Phase diagram of strong matter at finite chemical potentials

Péter Kovács

KFKI Research Inst. of Part. and Nucl. Phys. of HAS, Theoretical Department

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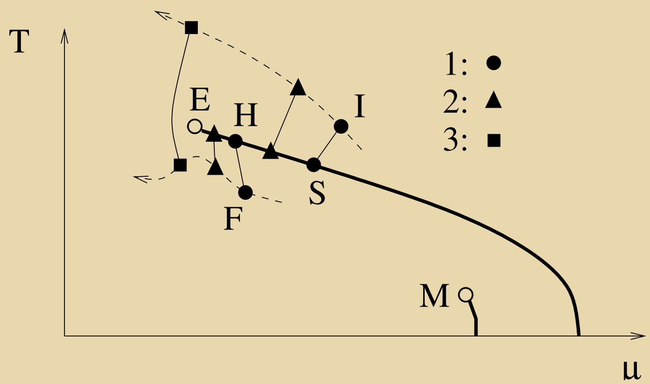
- Motivation for investigating QCD at non-zero μ_I, μ_Y
- The constituent quark model and its parameterization
- Introduction of the chemical potentials: μ_B, μ_I, μ_Y
- Results at $\mu_I = \mu_Y = 0$
- Results at non-zero μ_I, μ_Y
- Conclusions

Collaborators: Zs. Szép, A. Patkós, T. Herpay

Relevance of the isospin and strange chemical potentials



- the CEP is experimentally accessible
- $\mu_B, \mu_I \neq 0$ in heavy ion collision experiments
- μ_B is tunable $\rightarrow$ beam energy, centrality
- μ_I is tunable $\rightarrow$ different isotopes of an element
- In some experiment even μ_Y plays role
- Focusing effect: if CEP exist it cannot be missed

Brookhaven AGS exp. Si+Au collision: at $\mu_B = 540$ MeV $\rightarrow$ $\mu_Y \approx 150$ MeV

CERN SPS exp. Pb+Pb collision:
at $\mu_B = 233 - 266$ MeV $\rightarrow$ $\mu_Y \approx 70 - 80$ MeV, $\mu_I \approx 12 - 13$ MeV

CMB exp. at FAIR will explore QCD phase diagram at high μ_B

Analogy to the QCD CEP $\rightarrow$ liquid-gas phase transition which is easy to hit

lattice simulations at finite chemical potential is very difficult
$\Longrightarrow$ not all the methods predict/find the CEP

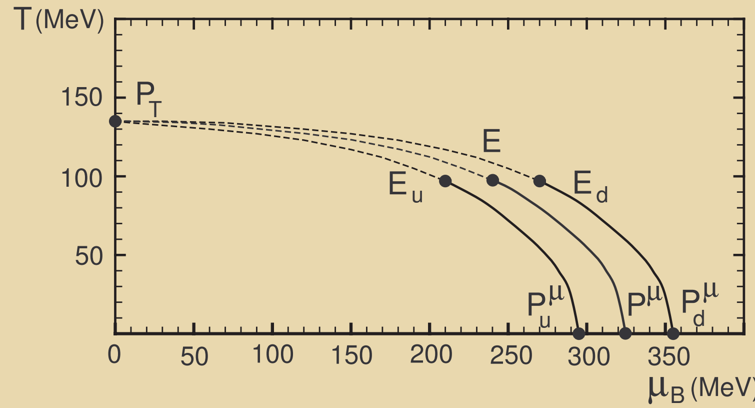
CEP found at: $(T, \mu_B)_{\text{CEP}} = (162 \pm 2, 360 \pm 40)$ MeV, volume: 12×4^3 and $m_\pi = m_\pi^{\text{phys}}$

Z. Fodor, S. D. Katz, JHEP 0404:050,2004

it is important to study the CEP and its μ_I, μ_Y dependence in effective models

Influence of μ_I on the $\mu_B - T$ diagram

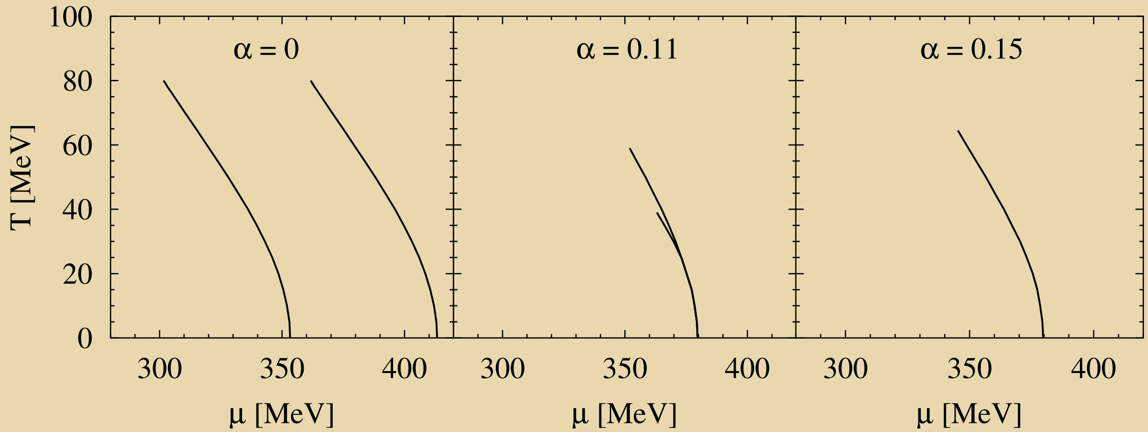
Barducci et. al, PLB **564**, 217



without $U(1)_A$ breaking $\rightarrow$ generic result
for low T μ_I induces two 1st order transitions
$\Longrightarrow$ 2 critical endpoints

the structure cease to exist in case of a sufficiently strong $U(1)_A$ breaking

Frank et. al, PLB **562**, 221



$SU_L(3) \times SU_R(3)$ symmetric chiral quark model

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}(\partial_\mu M^\dagger \partial^\mu M + m_0^2 M^\dagger M) - f_1 \left(\mathrm{Tr}(M^\dagger M)\right)^2 - f_2 \mathrm{Tr}(M^\dagger M)^2$$
$$- \; g\left(\det(M) + \det(M^\dagger)\right) + \epsilon_0\sigma_0 + \epsilon_3\sigma_3 + \epsilon_8\sigma_8 + \bar\psi\left(i\partial\!\!\!/ - g_F M_5\right)\psi.$$

