Phase diagram of strong matter in the linear sigma model

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Outline

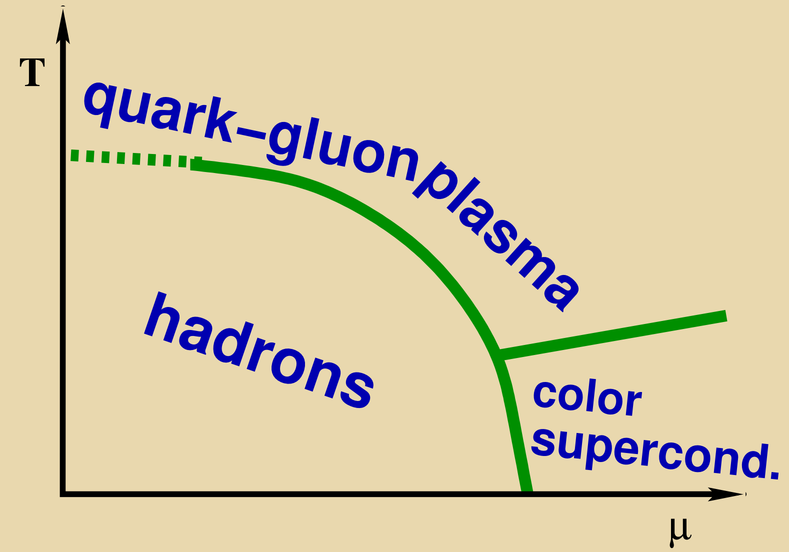
Motivation:
Mapping the boundary of the fist order phase transition in the (m_π, m_K, μ_B) space.

- Overview of the phases of the strongly interacting matter and the chiral symmetry
- Parameterization of the $SU_L(3) \times SU_R(3)$ linear sigma model on the $m_\pi - m_K$ mass plane
- Termodynamical calculations in the quasi–particle picture, restoration of chiral symmetry at finite temperature
- One–loop parameterization of the model
- The phase boundary in the (m_π, m_K) plane

Phases of QCD

Normal temperature/density:
strongly interacting matter → hadrons

Extreme temperature/density:
new phases → quark–gluon plasma, color superconductor



Increasing temperature:

formation of quark–gluon plasma → restoration of chiral symmetry

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Chiral symmetry is a global symmetry of the QCD so it can be investigated in effective models because they have the same global symmetry as the QCD

Chiral symmetry of QCD

$$L_{QCD} = -\frac{1}{2}\mathrm{Tr}G_{\nu\mu}G^{\nu\mu} + \sum_{i=1}^{N_f} (\bar{q}_{L_i}\gamma_\mu D^\mu q_{L_i} + \bar{q}_{R_i}\gamma_\mu D^\mu q_{R_i} + \bar{q}_{L_i} m_i q_{R_i} + \bar{q}_{R_i} m_i q_{L_i})$$

at low energy: $N_f = 3$ (u,d,s quarks)

$m_i = 0$:

symm. transformation: $q_{L,R} \to L, R\, q_{L,R},\ L, R \in U_{L,R}(3)$; "$L \pm R = V, A \in U_{V,A}(3)$"

$$U_L(3) \times U_R(3) \overset{U_A(1)}{\underset{\text{anomaly}}{\Longrightarrow}} SU_A(3) \times SU_V(3) \times U_V(1)$$

$SU_A(3)$ spontaneously broken for $T < T_c$:

$$SU_A(3) \times SU_V(3) \times U_V(1) \overset{T \le T_c}{\underset{\text{spont. breaking}}{\Longrightarrow}} SU_V(3) \times U_V(1)$$

Order parameters: $M^i_j = \langle \bar{q}^i_L q_{Rj} \rangle$ for $N_f \geq 2$.

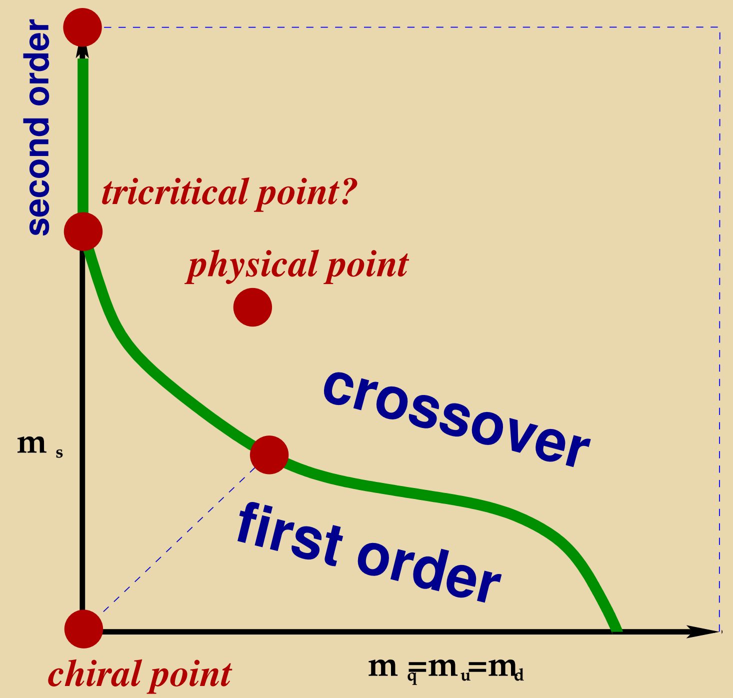
8 Goldstone bosons $\equiv(\pi,\ K,\ \eta)$. Their non zero masses arise from $m_{u,d,s} \neq 0$

isospin symmetry: $m_q \equiv m_u = m_d$, $m_\pi^2 \sim m_q$, $m_K^2 \sim m_q + m_s$, $m_\eta^2 \sim m_q + 2m_s$

