Resummation as renormalization scheme transformation

Antal Jakovác

Humboldt fellow (University of Wuppertal)
BME Fizikai Intézet

January 23, 2007

goal: computation of correlation functions:

$$\langle \hat{A}\hat{B} \rangle = \mathrm{Tr}\, \hat{\varrho}\hat{A}\hat{B} \qquad \hat{A}(t) = e^{iHt}\hat{A}(0)e^{-iHt}$$

Exact theory: path integral

goal: computation of correlation functions:

$$\langle \hat{A}\hat{B} \rangle = \mathrm{Tr}\, \hat{\varrho}\hat{A}\hat{B} \qquad \hat{A}(t) = e^{iHt}\hat{A}(0)e^{-iHt}$$

Exact theory: path integral

- for calculation: MC simulations

goal: computation of correlation functions:

$$\langle \hat{A}\hat{B} \rangle = \mathrm{Tr}\, \hat{\varrho}\hat{A}\hat{B} \qquad \hat{A}(t) = e^{iHt}\hat{A}(0)e^{-iHt}$$

Exact theory: path integral

- for calculation: MC simulations
- not all obervables are accessible

goal: computation of correlation functions:

$$\langle \hat{A}\hat{B}\rangle = \mathrm{Tr}\,\hat{\varrho}\hat{A}\hat{B} \qquad \hat{A}(t) = e^{iHt}\hat{A}(0)e^{-iHt}$$

Exact theory: path integral

- for calculation: MC simulations
- not all obervables are accessible

perturbation theory

goal: computation of correlation functions:

$$\langle \hat{A}\hat{B} \rangle = \mathrm{Tr}\, \hat{\varrho}\hat{A}\hat{B} \qquad \hat{A}(t) = e^{iHt}\hat{A}(0)e^{-iHt}$$

Exact theory: path integral

- for calculation: MC simulations
- not all obervables are accessible

perturbation theory

- UV safe (renormalization)

goal: computation of correlation functions:

$$\langle \hat{A}\hat{B} \rangle = \mathrm{Tr}\, \hat{\varrho}\hat{A}\hat{B} \qquad \hat{A}(t) = e^{iHt}\hat{A}(0)e^{-iHt}$$

Exact theory: path integral

- for calculation: MC simulations
- not all obervables are accessible

perturbation theory

- UV safe (renormalization)
- improveble (loop expansion)

goal: computation of correlation functions:

$$\langle \hat{A}\hat{B}\rangle = \mathrm{Tr}\,\hat{\varrho}\hat{A}\hat{B} \qquad \hat{A}(t) = e^{iHt}\hat{A}(0)e^{-iHt}$$

Exact theory: path integral

- for calculation: MC simulations
- not all obervables are accessible

perturbation theory

- UV safe (renormalization)
- improveble (loop expansion)
- often IR sensitive: becomes non-convergent

goal: computation of correlation functions:

$$\langle \hat{A}\hat{B} \rangle = \mathrm{Tr}\, \hat{\varrho}\hat{A}\hat{B} \qquad \hat{A}(t) = e^{iHt}\hat{A}(0)e^{-iHt}$$

Exact theory: path integral

- for calculation: MC simulations
- not all obervables are accessible

perturbation theory

- UV safe (renormalization)
- improveble (loop expansion)
- often IR sensitive: becomes non-convergent

gap equations

goal: computation of correlation functions:

$$\langle \hat{A}\hat{B} \rangle = \mathrm{Tr}\, \hat{\varrho}\hat{A}\hat{B} \qquad \hat{A}(t) = e^{iHt}\hat{A}(0)e^{-iHt}$$

Exact theory: path integral

- for calculation: MC simulations
- not all obervables are accessible

perturbation theory

- UV safe (renormalization)
- improveble (loop expansion)
- often IR sensitive: becomes non-convergent

gap equations

- coming from resummation or truncation of exact equations

goal: computation of correlation functions:

$$\langle \hat{A}\hat{B}\rangle = \mathrm{Tr}\, \hat{\varrho}\hat{A}\hat{B} \qquad \hat{A}(t) = e^{iHt}\hat{A}(0)e^{-iHt}$$

Exact theory: path integral

- for calculation: MC simulations
- not all obervables are accessible

perturbation theory

- UV safe (renormalization)
- improveble (loop expansion)
- often IR sensitive: becomes non-convergent

gap equations

- coming from resummation or truncation of exact equations
- IR safe (by construction)

goal: computation of correlation functions:

$$\langle \hat{A}\hat{B} \rangle = \mathrm{Tr}\, \hat{\varrho}\hat{A}\hat{B} \qquad \hat{A}(t) = e^{iHt}\hat{A}(0)e^{-iHt}$$