$M = \frac{1}{\sqrt{2}}\sum_{i=0}^{8}(\sigma_i + i\pi_i)\lambda_i$, $M_5 = \sum_{i=0}^{8}\frac{1}{2}(\sigma_i + i\gamma_5\pi_i)\lambda_i$ $\quad 3\times 3$ complex matrices

pseudo(scalar) fields: π_i, σ_i, constituent quark field: $\bar\psi = (u, d, s)$

Gell-Mann matrices: $\lambda_0 := \sqrt{\frac{2}{3}}\mathbf{1}$, $\lambda_i \; : \; i = 1\ldots 8.$

determinant breaks $U_A(1)$ symmetry

explicit symmetry breaking: external fields $\epsilon_0, \epsilon_3, \epsilon_8 \neq 0 \Longleftrightarrow m_u \neq m_d \neq 0, m_s \neq 0$

broken symmetry phase: three condensates $(\langle\sigma_0\rangle, \langle\sigma_8\rangle), \langle\sigma_3\rangle \longleftrightarrow (x, y), v_3$

x: non-strange, y: strange

fermion masses: $M_u = \frac{g_F}{2}(x + v_3)$, $M_d = \frac{g_F}{2}(x - v_3)$, $M_s = \frac{g_F y}{\sqrt{2}}$

technical difficulty: mixing in the $0, 3, 8$ sector

parameters determined from the $T = 0$ mass spectrum

Parameterization and thermodynamics at one-loop level

13 unknown parameters:

couplings	m_0^2, f_1, f_2, g, g_F
condensates	x, y, v_3
external fields	$\epsilon_x, \epsilon_y, \epsilon_3$
renormalization scales	l_f, l_b

resummation using optimized perturbation theory   Chiku & Hatsuda, PRD58:076001

change: $-m_0^2 \to m^2 \quad \Rightarrow \quad \mathcal{L}_{mass} = \frac{1}{2}m^2 \mathrm{Tr} M^\dagger M - \frac{1}{2}\underbrace{(m_0^2 + m^2)}_{\Delta m^2\text{: one-loop counterterm}}\mathrm{Tr} M^\dagger M$

fastest apparent convergence $\quad M_\pi^2 = iG^{-1}(p^2{=}0)\big|_{\text{1-loop}} \stackrel{!}{=} m_\pi^2\big|_{\text{tree}} \Longrightarrow$ equation for the effective mass:

$$m^2 = -m_0^2 + \Sigma_\pi(p=0, m_i(m^2), M_q)$$

From the tree-level pion mass: $m^2 = m_\pi^2 - (4f_1 + 2f_2)x^2 - 4f_1 y^2 - 2gy$

$\Longrightarrow$ introducing into the other tree-level masses

$\Longrightarrow$ self-consistent gap equation for the pion mass

Set of coupled nonlinear equations (for $v_3 = 0$):

(1) gap-equation: $m_\pi^2 = -m_0^2 + (4f_1 + 2f_2)x^2 + 4f_1 y^2 + 2gy + \mathrm{Re}\Sigma_\pi(p=0, m_i(m_\pi), M_u)$

(2) pole-mass M_K from:

$$M_K^2 = -m_0^2 + 2(2f_1 + f_2)(x^2 + y^2) + 2f_2 y^2 - \sqrt{2}x(2f_2 y - g) + \mathrm{Re}\Sigma_K(p^2 = M_K^2, m_i)$$

(3) FAC criterion for M_K: $\Sigma(p^2 = M_K^2) = 0$

(4) pole-mass M_η from:

$$\mathrm{Det}\begin{pmatrix} p^2 - m_{\eta xx}^2 - \Sigma_{\eta xx}(p^2, m_i) & -m_{\eta xy}^2 - \Sigma_{\eta xy}(p^2, m_i) \\ -m_{\eta xy}^2 - \Sigma_{\eta xy}(p^2, m_i) & p^2 - m_{\eta yy}^2 - \Sigma_{\eta yy}(p^2, m_i) \end{pmatrix}\Bigg|_{p^2 = M_\eta^2, M_{\eta'}} = 0$$

(5) tree-level PCAC: $x = f_\pi$

(6) From non-strange quark mass: $g_F = \frac{2M_u}{x}$

(7) From strange quark mass: $y = \frac{\sqrt{2}M_s}{g_F}$

(8) EOS for x:

$$\epsilon_x = -m_0^2 x + 2gxy + 4f_1 x y^2 + 2(2f_1 + f_2)x^3 + \sum_{\alpha,i,j} t^x_{\alpha_{i,j}}\langle\alpha_i\alpha_j\rangle + \frac{g_F}{2}(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)$$

(9) EOS for y: $\epsilon_y = -m_0^2 y + gx^2 + 4f_1 x^2 y + 4(f_1 + f_2)y^3 + \sum_{\alpha,i,j} t^y_{\alpha_{i,j}}\langle\alpha_i\alpha_j\rangle + \frac{g_F}{\sqrt{2}}\langle\bar{s}s\rangle$

Differences in case of isospin breaking

New variable: v_3
Equation for $v_3 \longrightarrow$ third EoS:

$$\langle\frac{\partial\mathcal{L}}{\partial\sigma_3}\rangle = 0 \tag{1}$$

Even if $\epsilon_3 = 0$ ($\Longrightarrow v_3 = 0$ at $T = 0$) non zero μ_I will generate v_3 at non zero temperature

Consequence: charged and neutral particle masses will be different at tree level

If explicit isospin breaking is also introduced another equation is needed:

$$m_{\pi^+,\mathrm{tree}} - m_{\pi^0,\mathrm{tree}} = 4.594MeV \tag{2}$$

This equation will determine v_3 at $T = 0$ and EoS for v_3 at $T = 0$ will determine ϵ_3

Introduction of chemical potentials

21 particles:

	pseudoscalars	$\pi^0, \eta, \eta', \pi^+, \pi^-, K^+, K^-, K^0, \bar{K}^0$
	scalars	$\sigma, a_0^0, f_0, a_0^+, a_0^-, \kappa^+, \kappa^-, \kappa^0, \bar{\kappa}^0$
	fermions	m_u, m_d, m_s

Lagrangian is invariant under

$$\begin{aligned} M &\rightarrow e^{-i\alpha_G G} M e^{i\alpha_G G} = M - i\alpha_G [G, M] + \mathcal{O}(\alpha_G^2), \\ \psi &\rightarrow e^{-i\alpha_G G}\psi = \psi - i\alpha_G \psi + \mathcal{O}(\alpha_G^2), \end{aligned}$$

where G can be $B = \sqrt{\frac{3}{2}}\lambda_0$, $I = \frac{1}{2}\lambda_3$ and $Y = \frac{1}{\sqrt{3}}\lambda_8$