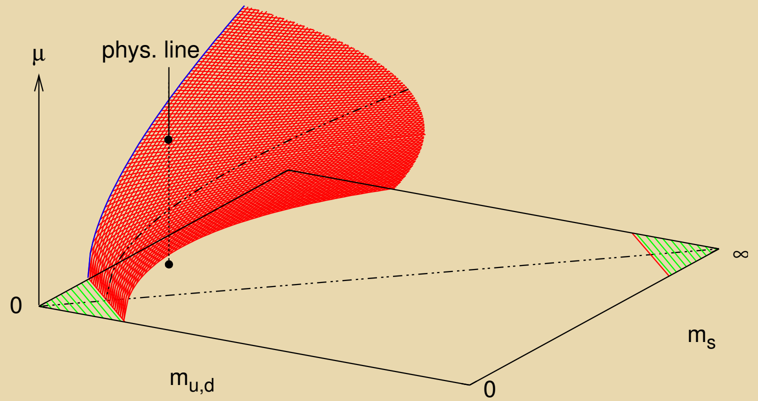
Phase diagrams of the chiral symmetry

In case of isospin symmetry $m_q \equiv m_u = m_d$

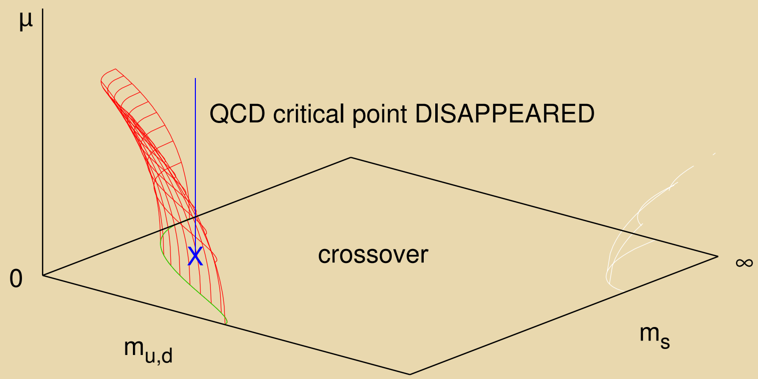
- Physical point: crossover
- Chiral point: 1$^{\mathrm{st}}$ order
- $m_s \to \infty$: 2$^{\mathrm{nd}}$ order
- Existence of tricritical point?
- $m_s = m_q$: lattice QCD results



$\mu_B \neq 0$:



?



F. Karsch, J.Phys. G31(2005); O. Philipsen, Ph. de Forcrand, JHEP 0701(2007)

$SU_L(3) \times SU_R(3)$ linear sigma model

$\bar{q}q$ bound states: mesons are transformed as $\bar{3} \otimes 3 = (8,1) \oplus (1,8)$

$\sigma_i(0^+)$: scalar nonet (1,8)	$\pi_i(0^-)$: pseudoscalar nonet (8,1)

they can be put into a 3×3 matrix: $M := \frac{1}{\sqrt{2}} \sum_{i=0}^{8} (\sigma_i + i\pi_i)\lambda_i$

The Lagrangian:

$$L(M) = \frac{1}{2}\mathrm{Tr}(\partial_\mu M^\dagger \partial^\mu M + \mu_0^2 M^\dagger M) - f_1 \left(\mathrm{Tr}(M^\dagger M)\right)^2 - f_2 \mathrm{Tr}(M^\dagger M)^2 - $$
$$-g\left(\det(M) + \det(M^\dagger)\right) + \epsilon_0\sigma_0 + \epsilon_8\sigma_8 + \epsilon_3\sigma_3$$

- two independent four–point couplings ($f_1,\ f_2$)
- three–point coupling: $g \det(M) \rightarrow U_A(1)$ anomaly
- symmetry breaking external fields: $\epsilon_0,\ \epsilon_8,\ \epsilon_3$

ext. fields	remaining symm.	broken symm.	quark, meson masses
$\epsilon_0 \neq 0$	$SU_V(3) \times U_V(1)$	$SU_A(3)$	$\epsilon_0 \sim m_q = m_s \sim m_\pi^2 = m_K^2 = m_\eta^2$
$\epsilon_{0,8} \neq 0$	$SU_V(2) \times U_V(1)$	$U_Y(1)$	$\epsilon_8 \sim m_s - m_q \sim m_K^2 - m_\pi^2/2$
$\epsilon_{0,8,3} \neq 0$	$U_V(1)$	$SU_V(2)$	$\epsilon_3 \sim m_u - m_d \sim m_{\pi\pm}^2 - m_{\pi 0}^2$

Ground state of the broken phase

$\langle \mathrm{Re\,Diag}\,(M)\,\rangle_0 \neq 0$ and defining pure non–strange, strange fields:

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_3\end{pmatrix} = \frac{1}{\sqrt{3}}\begin{pmatrix}\sqrt{2} & 1 & 0\\ 1 & -\sqrt{2} & 0\\ 0 & 0 & \sqrt{3}\end{pmatrix}\begin{pmatrix}\sigma_0\\ \sigma_8\\ \sigma_3\end{pmatrix} \quad \text{and} \quad \begin{aligned} x &\equiv \langle\sigma_x\rangle_0\\ y &\equiv \langle\sigma_y\rangle_0\\ v_3 &\equiv \langle\sigma_3\rangle_0\end{aligned}$$

then shifting the fields by their expectation value in the Lagrangian,

$$M \longrightarrow M - \begin{pmatrix} x+v_3 & 0 & 0\\ 0 & x-v_3 & 0\\ 0 & 0 & y\end{pmatrix} \Longrightarrow$$