Exact theory: path integral

- for calculation: MC simulations
- not all obervables are accessible

perturbation theory

- UV safe (renormalization)
- improveble (loop expansion)
- often IR sensitive: becomes non-convergent

gap equations

- coming from resummation or truncation of exact equations
- IR safe (by construction)
- renormalization?

goal: computation of correlation functions:

$$\langle \hat{A}\hat{B} \rangle = \mathrm{Tr}\, \hat{\varrho}\hat{A}\hat{B} \qquad \hat{A}(t) = e^{iHt}\hat{A}(0)e^{-iHt}$$

Exact theory: path integral

- for calculation: MC simulations
- not all obervables are accessible

perturbation theory

- UV safe (renormalization)
- improveble (loop expansion)
- often IR sensitive: becomes non-convergent

gap equations

- coming from resummation or truncation of exact equations
- IR safe (by construction)
- renormalization?
- improvability?

Solution: if we could embed the resummed equations into the framework of perturbation theory, all its difficulties would be solved.

Solution: if we could embed the resummed equations into the framework of perturbation theory, all its difficulties would be solved.

1. Perturbative resummation
2. Momentum dependence
3. Two loop scalar model
    - One loop order
    - Two loop level
    - Results
4. Conclusions

Outlines

If perturbation theory is non-convergent because of IR sensitive diagrams:

If perturbation theory is non-convergent because of IR sensitive diagrams:

- identify the source of the bad convergence

If perturbation theory is non-convergent because of IR sensitive diagrams:

- identify the source of the bad convergence
- resum the sensitive diagrams into an effective propagator/vertex

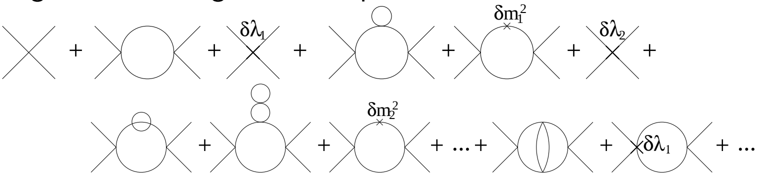
If perturbation theory is non-convergent because of IR sensitive diagrams:

- identify the source of the bad convergence
- resum the sensitive diagrams into an effective propagator/vertex
- use effective perturbation theory with the new propagators/vertexes: this should be IR safe

Example: tadpole mass resummation in Φ^4 theory

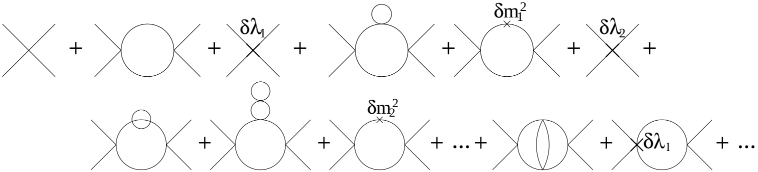
Example: tadpole mass resummation in Φ^4 theory

- Original set of diagrams for 4-point function

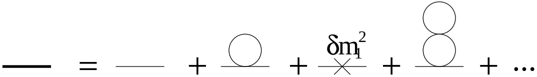


Example: tadpole mass resummation in Φ^4 theory

- Original set of diagrams for 4-point function

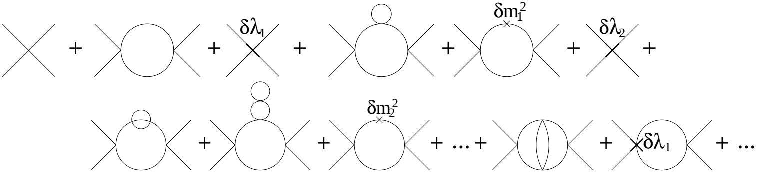


- Resum the tadpole

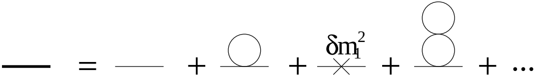


Example: tadpole mass resummation in Φ^4 theory

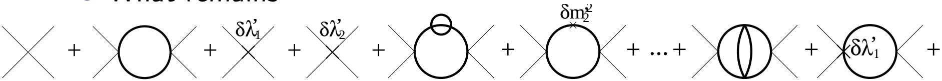
- Original set of diagrams for 4-point function