The conserved Noether currents:

$$J_\mu^G = -\frac{\delta L}{\delta(\partial^\mu M)_{ij}} i[G, M]_{j,i} - \frac{\delta L}{\delta(\partial^\mu M^+)_{ij}} i[G, M^+]_{j,i} - \frac{\delta L}{\delta(\partial^\mu \psi_i)} iG_{ij}\psi_j$$

The conserved charges:

$$Q^B = \frac{1}{3}(N_u + N_d + N_s - N_{\bar{u}} - N_{\bar{d}} - N_{\bar{s}}),$$

$$Q^I = \frac{1}{2}(N_u - N_{\bar{u}} - N_d + N_{\bar{d}} + N_{\kappa^+} - N_{\kappa^-} + N_{\bar{\kappa}^0} - N_{\kappa^0} + N_{K^+} - N_{K^-} + N_{\bar{K}^0} - N_{K^0})$$
$$+ \quad N_{a_0^+} - N_{a_0^-} + N_{\pi^+} - N_{\pi^-},$$

$$Q^Y = \frac{1}{3}(N_u - N_{\bar{u}} + N_d - N_{\bar{d}} - 2N_s + 2N_{\bar{s}}) + N_{\kappa^+} - N_{\kappa^-} + N_{\kappa^0} - N_{\bar{\kappa}^0} + N_{K^+} - N_{K^-} + N_{K^0} - N_{\bar{K}^0}$$

Statistical density matrix of the system:

$$\rho = \exp[-\beta(H - \mu_i N_i)]$$

The following chemical potentials can be introduced:

$$\begin{aligned}
\mu_u &= -\mu_{\bar{u}} = \frac{1}{3}\mu_B + \frac{1}{2}\mu_I + \frac{1}{3}\mu_Y, \\
\mu_d &= -\mu_{\bar{d}} = \frac{1}{3}\mu_B - \frac{1}{2}\mu_I + \frac{1}{3}\mu_Y, \\
\mu_s &= -\mu_{\bar{s}} = \frac{1}{3}\mu_B - \frac{2}{3}\mu_Y, \\
\mu_{a_0^+} &= \mu_{\pi^+} = -\mu_{a_0^-} = -\mu_{\pi^-} = \mu_I, \\
\mu_{\kappa^+} &= \mu_{K^+} = -\mu_{\kappa^-} = -\mu_{K^-} = \frac{1}{2}\mu_I + \mu_Y, \\
\mu_{\kappa^0} &= \mu_{K^0} = -\mu_{\bar{\kappa}^0} = -\mu_{\bar{K}^0} = -\frac{1}{2}\mu_I + \mu_Y
\end{aligned}$$

Finite temperature propagators of charged fields

For example the K^-, K^+ field operators:

$$K^-(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_\mathbf{p}}} \left(a^+(\mathbf{p}) e^{ip\cdot x} + b(\mathbf{p}) e^{-ip\cdot x} \right) \Big|_{p_0=E_\mathbf{p}},$$

$$K^+(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_\mathbf{p}}} \left(b^+(\mathbf{p}) e^{ip\cdot x} + a(\mathbf{p}) e^{-ip\cdot x} \right) \Big|_{p_0=E_\mathbf{p}}$$

The two-point functions:

$$G_{K^-}(y-x) := \langle T K^-(y) K^+(x) \rangle_\beta = \Theta(y_0 - x_0) \langle K^-(y) K^+(x) \rangle_\beta + \Theta(x_0 - y_0) \langle K^+(x) K^-(y) \rangle_\beta,$$

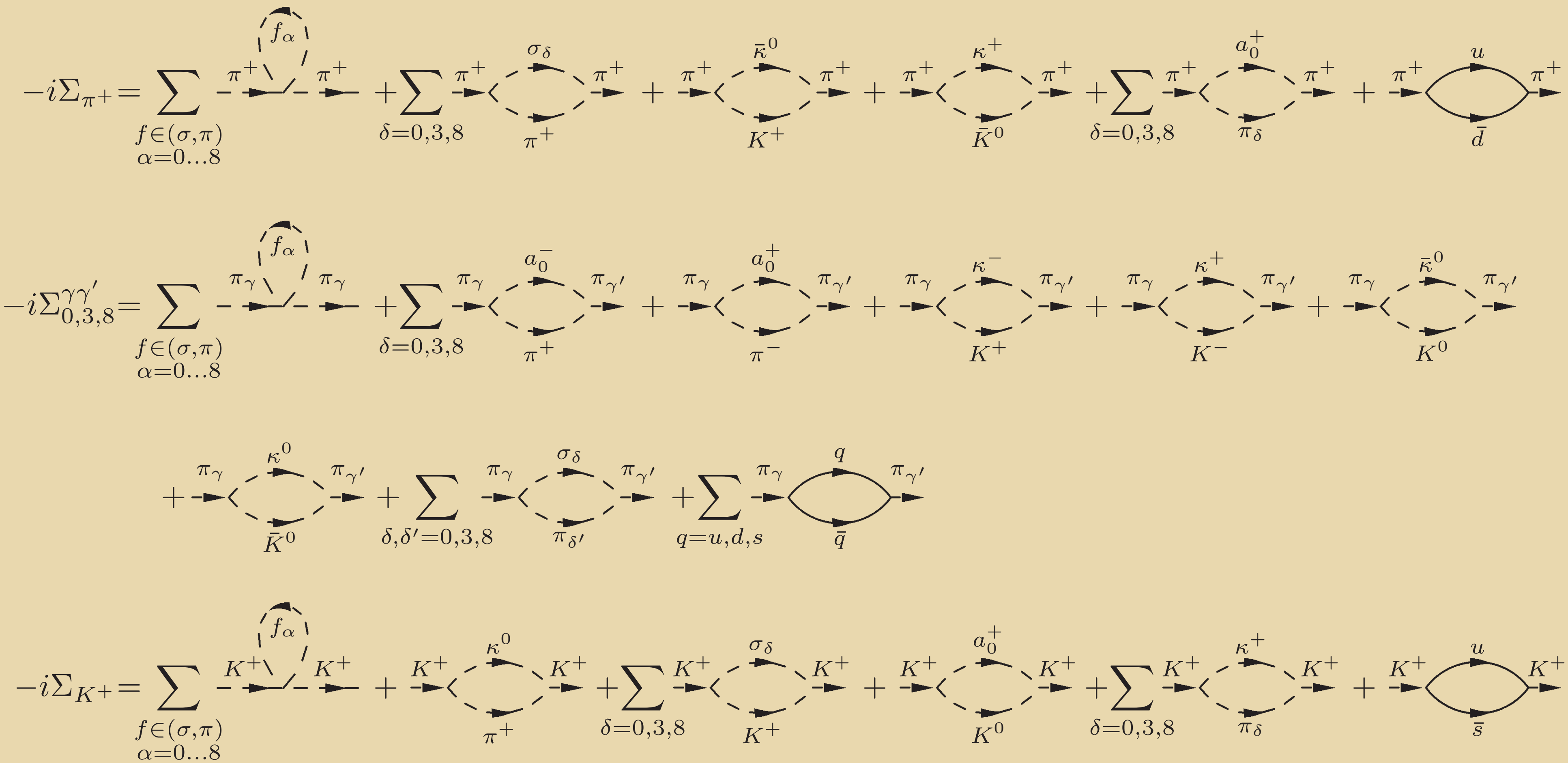
$$G_{K^+}(y-x) := \langle T K^+(y) K^-(x) \rangle_\beta = \Theta(y_0 - x_0) \langle K^+(y) K^-(x) \rangle_\beta + \Theta(x_0 - y_0) \langle K^-(x) K^+(y) \rangle_\beta,$$