- equations of state for x, y, v_3
- meson mass matrix
- new three point vertices

considering isospin symmetry $\epsilon_3 = 0 \Longrightarrow v_3 = 0$:

non-diagonal mass matrix:	tree level PCAC relations:	Ward identities:
$\begin{pmatrix} m^2_{\pi_{88}} & m^2_{\pi_{08}}\\ m^2_{\pi_{08}} & m^2_{\pi_{00}}\end{pmatrix}, \begin{pmatrix} m^2_{\sigma_{88}} & m^2_{\sigma_{08}}\\ m^2_{\sigma_{08}} & m^2_{\sigma_{00}}\end{pmatrix}$	$f_\pi m_\pi^2 = \epsilon_x$ $f_K m_K^2 = \frac{1}{2}(\epsilon_x + \sqrt{2}\epsilon_y)$	$\epsilon_x = m_\pi^2 x,$ $\epsilon_y = \frac{1}{\sqrt{2}}(m_K^2 - m_\pi^2)x + m_K^2 y$

The first step to the termodynamical calculations is the tree level parameterization with the above equations

Tree level parameterization

- Most of input data are from the well-known pseudoscalar sector, but the mixed scalar masses are unavoidable due to the degeneration of f_1 and μ through $M^2 = -\mu^2 + 4f_1(x^2 + y^2)$ $\longrightarrow$ assumptions A1, A2 for mixed scalars $\longrightarrow$ theoretical uncertainties
- This set of 8 coupled linear equations determine the parameters therefore $(f_1, f_2, \mu, g, \epsilon_x, \epsilon_y, x, y)$ depend on $m_\pi, m_K, f_\pi, f_K, \bar{m}_\eta^2 (\equiv m_\eta^2 + m_{\eta'}^2), A_{1,2}$.

inputs:		outputs:		predicts:
f_π, f_K	$\Longrightarrow$	x, y	$\Longrightarrow$	m_η, $m_{\eta'}$, θ_η, m_{a_0}, m_κ
m_π, m_K, $m_\eta^2 + m_{\eta'}^2$	$\Longrightarrow$	g, f_2, M^2		
A_1 & M^2, A_2 & M^2	$\Longrightarrow$	μ^2, f_1	$\Longrightarrow$	m_σ, m_{f_0}, θ_σ
$Eos_x = 0$, $Eos_y = 0$	$\Longrightarrow$	ϵ_x, ϵ_y		

Going away from the physical point, we have to keep in mind that $f_\pi, f_K, \bar{m}_\eta$ depend on m_π, m_K. The functions $f_\pi(m_\pi, m_K)$, $f_K(m_\pi, m_K)$, $\bar{m}_\eta(m_\pi, m_K)$ provided by $U(3)$ chiral perturbation theory (ChPT).

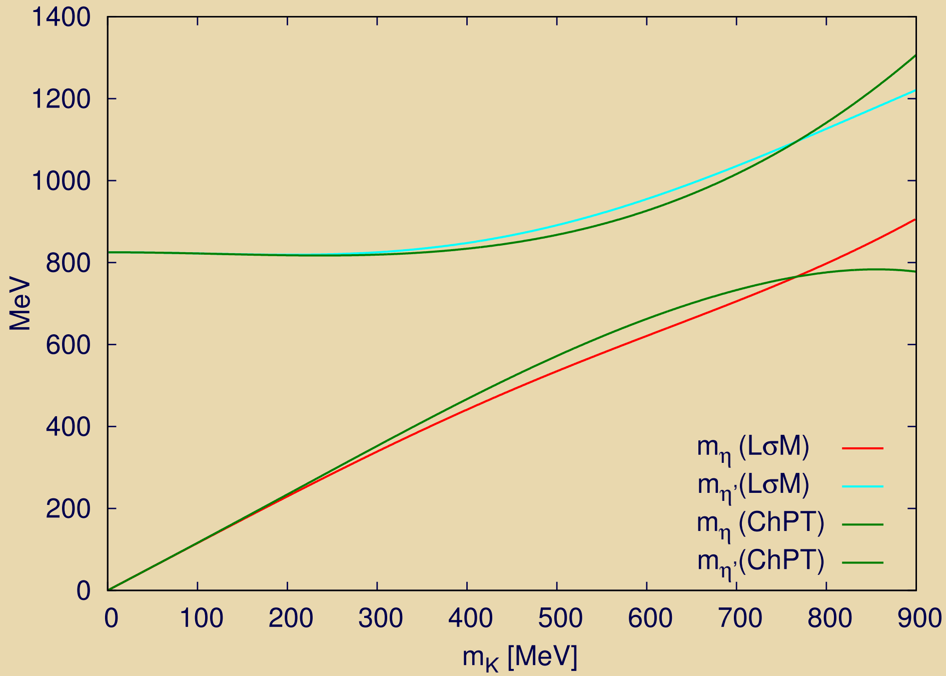
Going away from the physical point

The pion, kaon mass dependence of the decay constant,

$$f_\pi(m_\pi, m_K) = f\left[1 - \frac{1}{f^2}\left(2\mu_\pi + \mu_K - 4m_\pi^2(L_5 + L_4) - 8m_K^2 L_4\right)\right], \quad \mu_\alpha = \frac{m_\alpha^2}{16\pi^2}\ln\frac{m_\alpha}{4\pi f}$$

$$f_K(m_\pi, m_K) = f\left[1 - \frac{1}{f^2}\left(\frac{3}{4}\mu_\pi + \frac{3}{4}\mu_\eta + \frac{3}{2}\mu_K - 4m_\pi^2 L_4 - 4m_K^2(L_5 + 2L_4)\right)\right]$$

The m_K dependence of η, η' masses and the predictions of the LσM for m_π=0:



Remarkable agreement up to $m_K \approx 800$ MeV for $m_\pi = 0$.

Termodynamical calculations

Standard perturbation theory at finite T:

$$G_0(\mathbf{p})_{ij} = \left[\mathbf{p}^2 - m_{ij}^2(x(T), y(T))\right]^{-1} \text{ increasing } T \Longrightarrow m_{ij}^2 \to -\mu_0^2$$

Negative mass squares appear in the propagators at finite T!!
$\to$ Mass resummation needed!