- Resum the tadpole



- What remains



Lessons:

- in the resummation we have to include counterterm diagrams, too

Lessons:

- in the resummation we have to include counterterm diagrams, too
- after resummation we still have a pertrubative series

Lessons:

- in the resummation we have to include counterterm diagrams, too
- after resummation we still have a pertrubative series
- in the new pert. series new counterterms are needed

Lessons:

- in the resummation we have to include counterterm diagrams, too
- after resummation we still have a pertrubative series
- in the new pert. series new counterterms are needed
- in the new pert. series there are less diagrams
  (the ones which are resummed, are missing)

Continue with the last remark:

„in the new pert. series there are less diagrams
(the ones which are resummed, are missing)"

Continue with the last remark:

„in the new pert. series there are less diagrams
(the ones which are resummed, are missing)"

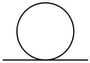
we can emulate this effect by defining a perturbation theory – by choosing appropriate counterterms –, where the necessary diagrams are missing.

Continue with the last remark:

„in the new pert. series there are less diagrams
(the ones which are resummed, are missing)”

we can emulate this effect by defining a perturbation theory – by choosing appropriate counterterms –, where the necessary diagrams are missing.

So let us define:

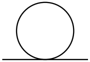
$$\delta\bar{m}^2 = -$$ 

Continue with the last remark:

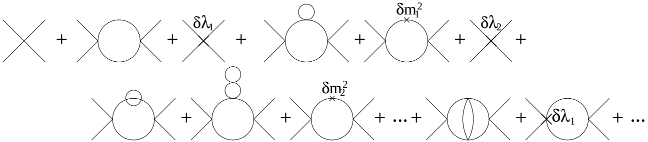
„in the new pert. series there are less diagrams
(the ones which are resummed, are missing)"

we can emulate this effect by defining a perturbation theory – by choosing appropriate counterterms –, where the necessary diagrams are missing.

So let us define:

$$\delta\bar{m}^2 = -$$ 

This kills some diagrams in the original set



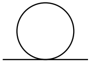
Continue with the last remark:

„in the new pert. series there are less diagrams
(the ones which are resummed, are missing)"

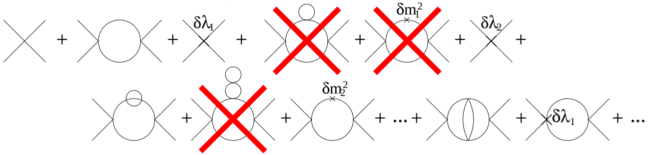
we can emulate this effect by defining a perturbation theory – by choosing appropriate counterterms –, where the necessary diagrams are missing.

So let us define:

$$\delta\bar{m}^2 = -$$



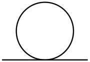
This kills some diagrams in the original set



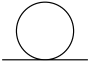
- Canceling diagrams by hand would result in a dubious perturbation theory.

- Canceling diagrams by hand would result in a dubious perturbation theory.
- But canceling diagrams by counterterms means just a different scheme.

- Canceling diagrams by hand would result in a dubious perturbation theory.
- But canceling diagrams by counterterms means just a different scheme.
- We have to ensure that the physics is unchanged when we use the other scheme. Require that the bare Lagrangian be the same:

$$m_0^2 = m^2_{\rm orig} + \delta m^2 = \bar{m}^2 + \delta \bar{m}^2 = \bar{m}^2 -$$  ,

or, rearranging:

$$\bar{m}^2 = m^2_{\rm orig} +$$  $$+ \delta m^2,$$

which is just the tadpole equation.

Conclusion:

- mass resummation & effective perturbation theory is equivalent with ordinary perturbation theory in a specific scheme (on-mass-shell scheme)

Conclusion:

- mass resummation & effective perturbation theory is equivalent with ordinary perturbation theory in a specific scheme (on-mass-shell scheme)
- gap equation corresponds to the renormalization scheme changing transformation

Conclusion:

- mass resummation & effective perturbation theory is equivalent with ordinary perturbation theory in a specific scheme (on-mass-shell scheme)
- gap equation corresponds to the renormalization scheme changing transformation
- benefit: no UV problem for resummation!

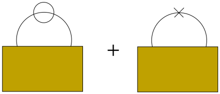
Conclusion:

- mass resummation & effective perturbation theory is equivalent with ordinary perturbation theory in a specific scheme (on-mass-shell scheme)
- gap equation corresponds to the renormalization scheme changing transformation
- benefit: no UV problem for resummation!
- easily extendible to static vertex resummation

What to do in case of momentum dependent resummation?