In momentum space the finite temperature propagators:

$$G_{K^-}(k) = \frac{i}{2E_\mathbf{k}} \left[\frac{1 + n_{K^-}(E_\mathbf{k})}{k_0 - E_\mathbf{k} + i\epsilon} - \frac{n_{K^-}(E_\mathbf{k})}{k_0 - E_\mathbf{k} - i\epsilon} - \frac{1 + n_{K^+}(E_\mathbf{k})}{k_0 + E_\mathbf{k} - i\epsilon} + \frac{n_{K^+}(E_\mathbf{k})}{k_0 + E_\mathbf{k} + i\epsilon} \right]$$

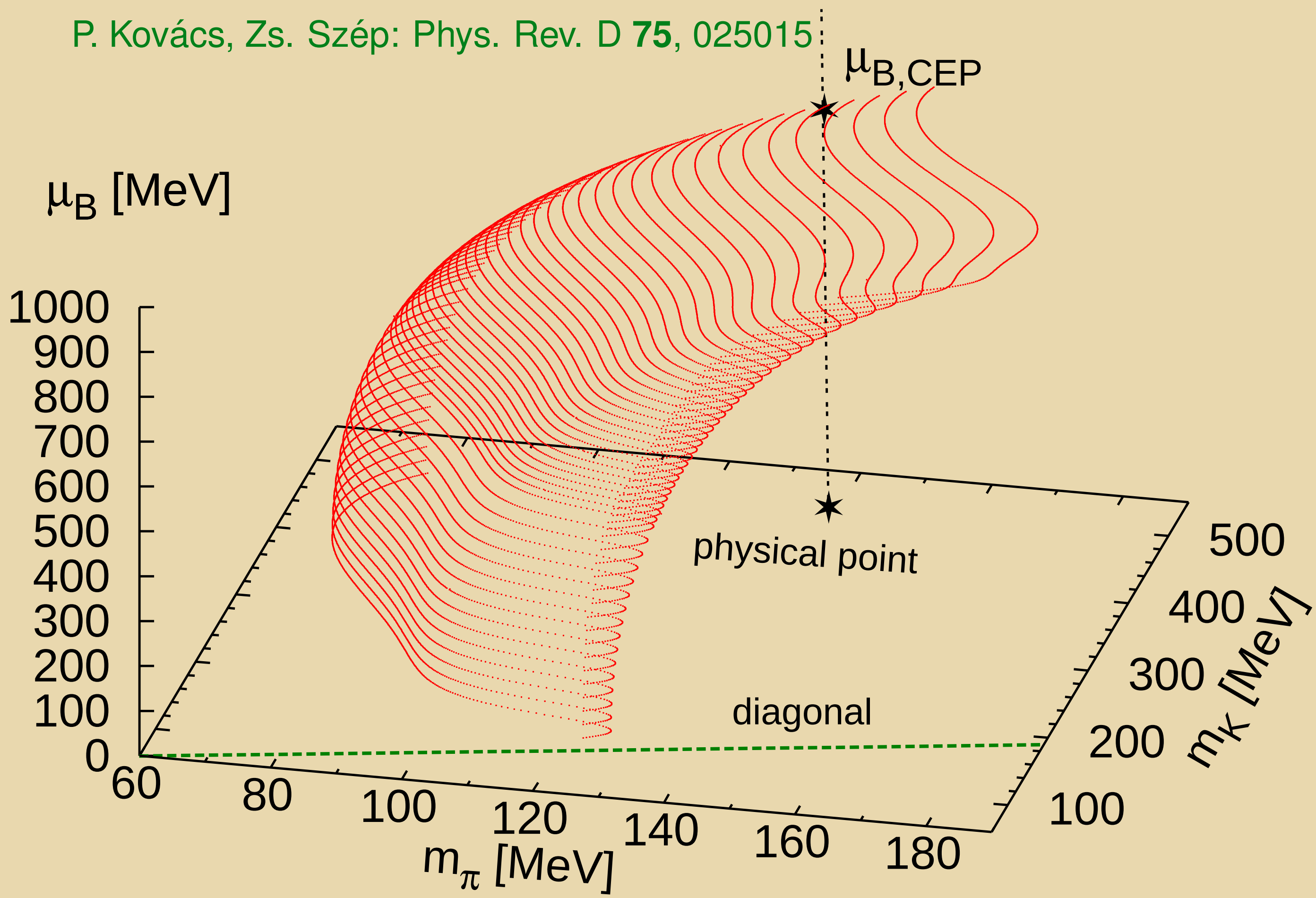
$$G_{K^+}(k) = \frac{i}{2E_\mathbf{k}} \left[\frac{1 + n_{K^+}(E_\mathbf{k})}{k_0 - E_\mathbf{k} + i\epsilon} - \frac{n_{K^+}(E_\mathbf{k})}{k_0 - E_\mathbf{k} - i\epsilon} - \frac{1 + n_{K^-}(E_\mathbf{k})}{k_0 + E_\mathbf{k} - i\epsilon} + \frac{n_{K^-}(E_\mathbf{k})}{k_0 + E_\mathbf{k} + i\epsilon} \right]$$