Optimalised perturbation theory (OPT):

$$L_{mass} = -\frac{1}{2}M^2(T)\mathrm{Tr}M^\dagger M + \frac{1}{2}\overbrace{(\mu_0^2 + M^2(T))}^{\Delta m^2}\mathrm{Tr}M^\dagger M,$$

$M^2(T)$: T dependent effective mass

Δm^2: one–loop order counterterm

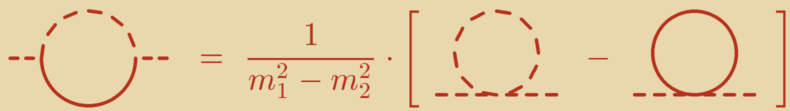
OPT replaces $-\mu^2 \to M^2(T)$ in the tree level masses
$\Longrightarrow$ appropriate choice of $M^2(T) \Longrightarrow m_i^2(M(T), x(T), y(T)) > 0$.

OPT preserves the renormalizibility and symmetries (Goldstone th., Ward ids.).

Quasi–particle picture

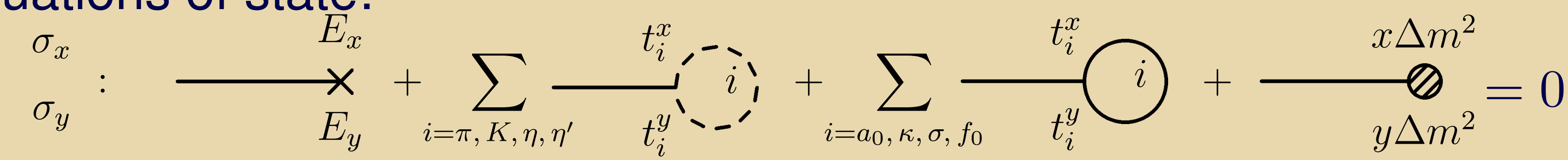
Determination of $M^2(T)$:

$$m_\pi^2 \stackrel{!}{=} M_\pi = G_1^{-1}\left[\omega = \mathbf{p} = 0, T\right] = m_\pi^2 - \Delta m^2 + \Sigma_\pi\left[\omega = \mathbf{p} = 0, T, m_i(m_\pi, x, y)\right]$$

in $\Sigma_\pi|_{p=0}$: 

$$I_{tp}(m,T) = \underbrace{\frac{1}{16\pi^2} m^2 \ln\frac{m^2}{l^2}}_{I_{tp}^l(m)} + \underbrace{\int \frac{d^3\mathbf{k}}{(2\pi)^3}\left(\frac{1}{\omega^2}\right)\frac{1}{e^{\beta\omega}-1}}_{I_{tp}^T(m,T)},$$

Equations of state:



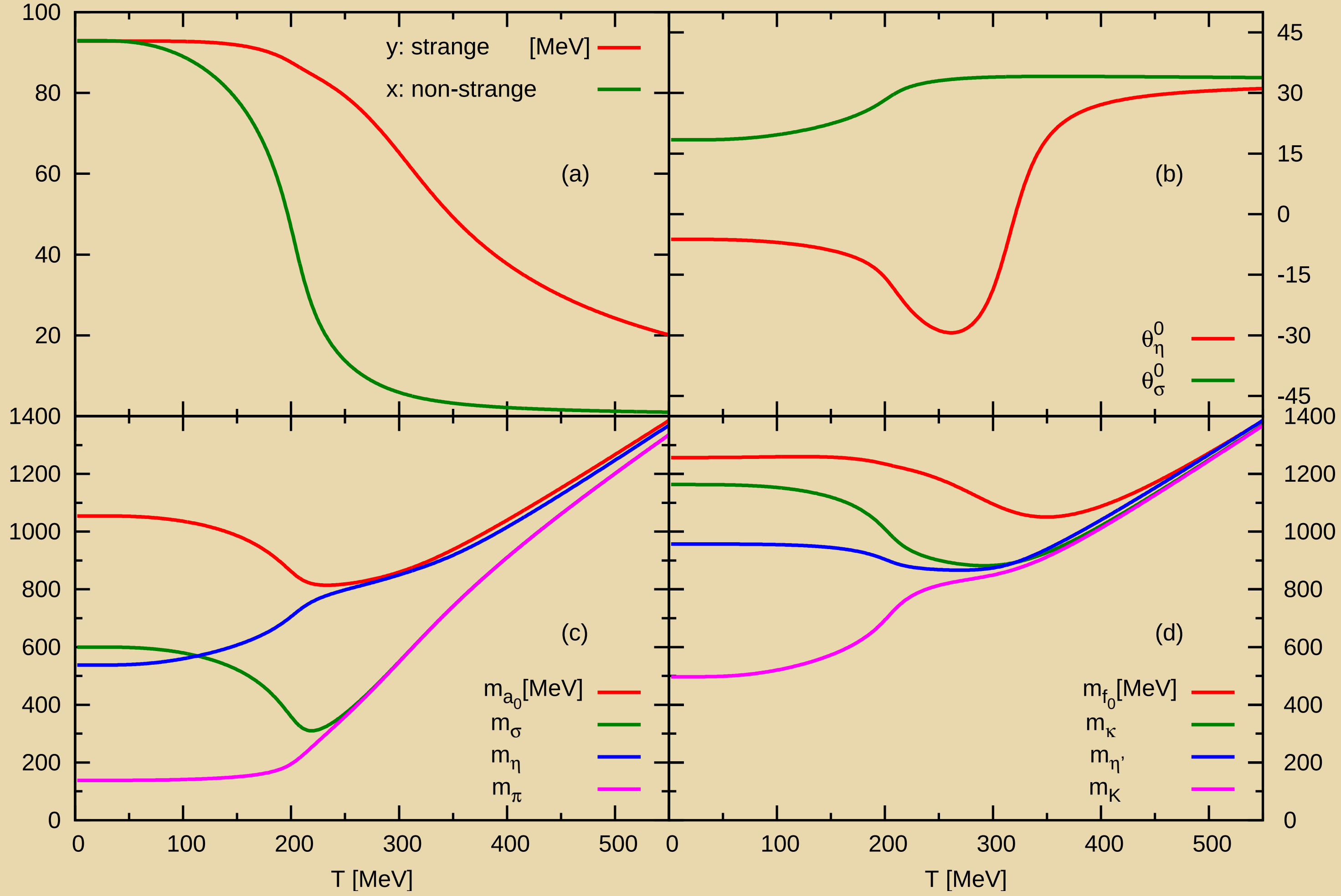
$$\epsilon_x = -\mu_0^2 + 2gxy + 4f_1xy^2 + (4f_1 + 2f_2)x^3 + \sum_i t_i^x(x,y) I_{tp}(m_\pi, m_i(x,y), T)$$