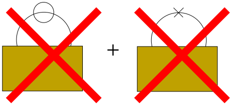
What to do in case of momentum dependent resummation?
eg.: 2PI resummation $\equiv$ self energy resummation:
work with an effective propagator, where all self-energy diagrams are resummed. In the effective perturbation theory there appears no self-energy-insertion diagrams, ie. no 2PI diagrams.

What to do in case of momentum dependent resummation?
eg.: 2PI resummation ≡ self energy resummation:
work with an effective propagator, where all self-energy diagrams are resummed. In the effective perturbation theory there appears no self-energy-insertion diagrams, ie. no 2PI diagrams.
The key point again: what are the missing diagrams?



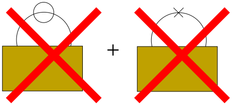
- if in a generic perturbation theory there is a self-energy insertion

What to do in case of momentum dependent resummation?
eg.: 2PI resummation ≡ self energy resummation:
work with an effective propagator, where all self-energy diagrams are resummed. In the effective perturbation theory there appears no self-energy-insertion diagrams, ie. no 2PI diagrams.
The key point again: what are the missing diagrams?



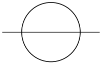
- if in a generic perturbation theory there is a self-energy insertion
- it is missing in the 2PI effective pert.th.

What to do in case of momentum dependent resummation?
eg.: 2PI resummation $\equiv$ self energy resummation:
work with an effective propagator, where all self-energy diagrams are resummed. In the effective perturbation theory there appears no self-energy-insertion diagrams, ie. no 2PI diagrams.
The key point again: what are the missing diagrams?



- if in a generic perturbation theory there is a self-energy insertion
- it is missing in the 2PI effective pert.th.

The same effect can be obtained using specific counterterm:

$$\delta m^2 = -$$


but momentum dependent counterterm??

Outlines

In principle we can introduce momentum dependent counterterms by rearranging the original Lagrangian as

$$\mathcal{L} = \frac{1}{2}\Phi K(i\partial)\Phi - \frac{\lambda_R}{24}\Phi^4 + \frac{1}{2}\Phi\delta K(i\partial)\Phi - \frac{\delta\lambda}{24}\Phi^4.$$

which is equivalent to the original Lagrangian if

$$Zp^2 - m_0^2 = K(p) + \delta K(p)$$

$\Rightarrow$ generic kernel and momentum dependent propagators come together

Consistency

A theory renormalizable with counterterms if (Weinberg-thm):

- overall divergence is local

Consistency

A theory renormalizable with counterterms if (Weinberg-thm):

- overall divergence is local
- after removing subdivergences the diagram is overall divergent or finite

Consistency

A theory renormalizable with counterterms if (Weinberg-thm):

- overall divergence is local
- after removing subdivergences the diagram is overall divergent or finite

these conditions are satisfied, if for $p \longrightarrow \infty$

$$G(p+k) = G(p) + k_\mu \partial^\mu G(p) + \dots,$$

the propagator can be power expanded „around infinity".

Within this class, all choices lead to consistent perturbation theory.

Consistency

A theory renormalizable with counterterms if (Weinberg-thm):

- overall divergence is local
- after removing subdivergences the diagram is overall divergent or finite

these conditions are satisfied, if for $p \to \infty$

$$G(p+k) = G(p) + k_\mu \partial^\mu G(p) + \dots,$$

the propagator can be power expanded „around infinity".

Within this class, all choices lead to consistent perturbation theory. It is satisfied for lot of propagators, including the exact one where we expect $G^{-1}(p) = p^2(\ln p^2)^n + m^2(\ln p^2)^n$ asymptotically.

Using this type of propagators, the divergence structure of $\delta K(p)$ is local:

$$\delta K_{\rm div}(p) = A_{\rm div} + B_{\rm div} p^2$$

- despite the kernel and the counterterms are momentum dependent, the divergences are local!

Using this type of propagators, the divergence structure of $\delta K(p)$ is local:

$$\delta K_{\mathrm{div}}(p) = A_{\mathrm{div}} + B_{\mathrm{div}} p^2$$

- despite the kernel and the counterterms are momentum dependent, the divergences are local!
- Only the finite parts can be nonlocal.

