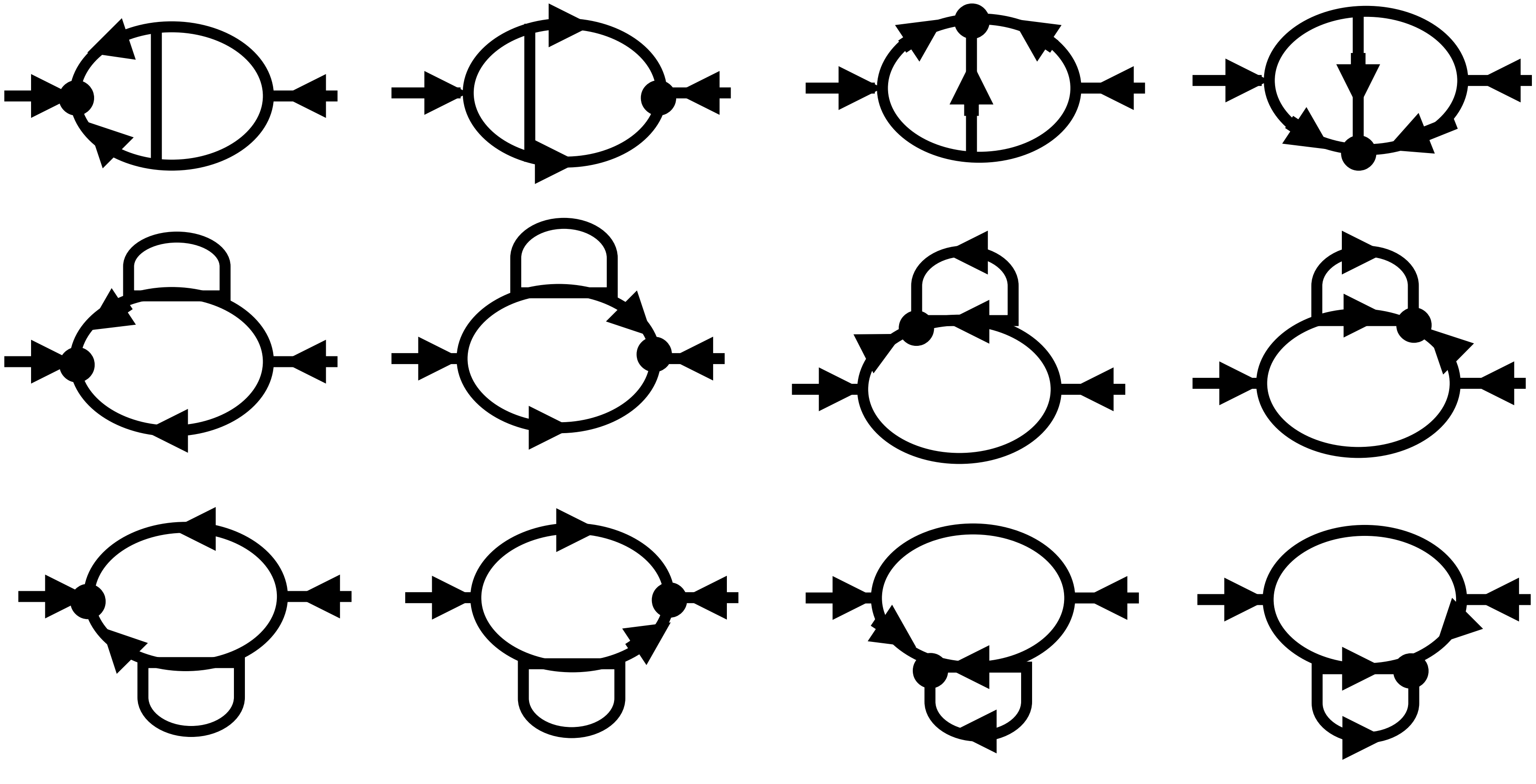
Photon production from QGP

g^2 order

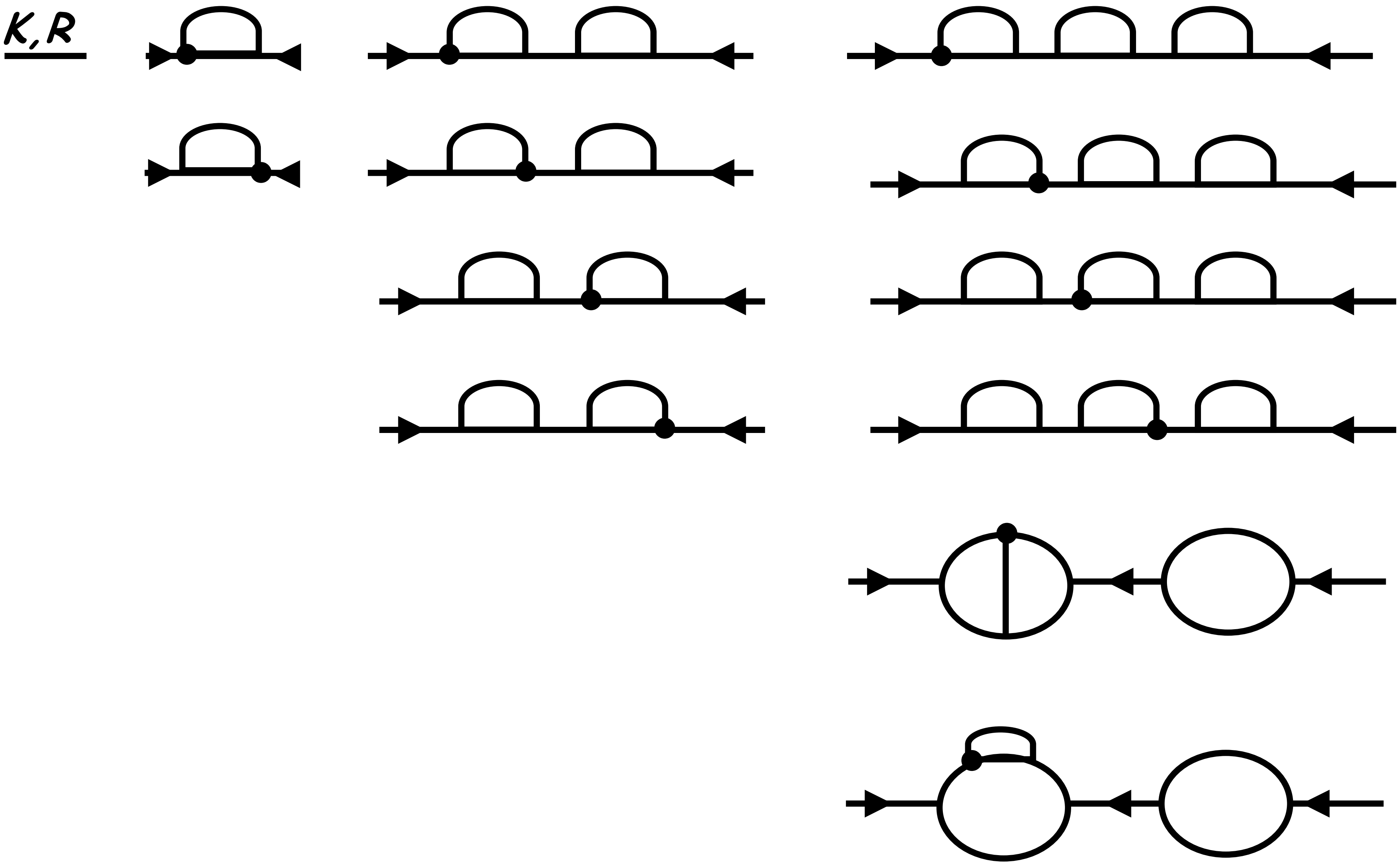


pairing of nonsymmetric diagrams

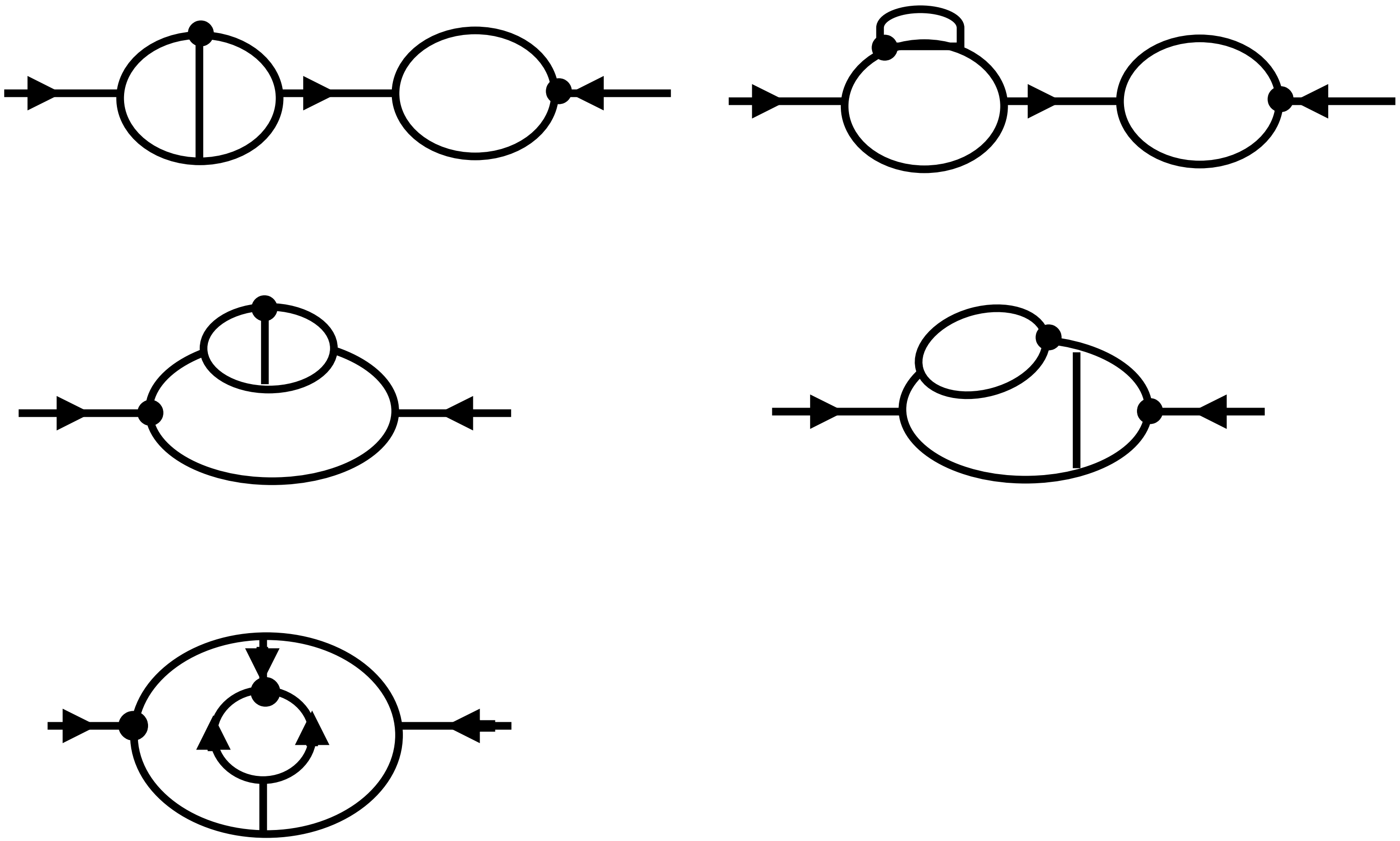
g^4 order



Sinks in Swinger-Dyson equation

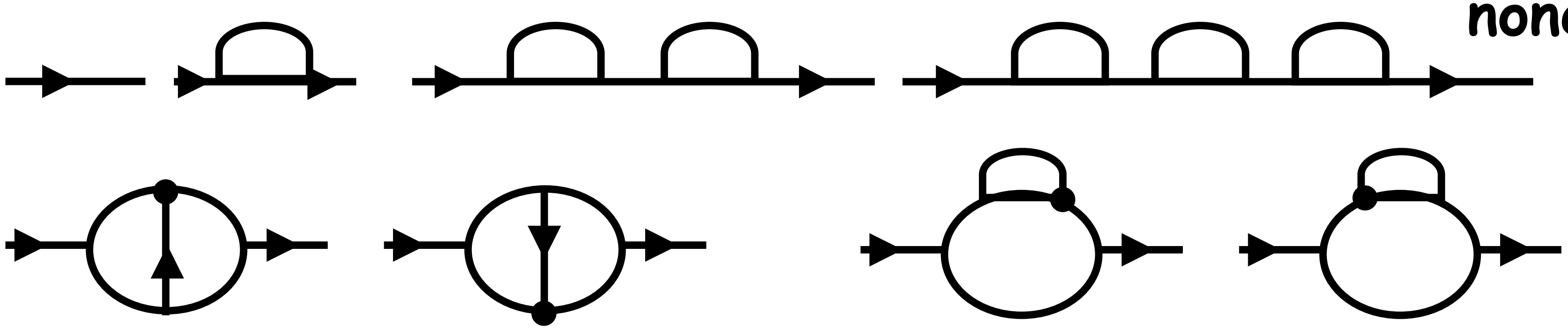


Double sinks

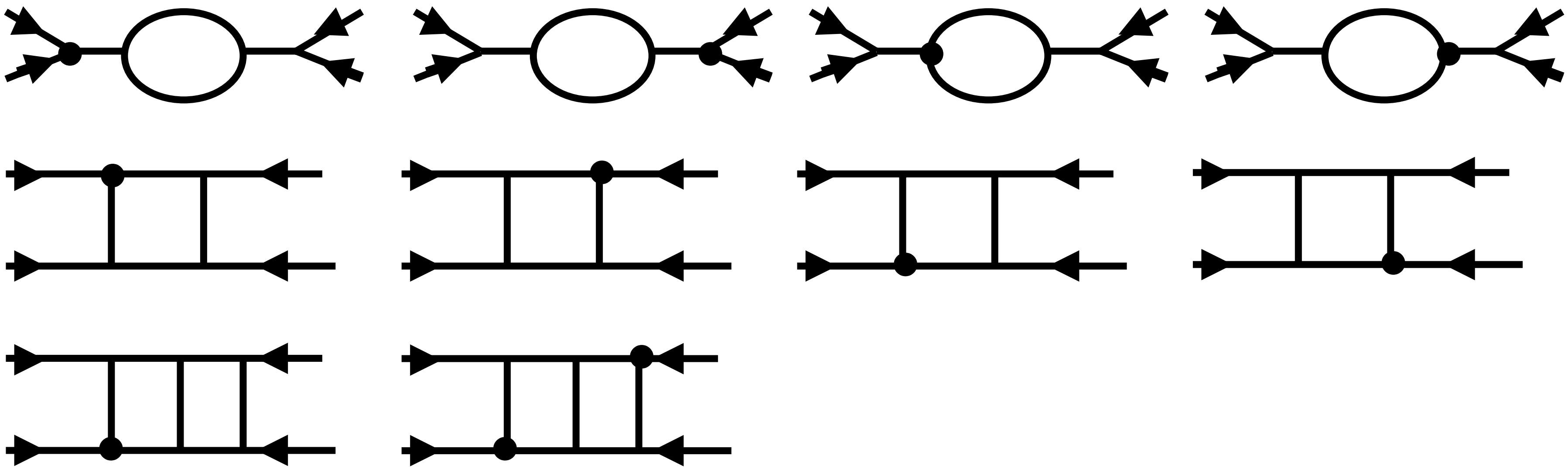


Sinks in retarded propagator

none

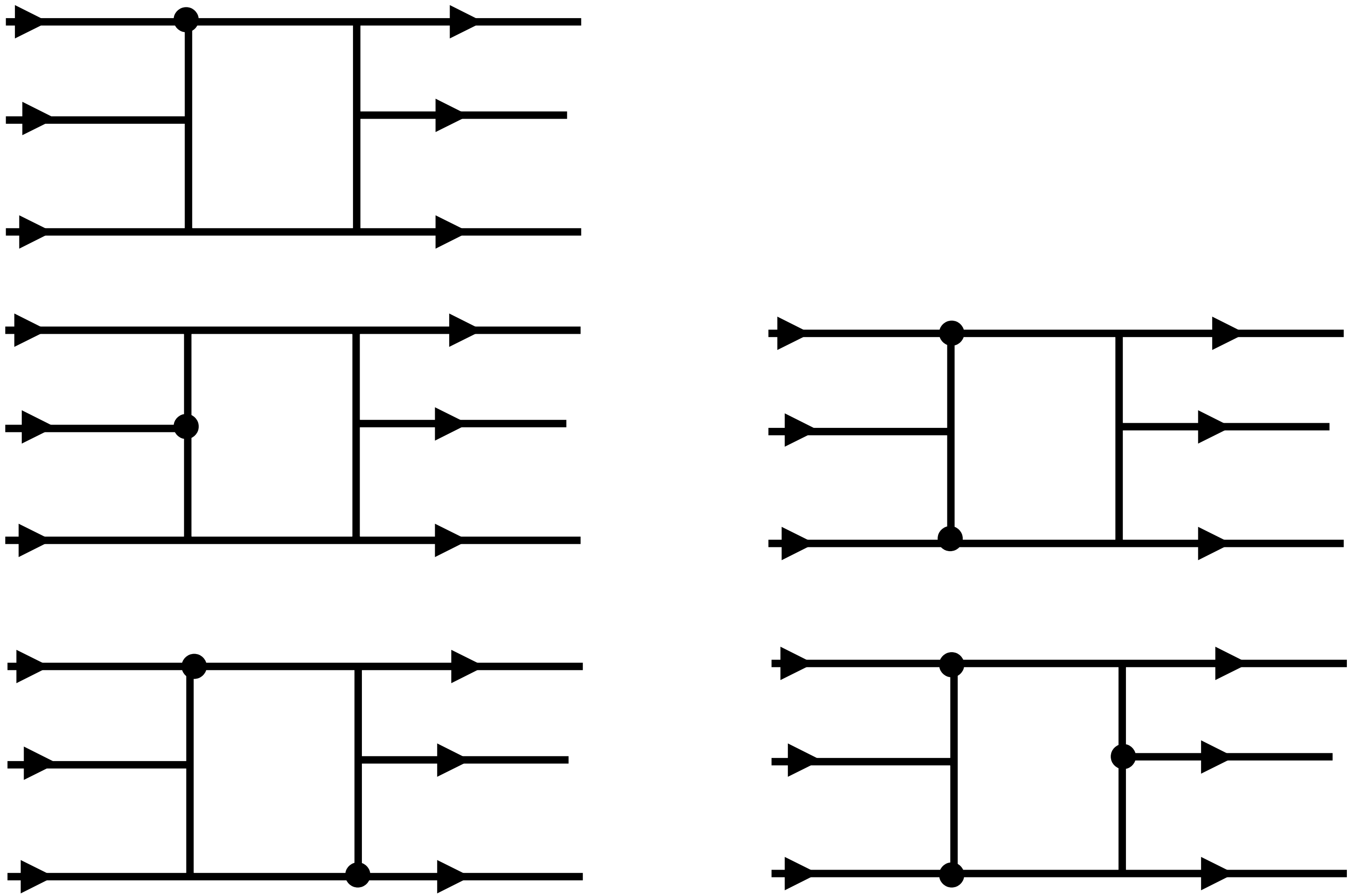


Sinks in 4-point Green functions



- dilepton production 2-particle scattering