$$\epsilon_y = -\mu_0^2 + 2gx^2 + 4f_1x^2y + (4f_1 + 4f_2)y^3 + \sum_i t_i^y(x,y) I_{tp}(m_\pi, m_i(x,y), T)$$

Quasi–particle approximation: $I_{tp}^l(m) \equiv 0$!!

Results in the physical point

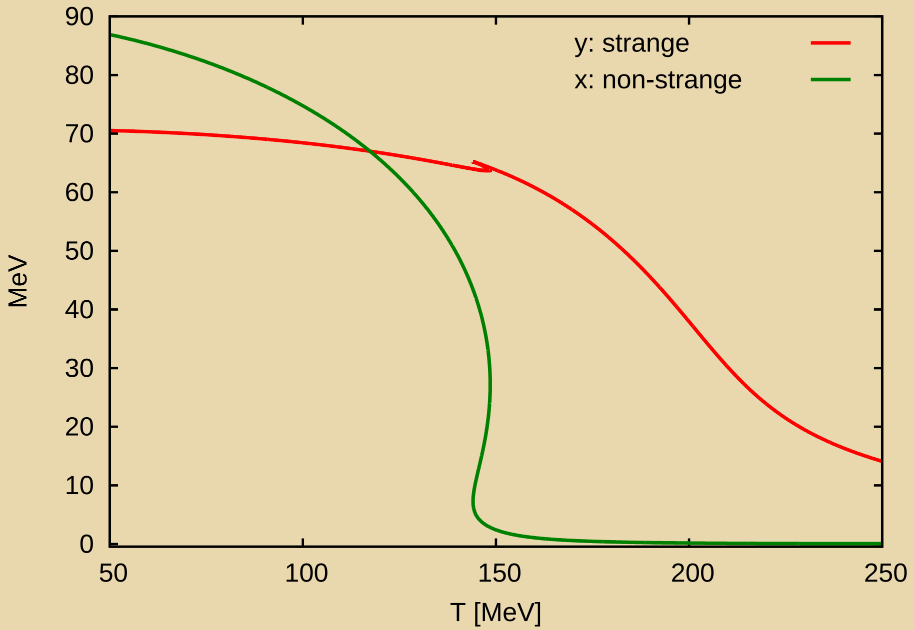
The restoration of chiral symmetry is continuous (crossover) in the physical point

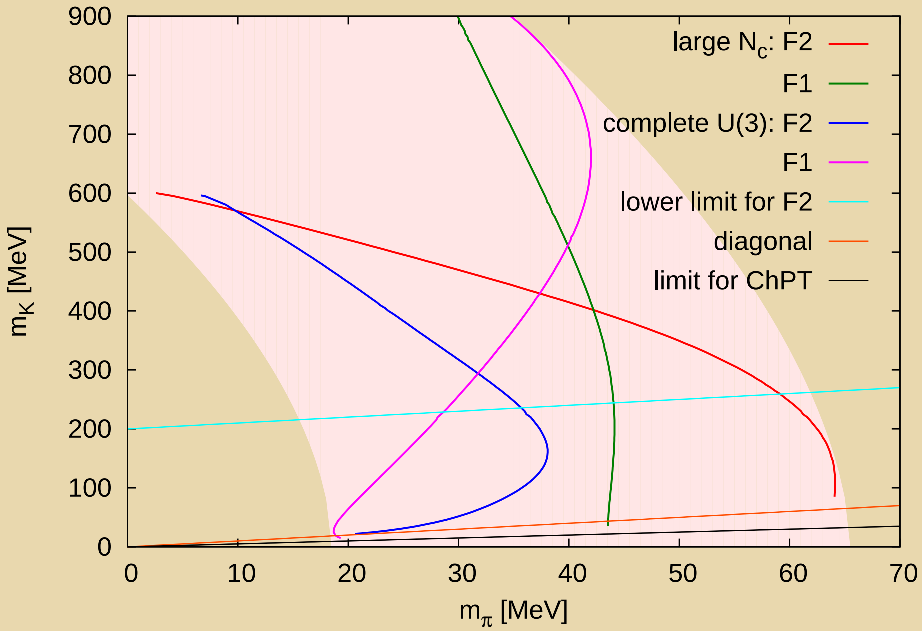


a) condensates; b) mass matrix mixing angles; c) and d) tree level masses

Phase diagram (quasi-particle appr.)

Decreasing pion, kaon masses:
crossover $\longrightarrow$ 1st phase transition.





1st order phase transition
for $m_\pi = 30$ MeV,$m_K = 400$ MeV

The phase boundaries for different parameterizations

Different parameterization $\longrightarrow$ different phase boundaries
$\longrightarrow$ the highlighted band indicates the “real” boundary
$\longrightarrow$ one–loop parameterization is needed.