Using this type of propagators, the divergence structure of $\delta K(p)$ is local:

$$\delta K_{\mathrm{div}}(p) = A_{\mathrm{div}} + B_{\mathrm{div}} p^2$$

- despite the kernel and the counterterms are momentum dependent, the divergences are local!
- Only the finite parts can be nonlocal.
- renormalization goes as usual: we determine $\delta Z, \delta m^2$ and $\delta\lambda$ order by order. Divergences can depend on the kernel!

Using this type of propagators, the divergence structure of $\delta K(p)$ is local:

$$\delta K_{\rm div}(p) = A_{\rm div} + B_{\rm div} p^2$$

- despite the kernel and the counterterms are momentum dependent, the divergences are local!
- Only the finite parts can be nonlocal.
- renormalization goes as usual: we determine $\delta Z, \delta m^2$ and $\delta\lambda$ order by order. Divergences can depend on the kernel!
- the local finite parts besides the infinities are ill-defined (as usual). They can be determined by fixing values of observables (renormalization conditions).

After subtracting the infinities, we have to satisfy

$$\zeta p^2 - m^2 = K(p) + \delta K_{\rm fin}(p)$$

where ζ, m^2 (and λ) is to be determined by the renorm. conditions.

Practical recipe:

- we choose a reference scheme where $\zeta = 1$ and we choose m^2 and λ comfortably.

After subtracting the infinities, we have to satisfy

$$\zeta p^2 - m^2 = K(p) + \delta K_{\mathrm{fin}}(p)$$

where ζ, m^2 (and λ) is to be determined by the renorm. conditions.
Practical recipe:

- we choose a reference scheme where $\zeta = 1$ and we choose m^2 and λ comfortably.
- K-scheme: work with kernel K to avoid certain IR divergences.

After subtracting the infinities, we have to satisfy

$$\zeta p^2 - m^2 = K(p) + \delta K_{\mathrm{fin}}(p)$$

where ζ, m^2 (and λ) is to be determined by the renorm. conditions.

Practical recipe:

- we choose a reference scheme where $\zeta = 1$ and we choose m^2 and λ comfortably.
- K-scheme: work with kernel K to avoid certain IR divergences.
- for renormalization condition we choose those observables, which can be well calculated both in the reference scheme and in the K-schemes. Calculate their values in the reference scheme. In the K-scheme, we determine ζ, m^2 and λ to have the same results for these observables.

After subtracting the infinities, we have to satisfy

$$\zeta p^2 - m^2 = K(p) + \delta K_{\mathrm{fin}}(p)$$

where ζ, m^2 (and λ) is to be determined by the renorm. conditions.

Practical recipe:

- we choose a reference scheme where $\zeta = 1$ and we choose m^2 and λ comfortably.
- K-scheme: work with kernel K to avoid certain IR divergences.
- for renormalization condition we choose those observables, which can be well calculated both in the reference scheme and in the K-schemes. Calculate their values in the reference scheme. In the K-scheme, we determine ζ, m^2 and λ to have the same results for these observables.
- alternatively: define the finite parts of the diagrams in a way that $\zeta = 1$ and the values of m^2 and λ do not change, but the 3 chosen observable still yields the same result.

2PI resummation: we have seen that the appropriate counterterm choice is:

$$\delta m^2(p) = -\Sigma(p, K),$$

where K is the kernel with which the self-energy is computed. The consistency condition therefore reads:

$$\zeta p^2 - \bar{m}^2 - \Sigma_{\mathrm{fin}}(p, K) = K(p),$$

a finite equation.
We define the „finite part" to satisfy

$$\Sigma_{\mathrm{fin}}(p, \mathrm{reference}) = \Sigma_{\mathrm{fin}}(p, K),$$

for two asymptotic momenta. Then we can fix $\zeta = 1$, and $\bar{m}^2 = m^2$ (reference scheme mass) for any kernel

$$p^2 - m^2 - \Sigma_{\mathrm{fin}}(p, K) = K(p).$$

Solution of 2PI equations at finite T

with successive approximation:

$$K^{n+1}(p) = p^2 - m^2 - \Sigma_{\text{fin}}[K^n](p).$$

We aim to calculate the spectral function: it determines all other propagators.
At finite T the propagator is a matrix – except the retarded prop.
Also $\varrho = -\frac{1}{2}\,\mathrm{Im}\,G_{\text{ret}}$.