Self-energies



Results at zero μ_I, μ_Y: critical surface and CEP

P. Kovács, Zs. Szép: Phys. Rev. D **75**, 025015

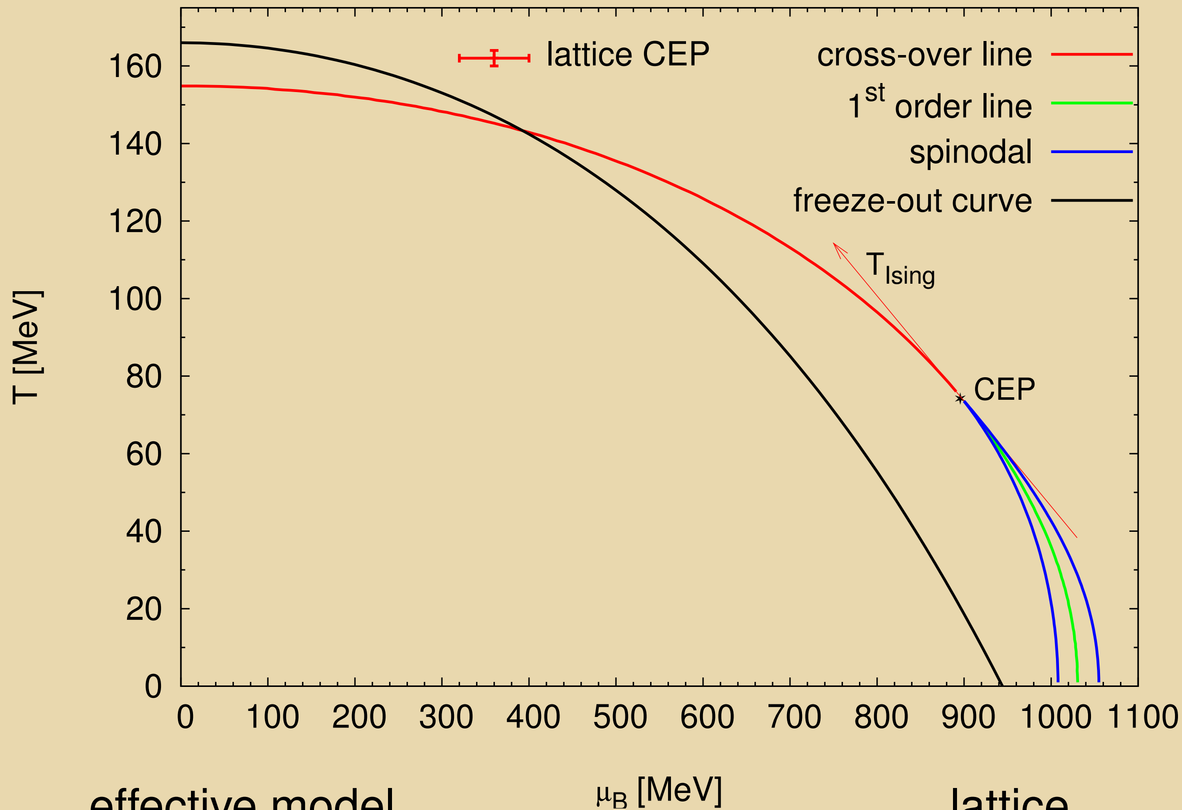


The surface bends towards the physical point $\Longrightarrow$ The CEP must exist

The continuation is reliable up to $m_K \approx 500$ MeV and above the diagonal

The CEP at the physical point of the mass plane

P. Kovács, Zs. Szép: Phys. Rev. D **75**, 025015



effective model

- $T_c(\mu_B = 0) = 154.84$ MeV
  $\Delta T_c(x\chi) = 15.5$ MeV
- $T_{CEP} = 74.83$ MeV
  $\mu_{B,CEP} = 895.38$ MeV
- $T_c \frac{d^2 T_c}{d\mu_B^2}\Big|_{\mu_B=0} = -0.09$

lattice

- $T_c(\mu_B = 0) = 151(3)$ MeV
  $\Delta T_c(\chi_{\bar{\psi}\psi}) = 28(5)$ MeV

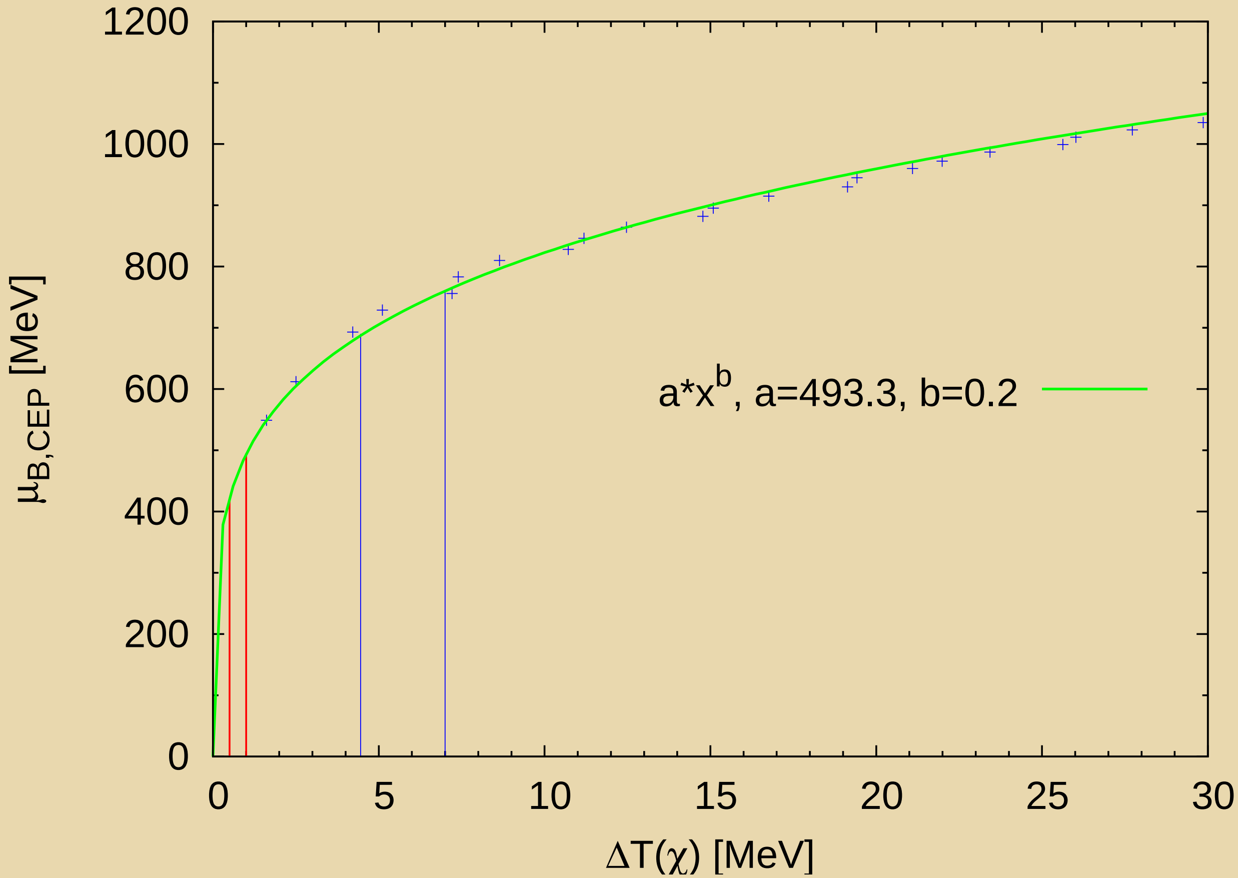
  Y. Aoki,*et al.*, PLB **643**, 46 (2006)
- $T_{CEP} = 162(2)$ MeV
  $\mu_{B,CEP} = 360(40)$ MeV
- $-0.058(2)$