One–loop parameterization

One–loop level propagators at zero temperature,

$$D_\alpha(p) = \frac{iZ_\pi^{-1}}{p^2 - m_\alpha^2 - \Sigma_\alpha(p^2, m_i, l)}, \qquad Z_\alpha^{-1} \equiv 1 - \left.\frac{\partial \Sigma_\alpha(p^2, m_i, l)}{\partial p^2}\right|_{p^2 = M_\alpha^2}$$

Physical mass defined as the pole of the propagators,

$$M_\alpha^2 = m_\alpha^2 + \mathrm{Re}\left\{\Sigma_\alpha(p^2 = M_\alpha^2, m_i, l)\right\}$$

The finite wave function renormalisation constant appears in the PCAC relations,

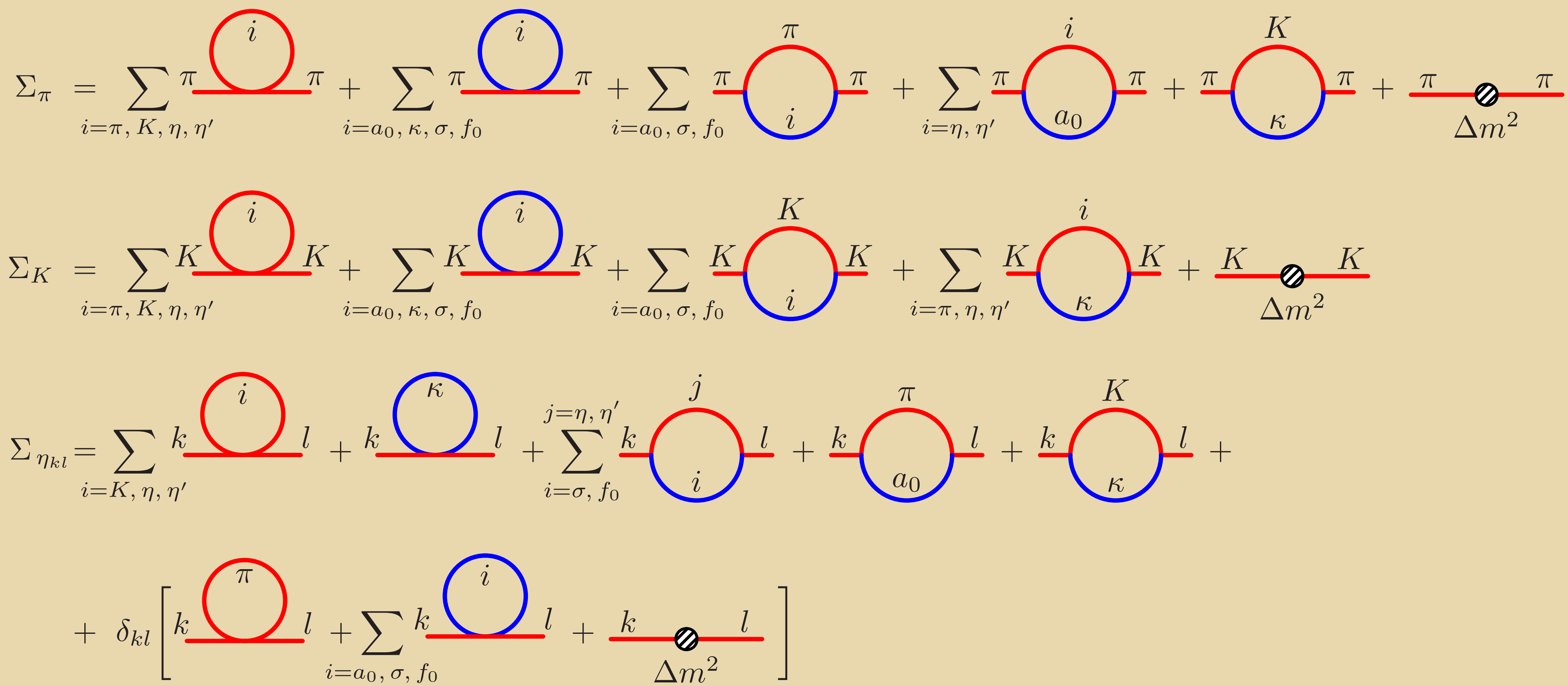
$$f_\pi M_\pi^2 = \epsilon_x \sqrt{Z_\pi}, \qquad f_K M_K^2 = \frac{1}{2}\left(\epsilon_x + \sqrt{2}\epsilon_y\right)\sqrt{Z_K}$$

The equations of states are also written in the terms of propagators, with the help of the Ward identities,

$$\epsilon_x = -\mathrm{i} D_\pi^{-1}(p=0) Z_\pi^{-1} x, \qquad \epsilon_y = \frac{-\mathrm{i}}{\sqrt{2}}\left(D_K^{-1}(p=0) Z_K^{-1}(x + \sqrt{2}y) - D_\pi^{-1}(p=0) Z_\pi^{-1} x\right)$$