Solution of 2PI equations at finite T

with successive approximation:

$$K^{n+1}(p) = p^2 - m^2 - \Sigma_{\rm fin}[K^n](p).$$

We aim to calculate the spectral function: it determines all other propagators.
At finite T the propagator is a matrix – except the retarded prop.
Also $\varrho = -\frac{1}{2}\,\mathrm{Im}\, G_{\rm ret}$.

algoritmus:

- $\varrho^n(p) \quad \Rightarrow \quad$ propagators

Solution of 2PI equations at finite T

with successive approximation:

$$K^{n+1}(p) = p^2 - m^2 - \Sigma_{\mathrm{fin}}[K^n](p).$$

We aim to calculate the spectral function: it determines all other propagators.
At finite T the propagator is a matrix – except the retarded prop.
Also $\varrho = -\frac{1}{2}\,\mathrm{Im}\,G_{\mathrm{ret}}$.

algoritmus:

- $\varrho^n(p) \quad \Rightarrow \quad$ propagators
- propagators $\quad \Rightarrow \quad$ retarded self energy (diagram calculation)

Solution of 2PI equations at finite T

with successive approximation:

$$K^{n+1}(p) = p^2 - m^2 - \Sigma_{\rm fin}[K^n](p).$$

We aim to calculate the spectral function: it determines all other propagators.
At finite T the propagator is a matrix – except the retarded prop.
Also $\varrho = -\frac{1}{2}\,\mathrm{Im}\,G_{\rm ret}$.

algoritmus:

- $\varrho^n(p) \quad \Rightarrow \quad$ propagators
- propagators $\quad \Rightarrow \quad$ retarded self energy (diagram calculation)
- renormalization: ensure the reference scheme values for the self-energies at two asymptotic momenta

Solution of 2PI equations at finite T

with successive approximation:

$$K^{n+1}(p) = p^2 - m^2 - \Sigma_{\rm fin}[K^n](p).$$

We aim to calculate the spectral function: it determines all other propagators.
At finite T the propagator is a matrix – except the retarded prop.
Also $\varrho = -\frac{1}{2}\,\mathrm{Im}\,G_{\rm ret}$.

algoritmus:

- $\varrho^n(p) \quad \Rightarrow \quad$ propagators
- propagators $\quad \Rightarrow \quad$ retarded self energy (diagram calculation)
- renormalization: ensure the reference scheme values for the self-energies at two asymptotic momenta
- retarded self-energy $\quad \Rightarrow \quad K^{n+1}_{\rm ret}$

Solution of 2PI equations at finite T

with successive approximation:

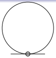
$$K^{n+1}(p) = p^2 - m^2 - \Sigma_{\rm fin}[K^n](p).$$

We aim to calculate the spectral function: it determines all other propagators.
At finite T the propagator is a matrix – except the retarded prop.
Also $\varrho = -\frac{1}{2}\,\mathrm{Im}\, G_{\rm ret}$.

algoritmus:

- $\varrho^n(p) \quad \Rightarrow \quad$ propagators
- propagators $\quad \Rightarrow \quad$ retarded self energy (diagram calculation)
- renormalization: ensure the reference scheme values for the self-energies at two asymptotic momenta
- retarded self-energy $\quad \Rightarrow \quad K_{\rm ret}^{n+1}$
- $K_{\rm ret}^{n+1} \quad \Rightarrow \quad \varrho^{n+1}$

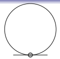
Outlines

Diagram: $\mathcal{T} =$  $\Rightarrow \quad \Sigma[K] = \frac{\lambda}{2}\mathcal{T}[K].$

Define:

$$\mathcal{T}_{\rm div} = \int_{k_0>0} \frac{d^4k}{(2\pi)^4}\varrho(k), \quad \mathcal{T}_{\rm fin} = \int_{k_0>0} \frac{d^4k}{(2\pi)^4} 2n(k_0)\varrho(k),$$

where n is the Bose-Einstein distribution.

Diagram: $\mathcal{T} =$  $\Rightarrow \quad \Sigma[K] = \frac{\lambda}{2}\mathcal{T}[K].$

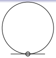
Define:

$$\mathcal{T}_{\mathrm{div}} = \int_{k_0>0} \frac{d^4k}{(2\pi)^4}\varrho(k), \quad \mathcal{T}_{\mathrm{fin}} = \int_{k_0>0} \frac{d^4k}{(2\pi)^4} 2n(k_0)\varrho(k),$$

where n is the Bose-Einstein distribution.