  Z. Fodor,*et al.*, JHEP 0404 (2004) 050

Dependence of the $\mu_{B,CEP}$ on the width of the susceptibility

$\mu_{\rm B,CEP} = 725(35)$ MeV $\longrightarrow$ non-physical quark mass

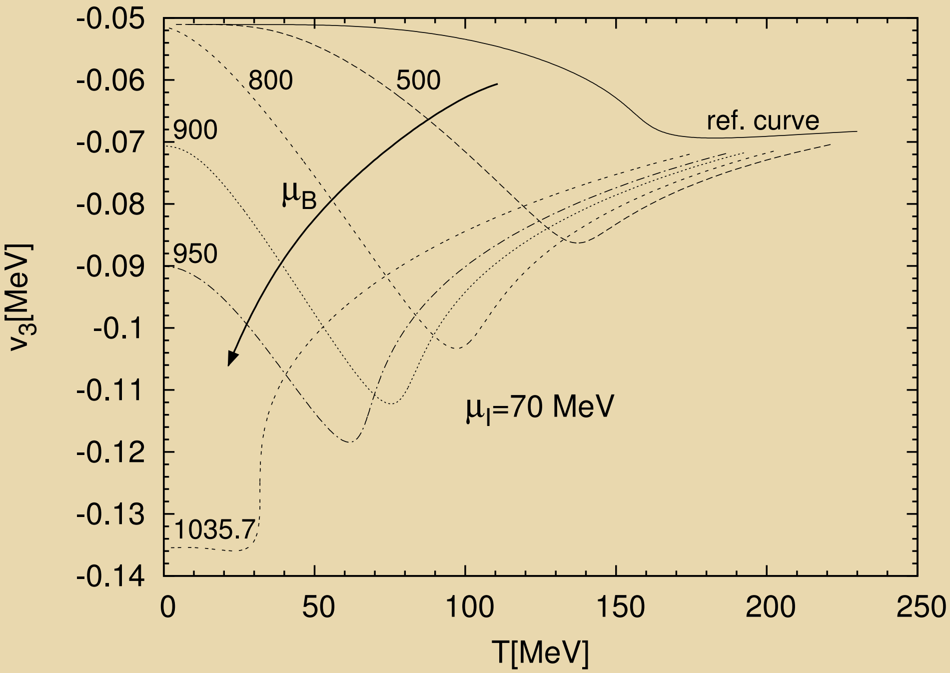
$\mu_{\rm B,CEP} = 360(40)$ MeV $\longrightarrow$ physical quark mass

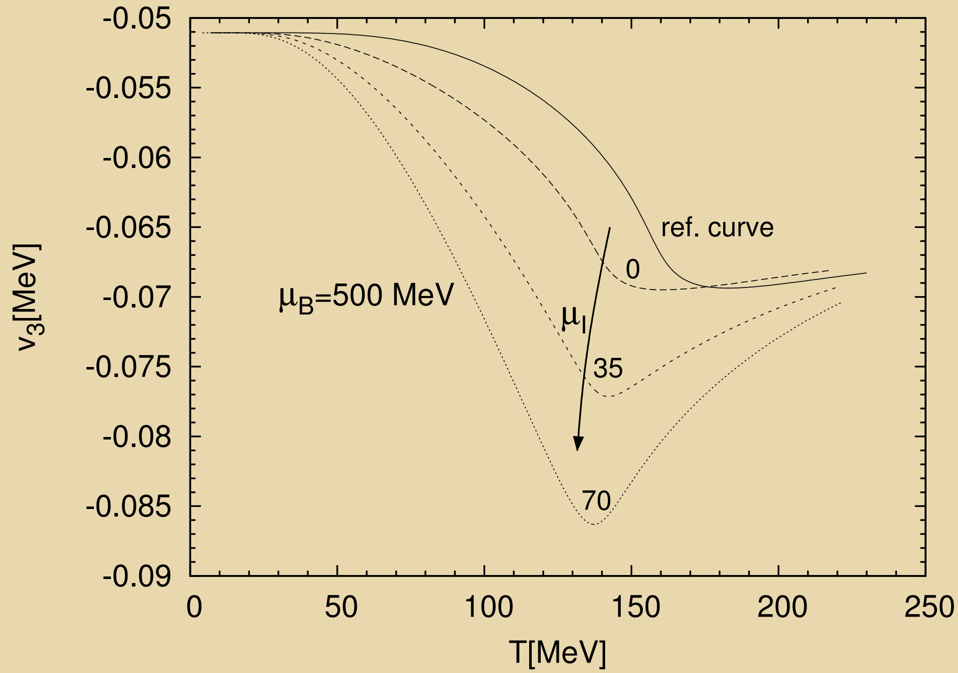


Preliminary lattice estimation by S. Katz: $\Delta T_c(\chi_{\bar\psi\psi}) \approx 0.5 - 1$ MeV
$\Delta T_c(\chi_{\bar\psi\psi}) \approx 2 - 4$ MeV

Since $\Delta T_c(\chi_{\bar\psi\psi}) \approx 28$ MeV at the physical point $\longrightarrow$ higher $\mu_{B,CEP}$ expected