One–loop parameterization

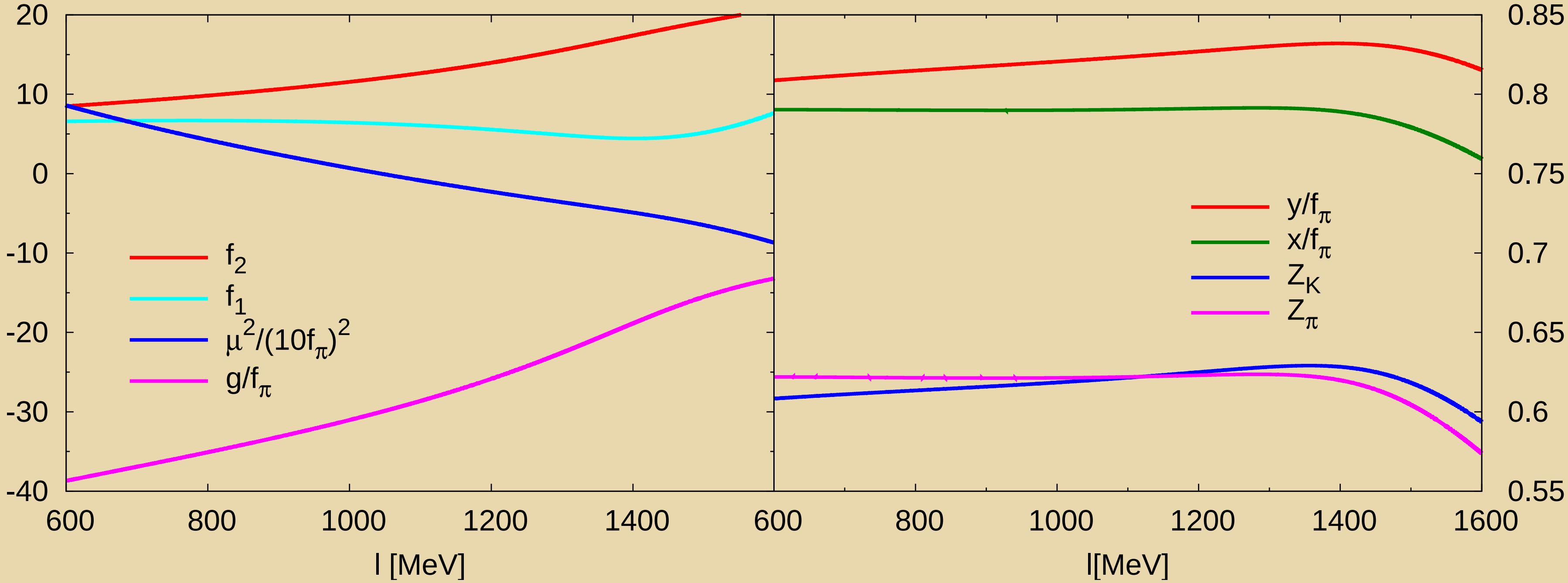
The self energies contain tadpole and bubble diagrams:



The inputs of the parameterization now can be restricted to the pseudoscalar masses, decay constants. One doesn't need any assumptions for the scalar sector, but we had to solve numerically a set of 6 non-linear coupled equations.

Scale dependence of the parameters

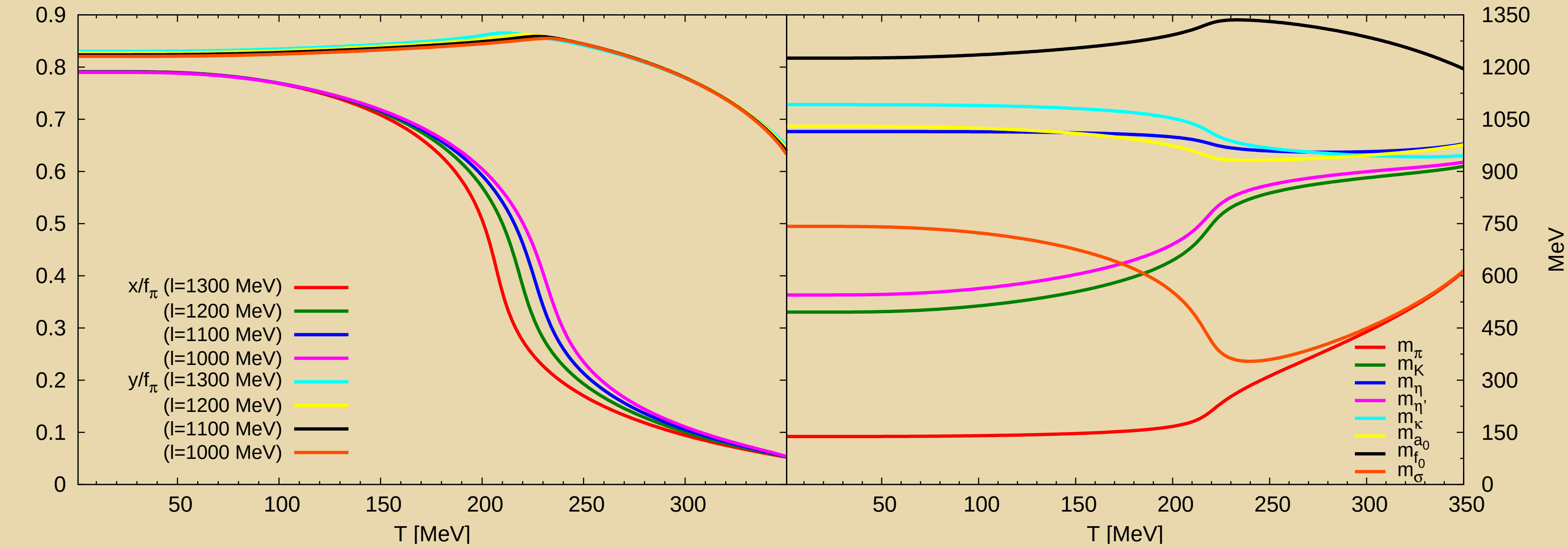
In the physical point:



parameters

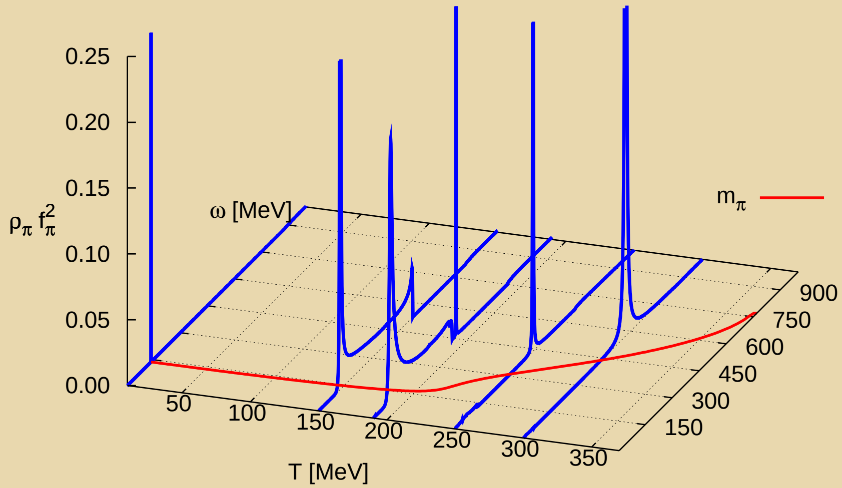
condensates, renormalisation constants

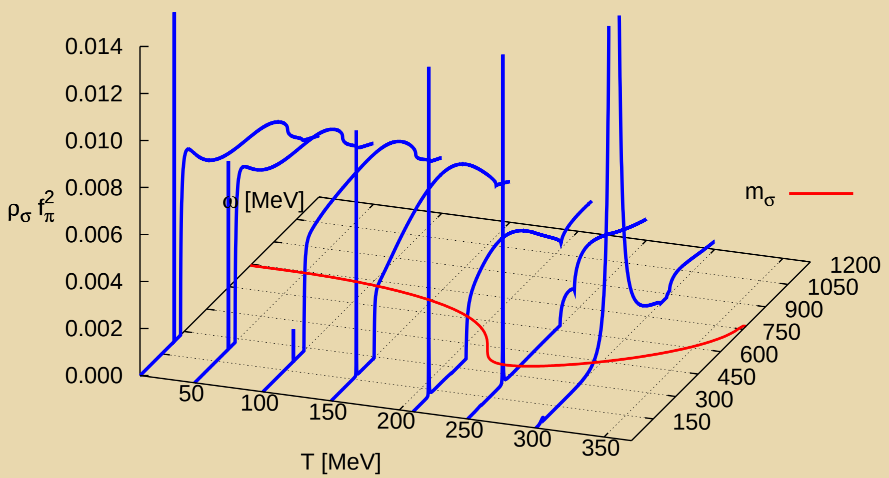
Temperature dependence in the physical point



condensates

masses

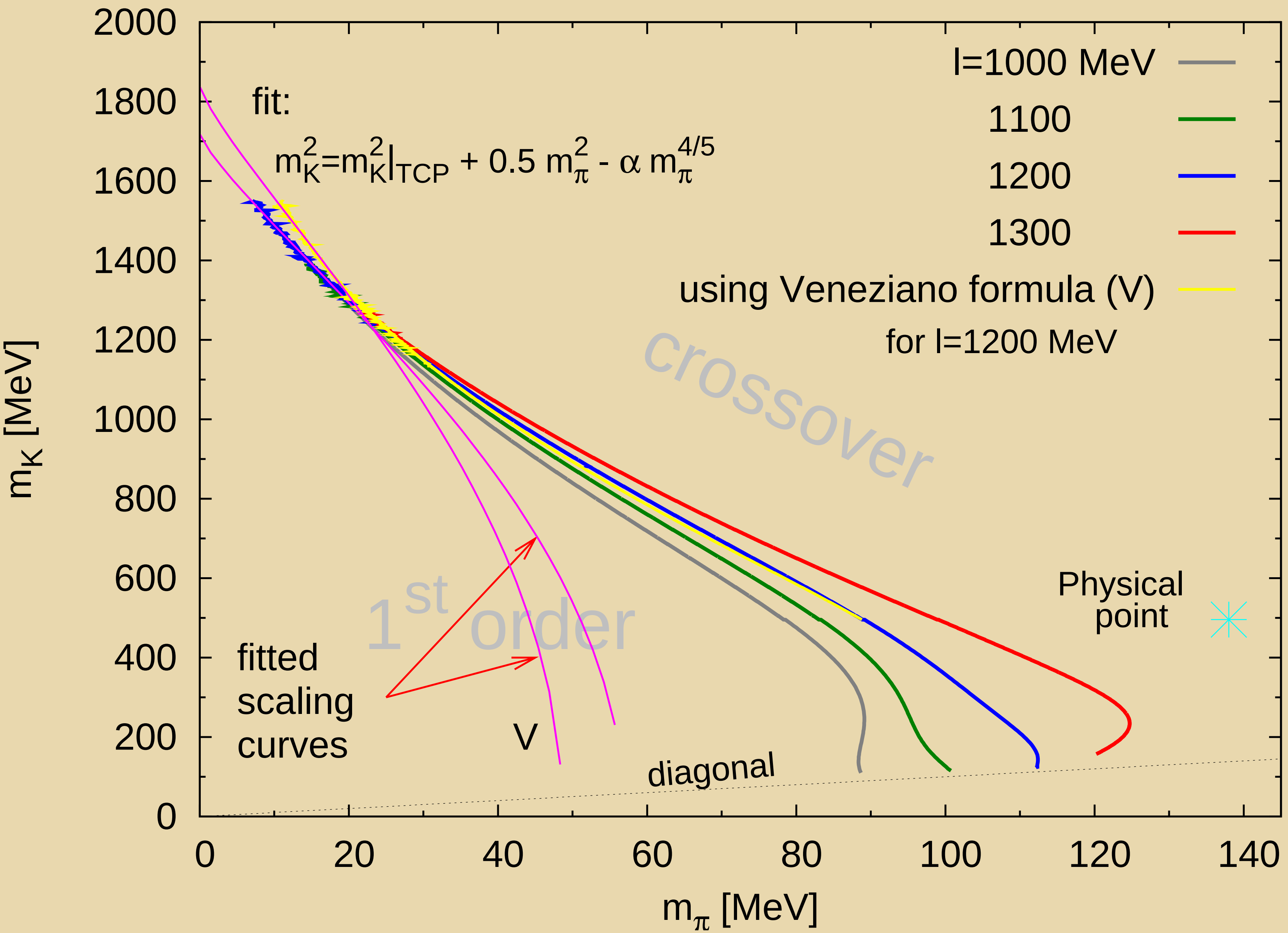




spectral function of the pion

spectral function of the sigma

Phase boundary on the $m_\pi - m_K$ plane



Phase boundary in the $m_\pi - m_K - \mu_B$ space

Adding constituent quarks to the linear sigma model $\longrightarrow$ chemical potentials can be intruduced and our phase boundary became a surface in the $m_\pi - m_K - \mu_B$ plane. P. Kovács, Zs. Szép: PRD 75, 02501