- reference scheme: 2PI scheme at $T = 0$. Because normalization $\mathcal{T}_{\mathrm{fin}}[K_{\mathrm{ref}}, T = 0] = 0$, ie. $\Sigma = 0$:

$$K_{\mathrm{ref}}(p) = p^2 - m^2$$

Diagram: $\mathcal{T} =$  $\Rightarrow \quad \Sigma[K] = \frac{\lambda}{2}\mathcal{T}[K].$

Define:

$$\mathcal{T}_{\mathrm{div}} = \int_{k_0>0} \frac{d^4k}{(2\pi)^4}\varrho(k), \quad \mathcal{T}_{\mathrm{fin}} = \int_{k_0>0} \frac{d^4k}{(2\pi)^4} 2n(k_0)\varrho(k),$$

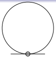
where n is the Bose-Einstein distribution.

- reference scheme: 2PI scheme at $T = 0$. Because normalization $\mathcal{T}_{\mathrm{fin}}[K_{\mathrm{ref}}, T = 0] = 0$, ie. $\Sigma = 0$:

$$K_{\mathrm{ref}}(p) = p^2 - m^2$$

- finite T: $\Sigma[K, T = 0] = \Sigma[K_{\mathrm{ref}}, T = 0]$ satisfied trivially $\Rightarrow \quad \zeta = 1$, $m_R = m$, and

$$K(p) = p^2 - m^2 - \frac{\lambda_T}{2}\mathcal{T}_{\mathrm{fin}}[K, T].$$

Diagram: $\mathcal{T} =$  $\Rightarrow \quad \Sigma[K] = \frac{\lambda}{2}\mathcal{T}[K].$

Define:

$$\mathcal{T}_{\rm div} = \int_{k_0>0} \frac{d^4k}{(2\pi)^4}\varrho(k), \quad \mathcal{T}_{\rm fin} = \int_{k_0>0} \frac{d^4k}{(2\pi)^4} 2n(k_0)\varrho(k),$$

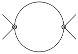
where n is the Bose-Einstein distribution.

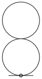
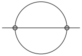
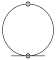
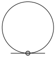
- reference scheme: 2PI scheme at $T = 0$. Because normalization $\mathcal{T}_{\rm fin}[K_{\rm ref}, T = 0] = 0$, ie. $\Sigma = 0$:

$$K_{\rm ref}(p) = p^2 - m^2$$

- finite T: $\Sigma[K, T = 0] = \Sigma[K_{\rm ref}, T = 0]$ satisfied trivially $\Rightarrow \quad \zeta = 1$, $m_R = m$, and

$$K(p) = p^2 - m^2 - \frac{\lambda_T}{2}\mathcal{T}_{\rm fin}[K, T].$$

- Also determine $\delta\lambda_1$ from the divergence of $I =$ 

$$\Sigma_2[K](p) = \frac{\lambda^2}{4}$$  $$+ \frac{\lambda^2}{6}$$  $$- \frac{\lambda}{2}\delta K_1$$  $$+ \frac{\delta\lambda_1}{2}$$  .

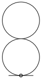
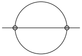
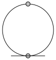
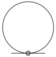
and take δK_1, $\delta\lambda_1$ from the one-loop result.

- reference scheme (2PI scheme at $T = 0$)

$$K_{\mathrm{ref}}(p) = p^2 - m^2 - \frac{\lambda^2}{6} S_{\mathrm{fin}}[K_{\mathrm{ref}}, T = 0](p),$$

where S is the sunset diagram, satisfying

$$S_{\mathrm{fin}}(p^2 = m^2) = 0, \qquad \partial_{p^2} S^{\mathrm{fin}}(p^2 = m^2) = 0$$

$\Sigma_2[K](p) = \frac{\lambda^2}{4}$  $+ \frac{\lambda^2}{6}$  $- \frac{\lambda}{2}\delta K_1$  $+ \frac{\delta\lambda_1}{2}$  .

and take δK_1, $\delta\lambda_1$ from the one-loop result.

- reference scheme (2PI scheme at $T=0$)

$$K_{\rm ref}(p) = p^2 - m^2 - \frac{\lambda^2}{6} S_{\rm fin}[K_{\rm ref}, T=0](p),$$

where S is the sunset diagram, satisfying

$$S_{\rm fin}(p^2=m^2)=0, \qquad \partial_{p^2} S^{\rm fin}(p^2=m^2)=0$$

- Numerically: compute total contribution and subtract ap^2+b function to satsify the ren. conditions.