Temperature dependence of v_3





On the left Fig.: μ_B dependence at a given μ_I

Lowest curve corresponds to a CEP

v_3 at $T = 0$ significantly depend on μ_B

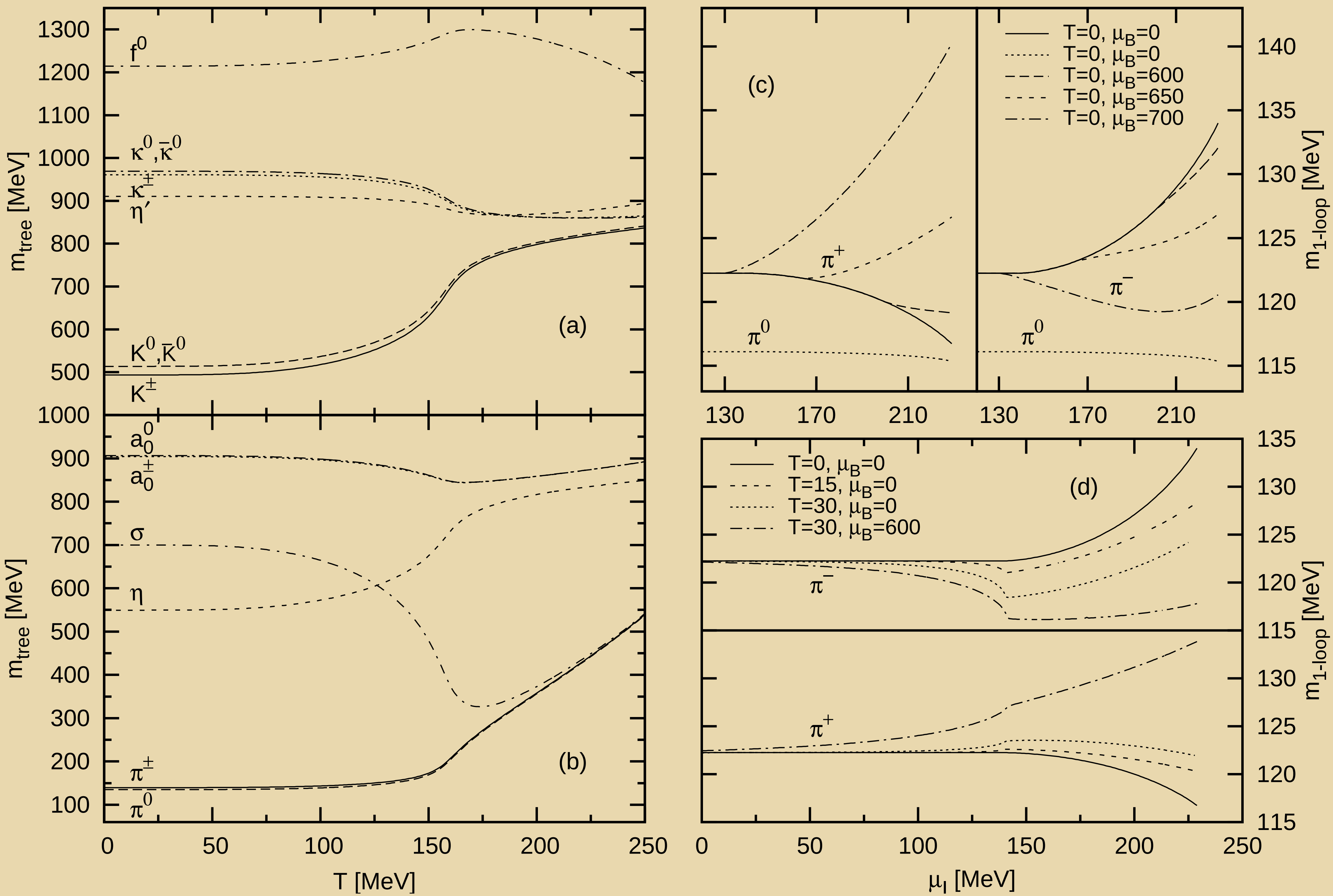
On the right Fig.: μ_I dependence at a given μ_B

Increasing of either μ_B or $\mu_I \longrightarrow$ influence of v_3 becomes stronger

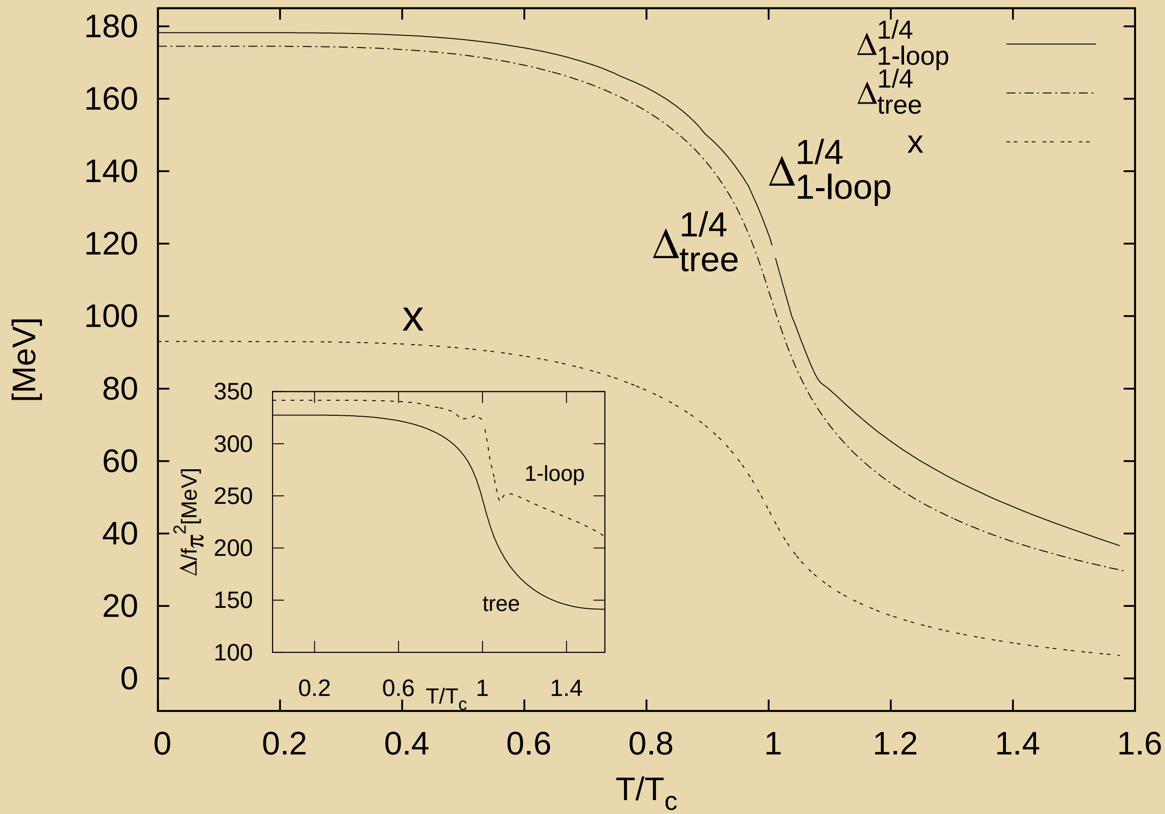
CEP at $\mu_I = 0$: $T_{\text{CEP}} = 63.08$ MeV, $\mu_{B,\text{CEP}} = 960.8$ MeV$\rightarrow$ large diff. to case $v_3 = 0$