Bhabha scattering



Large X_0 = t limit

$$\int_0^x \frac{dy}{y^2}\left(1-\cos yt\right)p(y) \overset{t\to\infty}{=} \frac{\pi}{2}tp(0)+p'(0)\left[\ln(\mu t)+\gamma\right]+O(1)$$

$$\int_0^\infty \frac{dy}{y}\left(1-\cos yt\right)p(y) \overset{t\to\infty}{=} p(0)\left[\ln(\mu t)+\gamma\right]+O(1)$$

$$\int_{-A}^\infty \frac{dy}{y^2}\left(1-\cos yt\right)p(y) \overset{t\to\infty}{=} \pi\, tp(0)+O(1)$$

$$for \quad A\rangle 0$$

D. Bojanovski at all., Phys. Rev. D 60 (1999) 065003
-in relativistic theories subleading terms may have infinite coefficients
-naive recipe for “adiabatic switching”:
keep only leading terms

Conclusion

- out of equilibrium quantum field theories expressed in terms of retarded propagators display very specific time ordering of equal time diagrams contributing to particle number:

- all external vertices are set at maximal time

- there are no internal vertices with locally maximal time

- there are no closed loops

Conclusion – continued

- there is at least one vertex ("sink") with locally minimal time

- pinching is connected with sink vertices

- at sink vertices energy is not conserved

- regulation at equal time limit i.e. elimination of energy conserving term at sink

Conclusion – continued

- enabled expansion around large time diagrams with one sink-vertex allow leading order contribution linear in time; these contributions enable connection with scattering theory with constant cross sections

- diagrams with n sink-vertices allow the n-th power of time-in linear response theories one expects vanishing of such contributions owing to the detailed balance theorem

Conclusion – continued

- retarded propagator is regulated only when it is a subgraph of multipoint equal time Greens function

- finite time path by regularisation of sink vertices and the expansion around large times completely replaces infinite (Keldysh) time path !

References

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