- finite T solution

$$K(p) = p^2 - m^2 - \frac{\lambda}{2}\mathcal{T}_{\rm fin}[K,T] - \frac{\lambda^2}{6}S_{\rm fin}[K,T](p),$$

where to determine the finite value of S, we have to ensure the asymptotic values to be the same as in the reference scheme:

$$S_{\rm fin}[K, T=0](p_{\rm as}) = S_{\rm fin}[K_{\rm ref}, T=0](p_{\rm as}).$$

- finite T solution

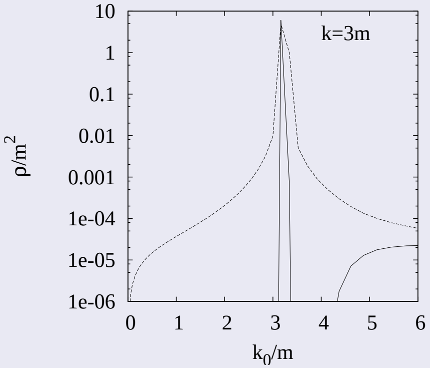
$$K(p) = p^2 - m^2 - \frac{\lambda}{2}\mathcal{T}_{\rm fin}[K,T] - \frac{\lambda^2}{6}S_{\rm fin}[K,T](p),$$

where to determine the finite value of S, we have to ensure the asymptotic values to be the same as in the reference scheme:

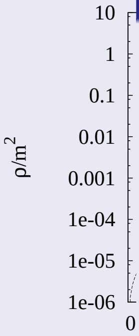
$$S_{\rm fin}[K,T=0](p_{\rm as}) = S_{\rm fin}[K_{\rm ref},T=0](p_{\rm as}).$$

- Numerically: determine the complete contribution and subtract from it an Ap^2+B function, to satisfy the as. condition. This function has to be used also at finite T (overall div. is T-independent)

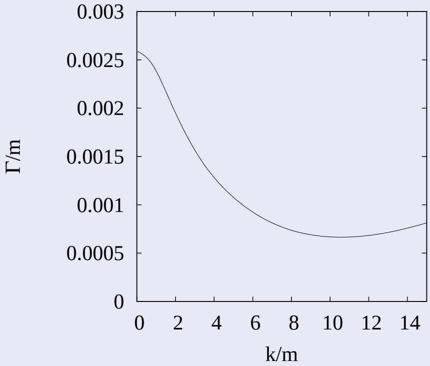
spectral function at $k = 3m$ momentum



spectral function at $k = 3m$ momentum

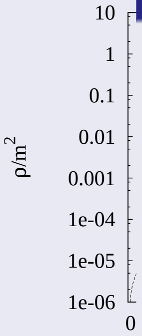


quasiparticle width



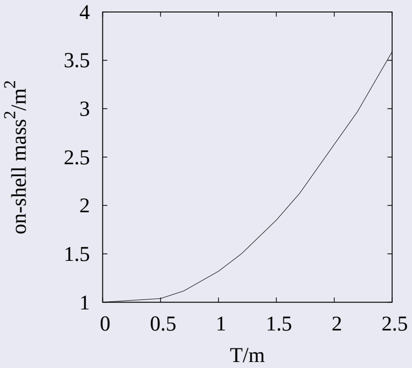
Results

spectral function at $k = 3m$ momentum



quasiparticle width

Debye mass



Outlines

To resum AND renormalize an IR sensitive theory:

- choose an IR safe renormalization scheme: counterterms should remove IR sensitivity. Do physical calculations in this IR safe perturbation theory.

To resum AND renormalize an IR sensitive theory:

- choose an IR safe renormalization scheme: counterterms should remove IR sensitivity. Do physical calculations in this IR safe perturbation theory.
- we can use momentum dependent counterterms together with new kernels: the divergences remain local

To resum AND renormalize an IR sensitive theory:

- choose an IR safe renormalization scheme: counterterms should remove IR sensitivity. Do physical calculations in this IR safe perturbation theory.
- we can use momentum dependent counterterms together with new kernels: the divergences remain local
- define a reference scheme with simple renormalization conditions

To resum AND renormalize an IR sensitive theory:

- choose an IR safe renormalization scheme: counterterms should remove IR sensitivity. Do physical calculations in this IR safe perturbation theory.
- we can use momentum dependent counterterms together with new kernels: the divergences remain local
- define a reference scheme with simple renormalization conditions
- determine the relation to a reference scheme by matching of (in both scheme) IR safe quantities