Reason: x and v_3 related $\longrightarrow$ common transition point

Tree-level and 1-loop pole masses



Estimation of the topological susceptibility



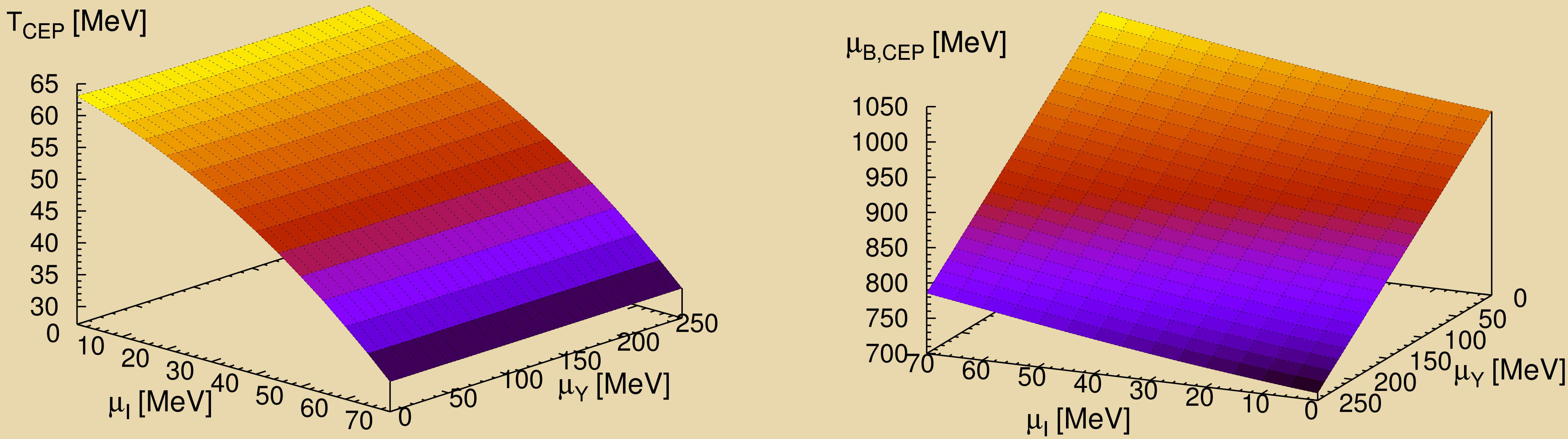
$$\Delta = \frac{1}{6}(m_\eta^2 + m_{\eta'}^2 - 2m_K^2)f_\pi^2$$

This is an estimation of $\chi_T(T)$ the topological susceptibility trough Witten-Veneziano formula: $\frac{2N_f}{f_\pi^2}\chi_T = m_\eta^2 + m_{\eta'}^2 - 2m_K^2$

$\rightarrow$ Connection with the $U_A(1)$ anomaly

Doesn't mean the restoration of $U_A(1) \rightarrow \chi_T(T)$ is dominated by chiral restoration

Dependence of the CEP on μ_I, μ_Y



T_{CEP} is almost independent of μ_Y, but significantly depends on μ_I

$\mu_{B,\text{CEP}}$ has an almost linear dependence on both chemical potentials μ_I, μ_Y

As μ_Y is increased the phase transition at $T = 0$ becomes stronger

Simple explanation of the influence of μ_I, μ_Y

Approximation: Ideal quantum gas of the quasi-particle degrees of freedom

In the first order region $\rightarrow$ generalized Clausius-Clapeyron equations

Partition function:

$$\ln Z = V \sum_i \gamma_i (2s_i{+}1) \int \frac{d^3p}{(2\pi)^3} \left[\beta\omega_i + \ln(1 + \alpha_i e^{-\beta(\omega_i - \mu_i)}) + \ln(1 + \alpha_i e^{-\beta(\omega_i + \mu_i)}) \right]$$

Gibbs-Duhem relation: $dp = s dT + n_B d\mu_B + n_I d\mu_I + n_Y d\mu_Y$

In case of $dp|_{phase1} = dp|_{phase2}$:

$$\left.\frac{dT}{d\mu_B}\right|_{\mu_Y,\mu_I} = -\frac{\Delta n_B}{\Delta s}, \quad \left.\frac{dT}{d\mu_Y}\right|_{\mu_B,\mu_I} = -\frac{\Delta n_Y}{\Delta s}, \quad \left.\frac{dT}{d\mu_I}\right|_{\mu_B,\mu_Y} = -\frac{\Delta n_I}{\Delta s},$$

$$\left.\frac{d\mu_B}{d\mu_Y}\right|_{T,\mu_I} = -\frac{\Delta n_Y}{\Delta n_B}, \quad \left.\frac{d\mu_B}{d\mu_I}\right|_{T,\mu_Y} = -\frac{\Delta n_I}{\Delta n_B}$$

Obtained relations: $\Delta n_B, \Delta n_Y, \Delta s > 0$, $\Delta n_I < 0$

$\Delta n_B \approx \Delta n_Y$, $\Delta s > \Delta n_B$, $\Delta s > |\Delta n_I|$

Conclusions and outlook

- The 2^{nd} order surface was determined in the $m_\pi - m_K - \mu_B$ space using ChPT to obtain the m_π,m_K dependence of the couplings and of the constituent quark masses.
- The CEP is robustly predicted and at the physical point of the mass-plane was located at: $T_{CEP} = 74.83$ MeV $\mu_{B,CEP} = 895.38$ MeV.
- The dependence of the μ_B on the width of the susceptibility was investigated.
- μ_I dependence of different pole masses were obtained.
- The temperature dependence of the topological susceptibility was estimated.
- Effects of isospin and hyper chemical potential on the CEP was investigated. $T_{CEP} = 63.08$ MeV $\mu_{B,CEP} = 960.8$ MeV at $\mu_I = 0$ ($v_3 \neq 0$ at $T = 0$).
- A simple ideal gas model was established, which could explain the shifts of the CEP due to μ_I, μ_Y
- Possible continuation: Pion condensate, inclusion of the Polyakov-loop